Theory exam in Structural Dynamics 2019-03-08 kl.10-12

The test consists of 6 questions giving the maximum of 20 points. 7 points are required to pass the exam (and the course). Each question should be answered on a separate paper. No helping aids are permitted on this test, except calculator. Do not forget to write your name on each submitted paper.

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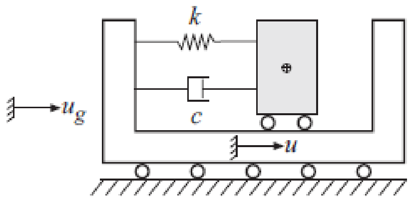
1) (3p)



Consider the SDOF system with m=5kg, k=250N/m, and c=3Ns/m. The frame is shaken by the displacement u_g and the displacement u measures the motion of the mass m (grey in the figure) relative to the frame.

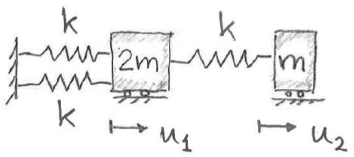
a) Determine by using free body diagrams the equation of motion for the mass m in terms of the mass horizontal displacement u. Hint: compare with eath quake response.

b) Calculate the natural angular frequency ω_n, natural frequency f_n , and the natural period time T_n .

c) Write the equation of motion in standard form using $c/m=2\zeta\omega_n$. Also calculate the damping ratio.

2) (3p)

Consider the two dof system with matrices defined as



$$\mathbf{m} = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 3k & -k \\ -k & k \end{bmatrix}$$

Show that the natural angular frequencies and the corresponding mode shapes for the system are:

$$\omega_1^2 = \frac{1}{2}\frac{k}{m} \quad \text{and} \quad \omega_2^2 = 2\,\frac{k}{m} \quad \text{with} \qquad \boldsymbol{\phi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \; , \quad \boldsymbol{\phi}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

3) (4p)

For the system in question 2) establish the uncoupled system of equations and solve for a steady state forcing.

a) Obtain the uncoupled (diagonal) system i.e. determine $\boldsymbol{M}^{\phi}$, $\boldsymbol{K}^{\phi}$, and $\boldsymbol{P}^{\phi}$, for a forcing $\boldsymbol{p}(t)=p(t)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

b) Solve for the modal coordinates $\boldsymbol{q}$ the forced steady state vibration problem with

$p(t) = p_0 \sin(\omega t)$ at a loading frequency $\omega^2 = \frac{k}{m}$.

c) Transform the modal amplitudes to obtain the physical displacement amplitudes $\boldsymbol{u}_0$ and comment upon the result.

4) (3p)

This question concerns model reduction in steady state and time stepping solutions.

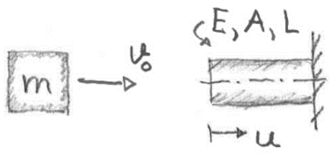
a) Explain the basic steps in the calculations when using modal truncation to obtain a numerical time stepping solution.

b) What are the main advantages of using modal truncation?

c) Shortly explain the difference between modal truncation and the use of Ritz vectors.

5) (3p)

Design a shock absorber using a linear elastic cylinder according to the figure. The requirement is that a mass of m=2kg hitting the shock absorber at a velocity of v_0=3m/s should be stopped to zero velocity over a distance of u_{max}=30mm.



a) Determine the length L to meet the requirements if the diameter D=50mm, Young's modulus E=1MPa. The spring stiffness is given by $k=EA/L$ with A being the cross sectional area.

b) What will be the maximum force F_{max} on the mass and the time t_{max} for the mass to come to zero velocity?

6) (4p)

Answer shortly the following questions from different topics:

a) In pulse loading of a SDOF system, the extreme cases are very short and very long pulses. How is pulse length characterized? What's the response of a very long pulse?

b) What is transmissibility? Give an example of how it can be interpreted.

c) Explain the concept "response history" in earth quake loading analysis. What is it and how is it established?

d) Why is rubber a good material for vibration isolators?