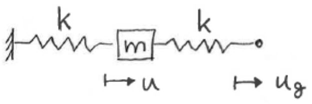
Theory exam in Structural Dynamics 2016-03-04 kl.10-12

The test consists of 6 questions giving the maximum of 20 points. 7 points are required to pass the exam (and the course). Each question should be answered on a separate paper. No helping aids are permitted on this test, except calculator. Do not forget to write your name on each submitted paper.

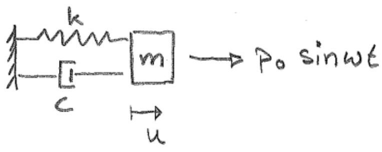
1) (3p)

Determine by using free body diagrams the equation of motion for the structure in terms of the displacement of the mass *u*. Note that *u* is here the total displacement.
Determine also the resonance frequency f_n of the system.

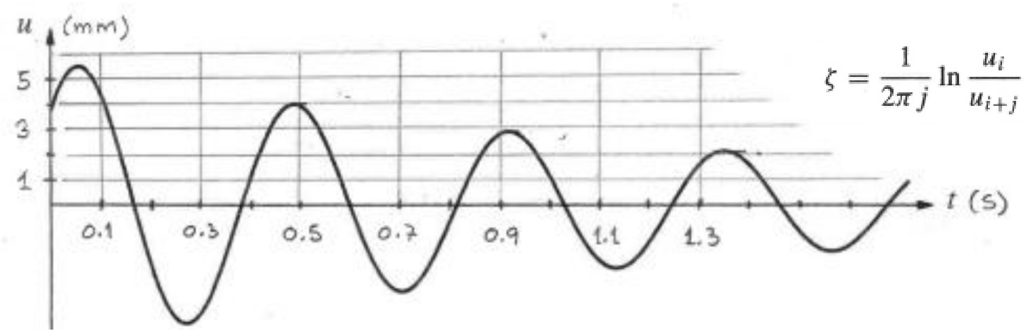


2) (4p)

Consider a damped single degree of freedom system in free and forced response.



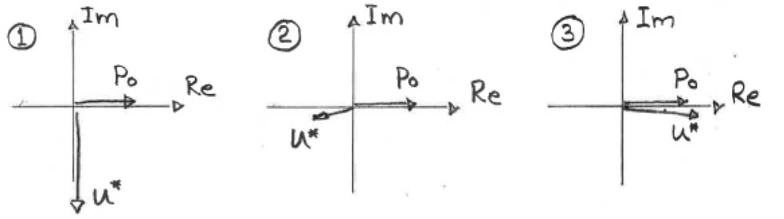
a) The figure shows the free response of the system (i.e. *p(t)*=0).



$$\zeta = \frac{1}{2\pi j} \ln \frac{u_i}{u_{i+j}}$$

Determine the natural angular frequency ω_n and the damping ratio ζ of the system. If the mass *m*=2kg what is then the stiffness of the spring?

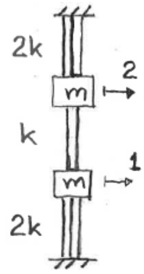
b) Stationary forced harmonic response to $p(t)=p_0 \sin\omega t$



The three figures show complex representations of the displacement for different frequencies ω.
Give your comment on the figures, one by one, concerning frequency, phase, and amplitude.

3) (4p)

Consider the two degree of freedom system below with system matrices defined as



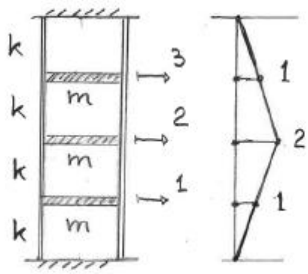
$$\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 3k & -k \\ -k & 3k \end{bmatrix}$$

a) Determine the natural angular frequencies and the corresponding mode vectors for the system.

b) Show that your mode vectors are both m- and k- orthogonal.

4) (4p)

Given the system in the figure,



$$\boldsymbol{k} = k \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

the natural frequencies and the first mode vector are

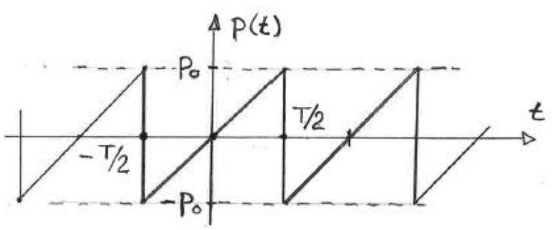
$$\omega_1^2 = (2 - \sqrt{2}\,)\,\frac{k}{m} \quad , \quad \omega_2^2 = 2\,\frac{k}{m} \qquad \omega_3^2 = (2 + \sqrt{2})\,\frac{k}{m}\,, \quad \text{and} \quad \boldsymbol{\phi}_1 = \begin{bmatrix} 1 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

a) Use the assumed shape $\boldsymbol{\Psi}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in the figure to estimate the lowest natural frequency of the system. Compare with the exact value.

b) Make your own assumption of the second eigenvector and determine an approximation of the two lowest natural frequencies. Compare again and comment.

5) (2p)

A single degree of freedom undamped system (*k* and *m* known) is exposed to a stationary saw-tooth loading according to the figure. The Fourier series development of the force is also given below.



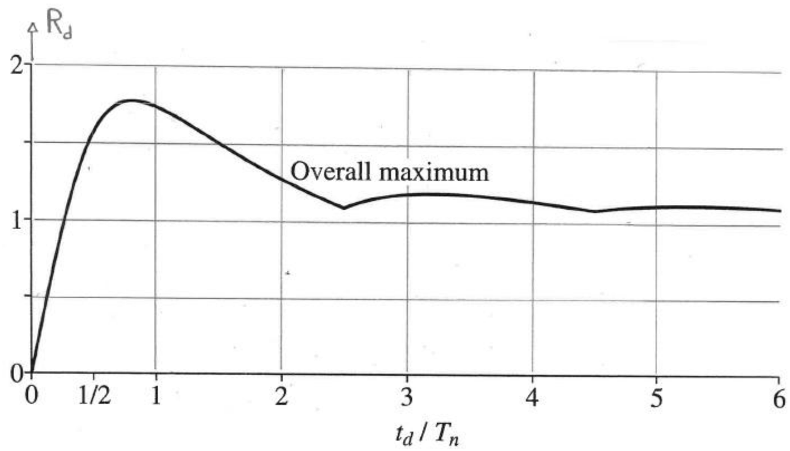
$$p(t) = \sum_{j=1}^{\infty} P_{oj} \sin j\omega t \quad \text{with} \quad P_{oj} = P_o \frac{2\cdot(-1)^{j+1}}{j\pi}$$

Calculate the stationary displacement response if the fundamental angular frequency of the loading $\omega = 2\omega_n/3$.

6) (3p)

Pulse loading for a spring and mass system under a half sinusoidal pulse is the subject of this question. The stiffness is k=10kN/m and the mass m=20kg.

a) The deformation response diagram concerning pulse loading for a half sinusoidal pulse is shown below.



Determine the maximum dynamic deflection u_0 of the system if the pulse duration t_d=0.2s and the force maximum is p_0=300N.

b) Determine again the maximum deflection if a very short and high force pulse, still half sinusoidal, act on the mass. The values for the pulse are now t_d=2ms and p_0=30kN, i.e. giving an approximate impulse I=40Ns being of about the same value as in case a).