Theory exam in Structural Dynamics 2014-03-04 kl.10-12

The test consists of 6 questions giving the maximum of 15 points. Each question should be answered on a separate paper. No helping aids are permitted on this test, except calculator. Do not forget to write your name on each submitted paper.

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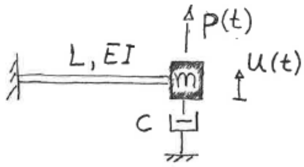
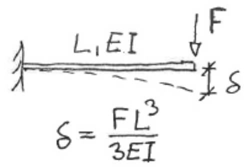
1) (2 p)

A point mass is connected to a light cantilever beam and a linear viscous damper according to the figure. The tip displacement δ of the beam when subjected to a force F is also given.



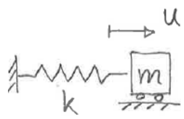


Determine by using free body diagrams the equation of motion for the structure.
Also give expressions for the natural angular frequency ω_n and the natural period time T_n .

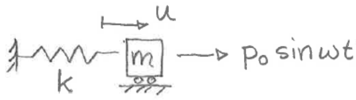
2) (2 p)

Free and forced response of a single degree of freedom (undamped) system should be determined. Parameter values; k =20kN/m and m =100kg.

a) The mass in the figure is displaced u_0=10mm and released from rest. Determine the displacement function $u(t)$. Give values of the amplitude (in mm) and frequency (in Hz).

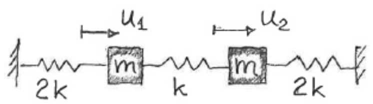


b) If the same system is subjected to a sinusoidal force according to the figure above. Determine the steady state displacement amplitude for the values below. Also determine the quasi-static displacement amplitude (in steady state) using the same force amplitude if the force amplitude p_0 =600N and the angular frequency ω =10 rad/s.



3) (4 p)

Consider the two degree of freedom system below with system matrices defined as

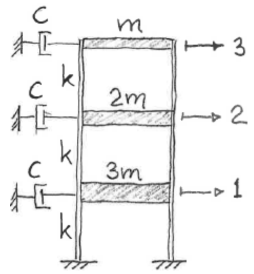


$$\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 3k & -k \\ -k & 3k \end{bmatrix}$$

a) Determine without using modal analysis the steady state displacements for harmonic loading acting only in dof 1 with an amplitude p_0 and a forcing angular frequency $\omega^2 = 3\frac{k}{m}$.

b) Determine the natural angular frequencies and the corresponding mode shapes for the system.

4) (2p)



The three-storey shear building has mass and stiffness matrices as

$$\boldsymbol{m} = m\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{k} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

a) Determine the damping matrix $\boldsymbol{c}$. Is it Rayleigh damping in this case? Motivate your answer.

b) Use a linear deflection shape to determine an approximation of the lowest natural angular frequency ω_1. (Put c=0.) Is this frequency higher or lower than the exact one? Note; it is not necessary to do the calculations of the exact frequency.

5) (2 p)

This concerns transmissibility of vibrations and conditions for application of tuned dampers.

A steel spring is to be used as a vibration isolator for vertical vibrations of a machine with a mass of m=100kg. The spring has a stiffness k=10kN/m.

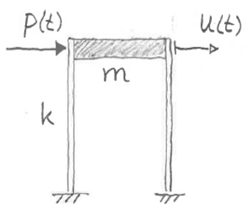
a) Decide how much (in %) of the disturbing force from the machine that runs through the spring down into the foundation at an angular frequency of the machine ω=50rad/s.

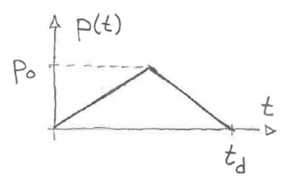
b) You have a rubber spring with stiffness k_d =500N/m to put on top of the machine in connection with another mass m_d. If you want to protect the machine from accidental loading at resonance, what would be your choice of m_d?

6) (3 p)

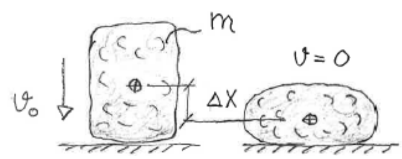
This question concerns pulse loading and impact.

a) A shear building with k=0.8MN/m and m=25·10^{3}kg is subjected to two cases of pulse loading according to the figure. The first load case is from a tsunami wave with a pulse duration of t_d=10s and with amplitude p_0=0.1MN. The second load case is a blast load with a pulse duration t_d=5ms and with amplitude p_0=2MN. Assume a symmetric triangular pulse and calculate an estimate of the maximum deflection u_0 of the building in the two cases (some damping is also present). Which is the worst case?





b) A sack of potatoes with m=50kg is dropped from five meters giving it a velocity v_0=10m/s when it hits a concrete floor. The force pulse between the floor and the sack is assumed to be symmetric and triangular according to a). Estimate the maximum force p_0 and the time duration t_d of the pulse if it comes to a stop in a distance Δx=0.2m.



Hint: Use the law of impulse and momentum and the law of work and kinetic energy.