Theory exam in Structural Dynamics 2018-03-02 kl.10-12

The test consists of 6 questions giving the maximum of 20 points. 7 points are required to pass the exam (and the course). Each question should be answered on a separate paper. No helping aids are permitted in this test, except calculator. Do not forget to write your name on each submitted paper.

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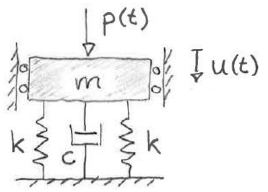
1) (4p)

Consider the single degree of freedom system in the figure.



a) Determine the equation of motion in terms of the mass vertical displacement using free body diagram.

b) Calculate the natural angular frequency ω_n and the corresponding natural period time T_n for m=100kg and k=200N/cm.

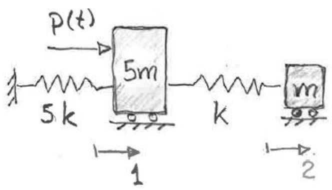
c) Determine the steady state displacement amplitude for harmonic loading with amplitude p_0=100N and loading frequency 5Hz. Set the damping to zero (i.e. c=0).
Also, are force and displacement in or out of phase?

d) Determine the quasi-static displacement amplitude for the same force amplitude as above.
Finally, determine also the static deflection.

2) (3p)

For the two degree of freedom system with matrices defined as

$$\mathbb{m} = m\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbb{k} = k\begin{bmatrix} 6 & -1 \\ -1 & 1 \end{bmatrix}$$



where only the heavier mass is driven by a force:

a) Calculate the steady state displacement response (without using modal analysis) for harmonic loading with amplitude p_0 and a loading angular frequency $\omega^2=k/m$.

b) Comment on the results.

3) (4p)

Modal analysis of the system in question 2):

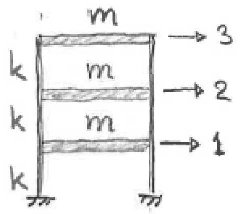
a) Determine natural frequencies.

b) Also determine the corresponding modes for the system.

c) Show k-orthogonality for the obtained mode vectors.

4) (3p)

The three story shear building in the figure, with m- and k-matrices below, has its lowest natural angular frequency $\omega_1 = 0.445\sqrt{k/m}$. The corresponding mode $\boldsymbol{\phi}_1$ is also given below:



$$\mathbb{m} = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbb{k} = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad \boldsymbol{\phi}_1 = \begin{bmatrix} 1 \\ 1.80 \\ 2.25 \end{bmatrix}$$

a) Use a linear approximation $\boldsymbol{\psi}$ of the first mode and determine the generalized single degree of freedom system for a load vector $\boldsymbol{p}$. Vectors are according to

$$\boldsymbol{\psi} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \boldsymbol{p} = p(t)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

b) Determine an approximation of the lowest natural angular frequency and compare with the exact solution. What requirement must the approximate solution fulfill?

c) Use the generalized single degree of freedom system to determine the steady state response in all degrees of freedom for a load $p(t) = p_0 \sin\omega t$ with $\omega = 0.3\sqrt{k/m}$.

5) (3p)

This question concerns numerical solutions with time stepping.

a) The three basic requirements on time stepping procedures are convergence, stability, and efficiency. Explain the meaning of these three concepts.

b) What is the basic difference between the Central difference method and the Newmark family methods?

c) What is common and essential for both methods in b) in order to start the iterations, (except for initial values of displacement and velocity)?

6) (3p)

Finally questions from different fields:

a) What is the fundamental assumption in calculating earth quake response for a spectrum of buildings?

b) Shortly explain the difference between modal truncation and the use of Ritz vectors.

c) What is Classical damping in terms of the properties of the C-matrix?