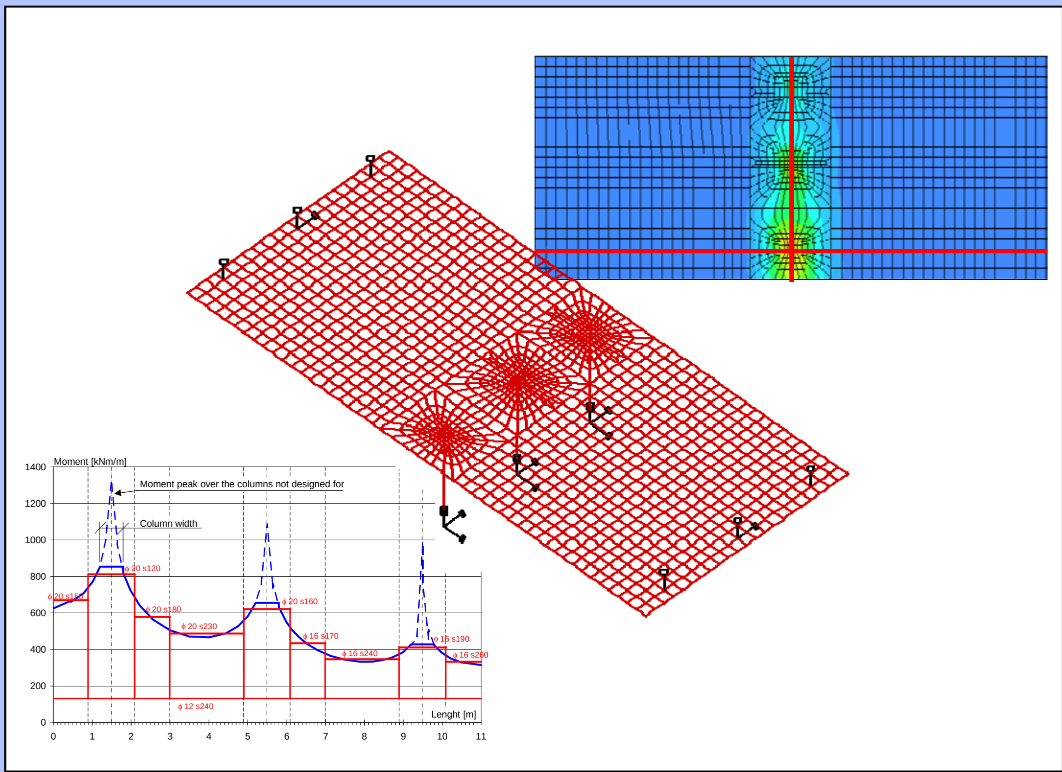

Recommendations for finite element analysis for the design of reinforced concrete slabs

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Recommendations for finite element analysis for the design of concrete slabs

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Foreword

In the bridge design community the usage of 3D finite element analyses has increased substantially in the last few years. Such analyses provide the possibility for a more accurate study of the structure than what is possible by using more traditional design tools. However, in order to use the full strength of the finite element method in daily design practice a number of critical issues have to be addressed. These issues are related either to the FE-modeling itself (geometry, support conditions, mesh density, etc.) or to the post processing of the obtained results (stress concentrations, choice of critical sections, distribution widths and so on). The purpose of this report is to address these problems and provide recommendations and guidelines for the practicing engineers.

The recommendations given here are based on what was found in literature combined with engineering judgement and considerations from engineering practice. The recommendations are believed to be conservative, implicating a potential for improvement based on increased knowledge on the response and distribution of shear in concrete slabs and how this is reflected by linear FE analysis. This also means that, in many cases, there may be other alternatives that are equally correct as the ones suggested in this report.

The authors want to express their gratitude to the members of the reference group. Their comments and suggestions have been invaluable in shaping the report.

Special thanks are also due to Marcus Davidson and dr. Per Kettil (Skanska Teknik) for having carefully read the manuscript and provided valuable comments and suggestions for improvement.

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Contents

1. Introduction

The finite element method is commonly used to design the reinforcement in concrete slabs. In order to simplify the analysis and to be able to use the superposition principle for evaluating the effect of load combinations, linear analysis is generally adopted even though concrete slabs normally have a pronounced non-linear response. In ultimate limit states this can be justified since concrete slabs normally have good plastic deformability. Theoretically, the design is based on the lower bound theorem of plasticity. Consequently since the design is based on a moment (and force) distribution that fulfils equilibrium, the load carrying capacity will be sufficient provided that the structure has sufficient plastic deformation capacity. In serviceability limit states, the use of linear analysis is based on the assumption that the redistribution of moments (and forces) due to concrete cracking is limited.

The present recommendations apply to concrete slab and shell structures subjected primarily to bending effects with limited membrane effects, i.e. slab and shell structures subjected to loading in a direction normal to the plane of the structure. In this case, the membrane forces are normally generated by temperature or shrinkage effects, by braking forces or (locally) by the geometrical effects of capitals or drop panels. The recommendations, in particular the provisions related to the distribution widths do not apply to slabs and shells subjected to in-plane loading (e.g. concrete walls and deep beams).

For the type of structures depicted above (i.e. plate and shell structures subjected to out of plane loading) a linear analysis is normally performed in order to determine the load effects that will further be used for the detailed design of the structure. However, in order to obtain a relevant basis for design a proper modelling and subsequently a proper interpretation and use of the FE analysis results is required. At a support for instance, the sectional forces and moments are needed in the sections where a failure mechanism may occur. For a concrete slab monolithically cast together with a supporting wall, the relevant moments in the slab are those at the face of the supporting wall. These considerations will influence the way in which the actual support is represented in the FE model, the mesh density around the support point and the choice of relevant result points. In addition, in linear models, unrealistic concentrations of moments and shear forces will occur due to necessary simplifications in the model. In order to obtain an economical design these concentrations need to be distributed over a certain width, here denoted as distribution or strip width.

Thus, three aspects of particular importance will be addressed in the present recommendations. These aspects refer to:

- Modelling of support conditions
- Choice of result sections
- Choice of distribution widths

In addition, a special chapter will be dedicated to cantilever slabs, which is a problem relevant to composite bridges or concrete beam bridges.

For the type of problems discussed in this report the authors have tried to provide an extensive coverage. However, due to the diversity of design problems it is virtually impossible to explicitly address all possible structural configurations, support and loading conditions. Thus, there are design situations not explicitly covered by the present recommendations or situations where the designer is required to interpret and adapt the recommendations instead of simply applying one or the other of the clauses. In all such situations it is necessary to determine a practical way of addressing the three issues listed above.

It should also be noted that the recommendations were primarily determined for bridge and tunnel structures. However, it is the authors' belief that the provisions can also be applied to other types of structures.

2. Modelling of support conditions

2.1. General aspects

The support conditions in a finite element model of a structure often have a decisive influence on the analysis results. Consequently, the modelling of the supports needs to be paid special attention.

In reality, the support from the foundation or from other structural parts provides stiffness with respect to both translation and rotation. In the structural model, this is often simplified to free or fixed translations or rotations at the supports. In many situations these simplifications can be motivated. However, in other cases such simplifications may have a critical influence on the analysis results. In such cases the supporting structure, or its supporting stiffness through translational or rotational springs, should be included in the model.

It is also important to ensure that support conditions are introduced in the model at their correct locations and in correct directions. Note that constrained degrees of freedom in the model will control the deformation and rotation distribution in the analysis. Consequently, a small shift in the direction or position of a constraint may shift the deformed shape, and hence the internal distribution and magnitude of internal stresses, moments and forces.

For slabs supported by bearings or columns, the support conditions are often modelled as concentrated at single nodes. The effect of this is that a singularity is introduced in the solution, with the sectional forces and moments tending to infinity upon mesh refinement. There are two principally different ways to deal with this problem: either the modelling of the support is improved so that the singularity is avoided or the results are evaluated in the failure-critical sections adjacent to the supports.

In most cases it is sufficient to model supports or connections to other structural parts in single points or lines. From the point of view of reinforcement design, the peak values that occur right at the connection are not of interest. Instead, design results are needed in critical sections adjacent to the supports. For instance, if a slab is monolithically connected to a column, the effects of the singularity in the slab may be disregarded. Instead the forces and moments transferred across cross-sections where potential failure mechanisms may occur at the border of the column should be used for reinforcement design. Modelling according to this alternative is described in Section 2.2. In Chapter 3, the choice of result sections for this alternative is treated.

In some cases it can be motivated to model the support conditions in more detail so that the stress transfer from the support to the slab is described in a more realistic way. This is the case, for example, if the support has a large minimum width compared to the slab thickness or the span length. A modelling alternative avoiding singularities at the supports is described in Section 2.3. If this modelling alternative is used, the maximum moments obtained in the slab should be used for design.

2.2. Modelling of supports at single points or lines

2.2.1. Modelling recommendations

Generally, for analyses used as basis for detailed design, it is recommended to model supports through prescribed boundary conditions in single points or along single lines. In this way unintended rotational restraints in the numerical model are avoided. Rombach (2004) examined different ways to model a wall support for a continuous one-way slab (see Figure 2.1) and compared the results with simple beam calculations. The wall provides vertical support but the connection cannot transfer any moments, i.e. it is a "hinged" support.

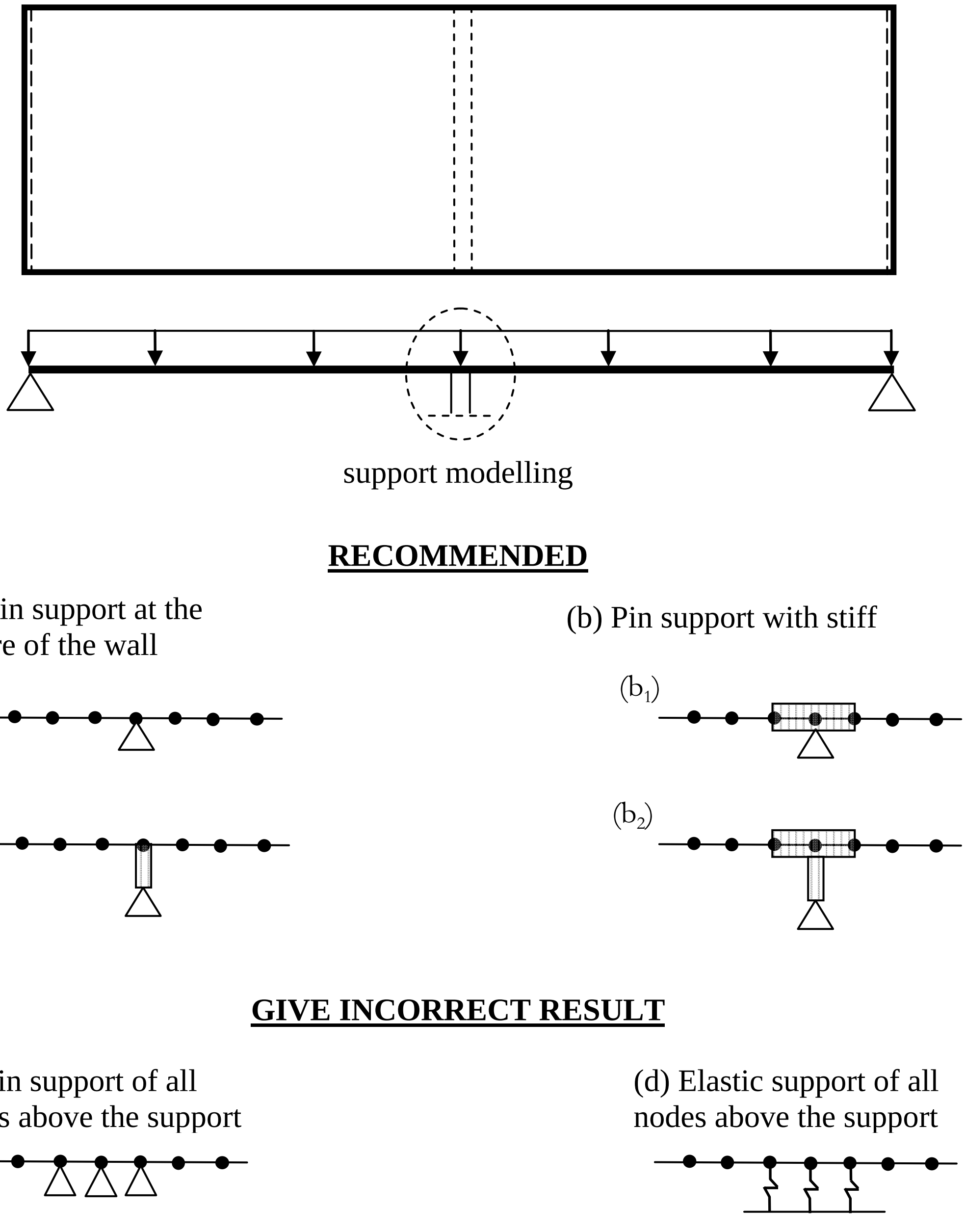


Figure 2.1 Different ways to model a "hinged" line support for a slab modelled with linear shell elements, adapted from Rombach (2004). The pin support (a) is recommended. The pin support with stiff couplings (b) also gives good results while the other alternatives give incorrect results.

The modelling alternative with a pin support at the centre of the wall (i.e. only the vertical displacements of the nodes situated at the centre line of the wall were restrained) corresponds best with beam theory. If the thickness of the slab is small, alternatives (a_1) may be used. However, in cases where horizontal restraints are present and the slab thickness is not negligible, a more correct modelling alternative (denoted as alternative (a_2) in the figure) is to include also a rigid link from the centre node of the slab to the support point.

The alternative (b) "pin support with stiff couplings" showed also a rather good correlation with beam theory. In this alternative the vertical displacements of the centre nodes are restricted. In addition, the centre nodes are rigidly linked to the adjacent nodes as shown in Figure 2.1. Note that in this case the rigid link should only include the displacements normal to the plane of the slab whereas the in-plane displacements should not be coupled in order to avoid over-constraining due to, for instance, temperature loads. As before, alternative (b_1) should be used if the slab thickness is small whereas alternative (b_2) should be used if the slab thickness is not negligible and horizontal restraints are present.

The other two modelling alternatives shown in the figure give incorrect results due to unintended rotational restraints. Rombach (2004) also examined different ways to model column supports of flat slabs which in essence are very similar to the ones shown in Figure 2.1. Similarly to wall supports, it was found that a pin (or ball) support at the axis of the column should be used, since all other modelling alternatives may cause unintended rotational restraints.

For the case of a monolithic connection between the wall\column and the slab, two modelling alternatives are shown in Figure 2.2.

Modelling alternative (a) involves a stiff coupling (rigid link) applied at the column top over a length equal to half the slab width. This modelling alternative gives results in good agreement with a continuum (solid) model if the slab thickness t and the wall (column) width a fulfil the following conditions:

$$\begin{cases} a < \min(l_{01}, l_{02}) \\ \frac{a}{t} < 2 \end{cases} \tag{2.1}$$

In equation (2.1) l_{01} and l_{02} represent the distances from the column centre to the points of zero moment on either side of the column. These distances can be evaluated for a load case involving the permanent loads only. In cases where the slab thickness is much smaller than the span, i.e. $a \ll \min(l_{01}, l_{02})$, the stiff coupling can for simplicity be left out and the wall or column be extended up to the centre of the slab.

Modelling alternative (b) gives higher moments at the supports and by consequence lower moments within the span. In addition one should be careful not to introduce over constraint out of the plane that can cause too high moments and membrane forces for temperature loading.

Apart from the alternatives presented in Figure 2.1 there are several other ways to model the monolithic connection. One possibility would be to use alternative (b) but not extent the rigid coupling over the whole width a of the wall (or column) while another possibility would be to increase the thickness of the slab over the connection zone thus accounting for the increase in stiffness within this region.

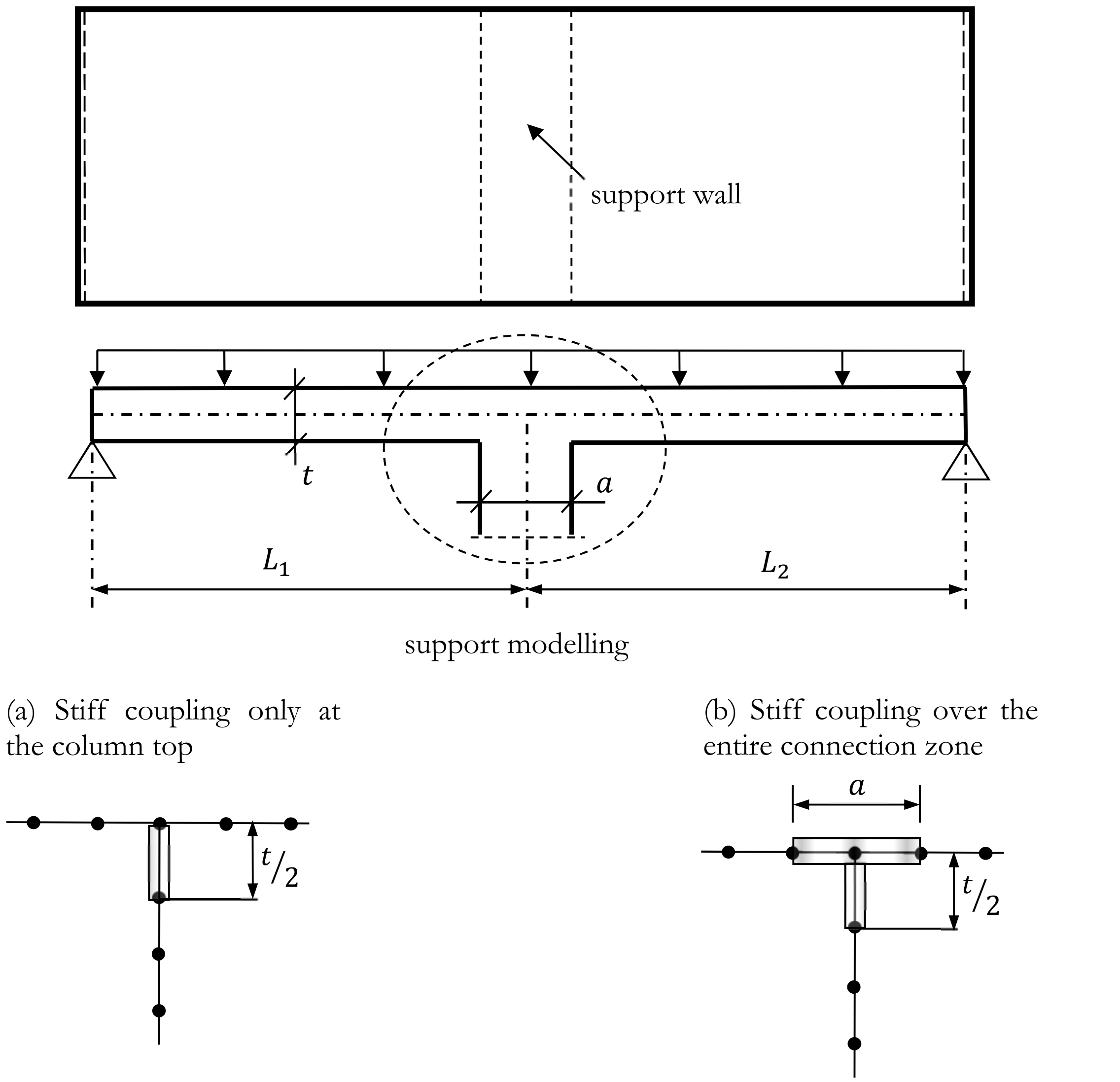


Figure 2.2 *Different ways to model a monolithic connection between the slab and the supporting wall\column.*

If the width a of the wall\column and the thickness t of the slab do not respect the conditions in equation (2.1), it may be motivated to model the support conditions in more detail so that the stress transfer from the support to the slab is described in a more realistic way. A modelling alternative avoiding singularities at the supports is described in Section 2.3.

When **designing concrete slabs** with irregular geometry, supported on concentrated and/or line supports, the same principles for modelling of the supports apply. However, due to the three-dimensional geometry of the slab, some additional considerations need to be taken. In general, the following way of modelling the supports can be recommended.

The slab is simply supported by a wall:

- This can be modelled as a line support along the centre-line of the wall. This means that only the vertical translations of the centre nodes of the wall are constrained. It should be noted that it is important to model the node positions along the centre-line exactly since even a small deviation from the correct position can introduce a significant rotational restraint.
- Where wall supports meet, a simply supported slab would often tend to lift from the support. If support reactions to prevent the uplift are not provided, a non-linear analysis

is generally needed. Alternatively, an iterative procedure with linear analyses in which the constraints are removed for the part of the line support where the slab tends to lift can also be adopted.

- The normal stiffness of the wall can sometimes have a large influence on the results for the slab, particularly if the slab is supported on walls that are interrupted. Here, it is recommended to include the wall in the model or to model its axial stiffness through translational springs.

The slab and the wall are monolithically connected:

- The wall is preferably included in the FE model. To assume that the wall will provide either a fixed or free rotational constraint to the slab is generally a too coarse assumption. When including the wall, the elements representing the slab will be rigidly connected to the wall elements along the line where their mid-surfaces meet. The rotational stiffness of the wall will provide a reasonably accurate support condition for the slab.
- If results are requested for the slab only, the opposite end of the supporting wall can be fixed or simply supported, depending on what is most appropriate. Since the purpose of the analysis is to obtain a distribution of internal forces and moments fulfilling equilibrium as a basis for the design rather than the exact elastic response, this will usually be a sufficiently good approximation. However, if the stiffness of the opposite edge connection (e.g. a foundation) is important, this can be modelled by applying spring supports at the opposite end of the supporting wall.

The slab is supported by columns or bearings:

- The column or bearing supports can often be modelled as point supports at the centre of the column or bearing.
- For bearings, translational and rotational degrees of freedom are prevented or given stiffness in accordance with the stiffness of the bearing.
- For columns providing a simple support for the slab only the vertical translation of the centre node of the column or bearing needs to be constrained. In the same manner, for slender interior columns cast together with the slab, it is sufficient to prevent vertical deflection over the centre of the column; this implies that the moments (and transversal forces) transferred from the column to the slab are negligible compared to the slab moments (and membrane forces). This simplification is in accordance with the provisions given in Eurocode 2 (SS-EN 1992-2:2005, Section 5.3.2.2 (2)).
- For edge or corner columns that are cast together with the slab it is recommended to include the columns in the model. The same applies to stiff interior columns. The column is then rigidly connected to the slab at the node where its centre-line meets the slab mid-surface. Similarly to walls, it is usually sufficient to model the support at the opposite edge of the column as free or fixed if results are requested for the slab only. For situations where the stiffness of the opposite edge connection is important this can be modelled through the use of spring supports.
- The coupling of the torsional degree of freedom of the column to the slab should also be considered. The difficulties arise from the fact that shell elements normally have only 5 degrees of freedom which do not include the rotation around the normal to the slab. Normally this coupling is not important for the load transfer mechanism between the slab and the column and can be disregarded. However this aspect should be assessed from case to case. If this coupling is relevant it can be modelled through the use of rigid links.

2.2.2. Mesh density at support regions

When performing a linear finite element analysis of a concrete slab, cross-sectional forces and moments become high at concentrated supports, and will tend to infinity upon mesh refinement.

However, when using the analysis as a basis for reinforcement design, the peak values of the sectional forces and moments are generally not of interest. Instead, the cross-sectional forces and moments in critical sections adjacent to the support are needed for design, see Chapter 3.

For this type of FE analysis it is recommended to model concentrated supports with constraints or connections to supporting elements at the centre nodes or along the centre lines of the supports, see Section 2.2. However, the critical sections are typically located at or close to the edge of the support, see Chapter 3. For example, for a concrete slab monolithically cast together with a concrete column, the critical sections with respect to bending moments in the slab are along the face of the column. In order to obtain sufficiently accurate results at these critical sections, the element mesh need to be sufficiently dense in the support region.

In Sustainable Bridges (2007), the influence region of the singularity at the point support of a slab subjected to a distributed load was studied with respect to the moments in the slab. The original element mesh was successively refined by dividing each element side in two. When comparing the moments close to the support point the difference is substantial. However, one element away from the support point in the coarser mesh, the difference was small and two elements away it was negligible (within a few percent). The same conclusion has also been reached by e.g. Rombach (2004). Based on this observation, it was recommended in Sustainable Bridges (2007) to have a mesh density corresponding to two first-order elements or one second-order element between the support point and the critical section.

When designing the slab reinforcement, averaged values of the bending moments over certain distribution widths perpendicular to the reinforcement direction will normally be used, see Chapter 4. The influence of the mesh density around a column on these averaged moment values will be much smaller than on the moment value at the critical section right at the edge of the column. Based on this observation, it is recommended that **the mesh density around the point support node in a slab (e.g. a column or an abutment), should be chosen such that there is at least one shell element regardless of order, between the support node and the critical cross-section.** Figure 2.3 illustrate an example of such a mesh refinement around column supports.

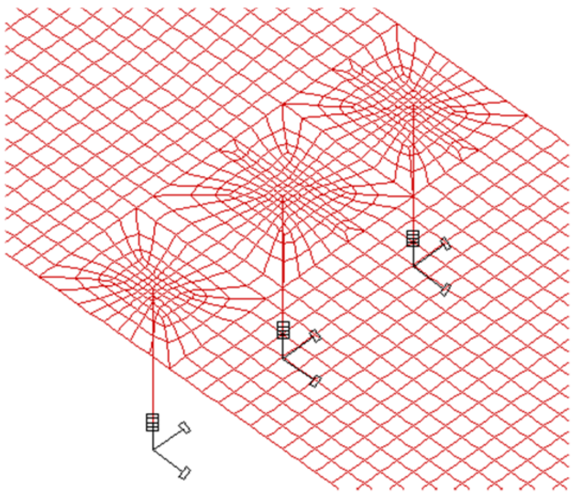


Figure 2.3 Example of mesh refinements around column supports of a bridge slab.

For situations where a slab is supported by a line support, there is no problem with singularities. However, the element mesh needs to be fine enough to give accurate results in the adjacent critical sections. Also here it is recommended to provide at least one element length between the line support node and the critical cross-section, regardless of the shell element order.

Alternatively, the maximum moment and shear force at the line support can be used as a conservative approximation.

2.3. Modelling that avoids singularities

An alternative to model the support given by a column or bearing in a single point, and evaluate the results in adjacent critical sections, is to remove the singularity by special modelling arrangements. The intent here is to describe the support pressure from the column or bearing towards the slab in order to obtain a more realistic moment distribution over the support.

One such solution is to replace the point reactions by surface loadings as shown in Figure 2.4.

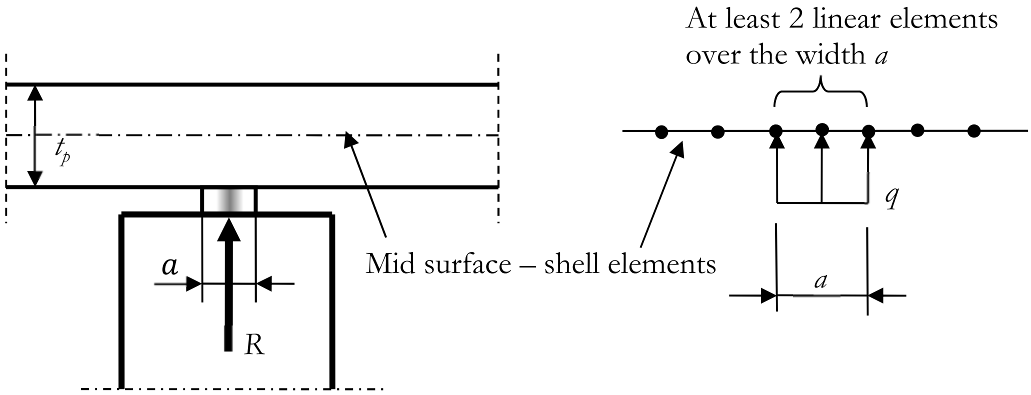


Figure 2.4 Point reactions replaced by surface loading.

For an elastomeric bearing, the surface load q can be approximated according to: $q = R / S$ where R is the support reaction force and S is the equivalent surface of the bearing. For more rigid supports and monolithic connections the support pressure will be concentrated towards the edges of the support surface. This can be taken into account after evaluation of the support pressure distribution. Alternatively, equally distributed pressure can be used as a conservative approximation. At least two first order elements should be used over the width a. For supports that are wide compared to the slab thickness, more elements are needed.

The general analysis procedure is as follows:

- The computations are first performed with a point support at the centre of the bearing in order to determine the reaction force.
- The surface pressure is then computed and applied upwards at the support.
- The analysis is then performed once again, now adding the computed surface pressure at the support, in order to determine the bending moments and shear forces. Note that in this case the actual reaction force at the support point should become (approximately) zero.

The solution presented above is not the only possible solution. An alternative approach is to model the bearing or support by springs according to the principle shown in Figure 2.5.

Note that the modelling alternative presented in Figure 2.5 does not introduce any spurious rotational restraints in the model. The stiffness properties of the springs can be determined from the stiffness properties of the support, e.g. a bearing. Note also that the spring stiffness must be different in the middle, on the side and at the corner of the support plate if discrete spring elements connecting the nodes are used to describe e.g. a constant surface stiffness.

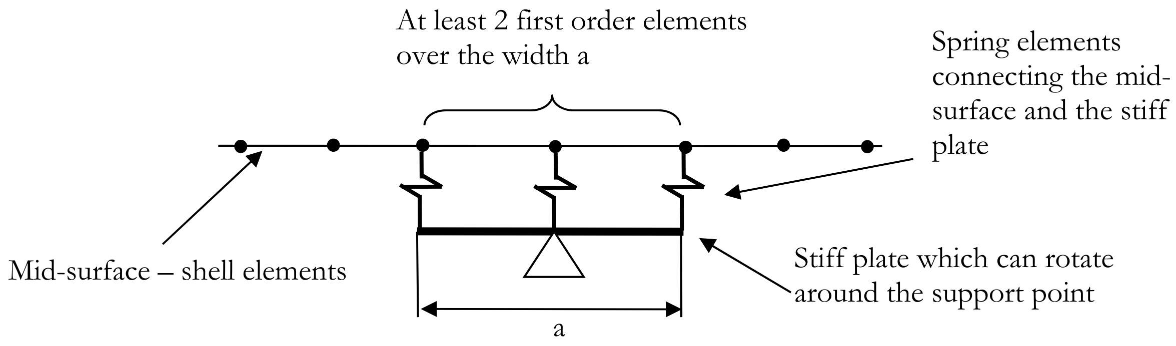


Figure 2.5 Bearing support modelled by spring elements.

The modelling alternative described in Figure 2.5 should also be adopted if the support has a large minimum width compared to the span length and\or slab thickness. Typically, this situation occurs if wide columns are monolithically connected to the slab (i.e. the width of the column does not fulfil conditions in Section 2.1). In this case the column should be included in the model and the stiff plate in Figure 2.5 should be rigidly connected to the column top.

Finally note that if the modelling alternatives presented in Figure 2.4 and Figure 2.5 are adopted, the maximum moments obtained in the slab should be used for design.

3. Choice of result sections

3.1. Result sections for moments

3.1.1. General

If we study the bending moments in a one-way slab over a support with a certain width, we will find that the bending moment has its maximum somewhere above the support, see Figure 3.1. The exact location and magnitude of the maximum moment will depend on the loading on the slab and the distribution of the support pressure towards the bottom of the slab. At the location where the maximum bending moment occurs a vertical bending crack will develop and eventually the reinforcement in this section will start to yield, possibly limiting the capacity with respect to bending. The same principle applies for a two-way slab; the critical sections for bending are situated at locations where the maximum moments occur.

If the supports are modelled to describe the support pressure in a more realistic way (see Section 2.3) the result sections for the moments are the sections where maximum moments are obtained in the FE analysis.

If, on the other hand, the supports are modelled in a simplified way, in single points or along discrete lines, the maximum bending moments obtained from the FE analysis will over-estimate the real moments. At locations where the slab is supported at single points it will even tend to infinity upon mesh refinement. Here, the locations of the result sections where FE results should be evaluated depend on the design and the actual stiffness of the slab-support connection. This is further treated in the following sections.

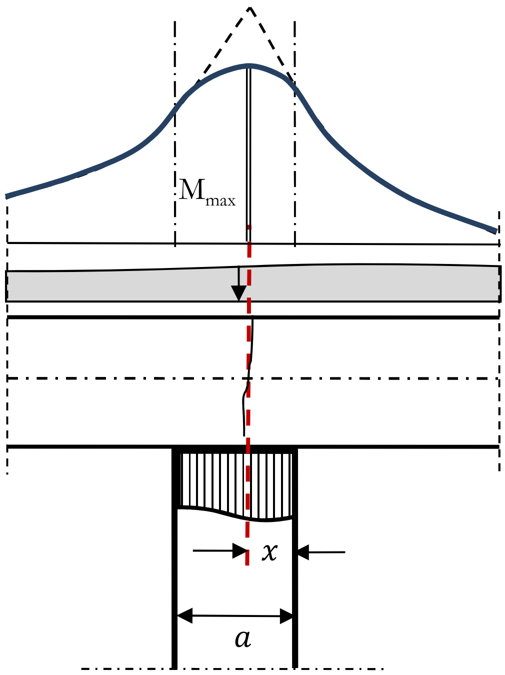


Figure 3.1 Bending moment variation and critical section for the bending moment in a slab with distributed support pressure.

3.1.2. Monolithic connections modelled in single points or lines

If the slab is monolithically connected with its supports, columns or walls, it can be shown that the maximum stresses do not occur inside the connection region but instead appear at the border of the connection. This corresponds with observations on failures in connection regions, and was shown for frame corners by e.g. Plos (1995), Johansson (2000) and Lundgren (1999). Note also that the cross-sectional moments and forces in the slab are defined as integrals of the stresses over the cross-section and do not have a clear interpretation inside a connection region. Instead, these regions must be seen as disturbed regions where beam or slab theory is not valid. A critical bending crack will form no closer to the theoretical support point than along the surface of the column or the wall. This is also where the tensile reinforcement will start to yield. Consequently, the critical cross-section for bending failure in the slab is along the surface of the column or wall, see Figure 3.2.

In Section 2.2.1 it was recommended to model monolithic connections between a slab and its supporting walls and columns along discrete lines and in single points. For a monolithic connection modelled in this way **the result section for moments can be taken as the section along the surface of the column or wall**, see Figure 3.2. This corresponds to the recommendations given in Eurocode 2 (SS-EN 1992-2:2005, Section 5.3.2.2), provided that the support width is smaller than the slab thickness.

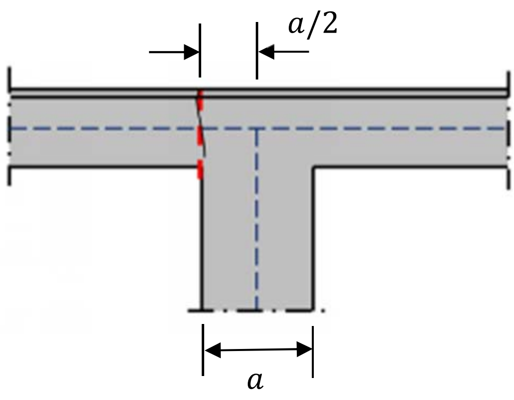


Figure 3.2 Result section for bending moments, for a monolithic connection modelled as a connection in a single point or along a discrete line between structural finite elements (typically beam and shell elements).

The width a of the column, in Figure 3.2, is the side length of a quadratic or rectangular column cross section. For a circular column, the real geometry can be approximated by an equivalent quadratic cross-section with

$$a = \frac{\sqrt{\pi}\phi}{2} \tag{3.1}$$

where ϕ is the diameter of the column.

3.1.3. Simple supports modelled in single points or lines

If the slab is simply supported on a wall or a column and the support transfers compression stresses only, the position of the critical section depends on the stiffness of the support. Blaauwendraad (2010) studied the support stress distribution for different support conditions for one-way slabs supported on walls. In case of a slab resting on a rigid support, the resultants of the support stresses for each support half will shift towards the edge of the support. As an approximation, the support resultants can be assumed to act at the face of the support. On the other hand, if the support is very weak, the support pressure will tend to approach a uniformly distributed support stress. These two cases can be seen as extremes regarding the support pressure distribution, see Figure 3.3. In case of column supports, the support pressure for rigid supports will tend to shift more towards the support edges than for walls.

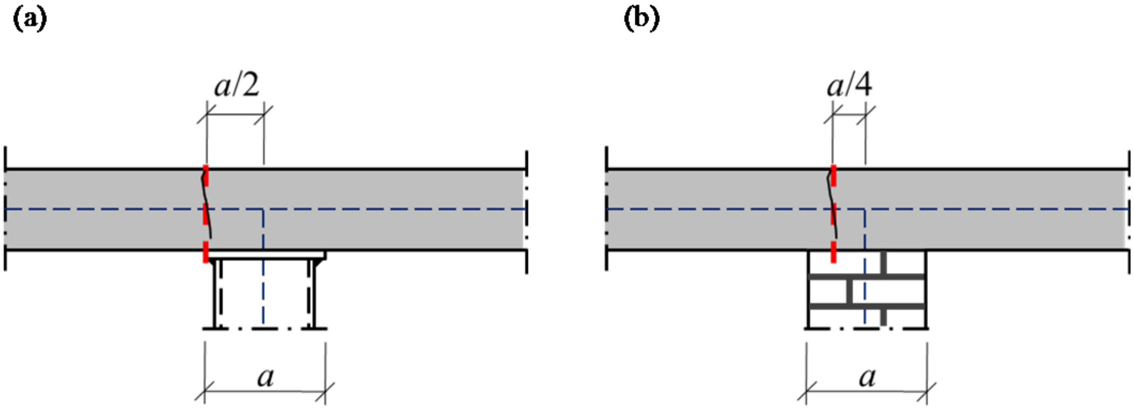


Figure 3.3 Result section for bending moments for a simple support modelled in a single point or along a discrete line between structural finite elements (typically beam and shell elements): (a) in case of rigid support and (b) in case of weak support. As a conservative approximation, case (b) can be assumed.

According to Eurocode 2 (SS-EN 1992-2:2005, Section 5.3.2.2), the design support moment, calculated with centre-to-centre distance between support points, can be reduced with

$$\Delta M = Ra/8 \tag{3.2}$$

where R is the support reaction and a is the support width. This corresponds to assuming an equally distributed support pressure. In Appendix A, the moment distributions in a continuous slab strip with equal span lengths are compared for different assumptions regarding the support pressure distribution. It is shown that the maximum support moment according to Eurocode's assumption can be conservatively approximated from the theoretical moment distribution (with discrete supports at the support centre) as the moment a distance $0.25a$ from the support point. Similarly, it is shown that the assumption of support resultants at the edges of the supports corresponds to the moments at the edges of the support in the theoretical moment distribution (i.e. $0.5a$ from the centre support point).

A weak support for a concrete slab could typically be a masonry wall or column. However, not even for this support condition we would obtain an equally distributed support pressure. For a concrete support of a one-way concrete slab that transfers both compressive and tensile strains, Blaauwendraad (2010) showed that the support resultant for each support half will be at 2/3 of the distance from the centre towards the edges, and for a steel support that transfers only compressive forces at 95 % of the distance towards the edges. Based on the knowledge of the support pressure distribution, the position of the support section to use together with a theoretical moment distribution with discrete supports at the support centre can be determined in a similar way to the calculations in Appendix A. For a point support, the support resultants will always be closer to the edges than for a line support with the same support stiffness.

Without any detailed evaluation of the real support pressure, a distributed support pressure can be assumed as a conservative approximation. This means that, for simple supports modelled in single points or lines, **the result section for moments can be taken as the section at half the distance between the centre and the edge of the column or wall**, see Figure 3.3. This corresponds to the recommendations given in Eurocode 2 (SS-EN 1992-2:2005, Section 5.3.2.2).

3.1.4. Bearing supports modelled in single points or lines

A bearing consists principally of an elastomeric material between two steel plates. Older types of bearings may instead consist of rollers between steel plates. The steel plate that is in direct contact with the concrete slab is usually very stiff. Such a case can thus be seen as a stiff support with reaction resultants acting at the edges of the support. The vertical flexibility of the elastomer then just adds to the vertical flexibility of the supporting structure. For this case **the result section can be assumed to be along the edge of the bearing top plate**, see Figure 3.4.

On the other hand, if the steel plate cannot be regarded as stiff, the support pressure will change towards a more distributed support pressure. For this case, and as a conservative assumption in general, **the result section for moments can be taken as the section at half the distance between the centre and the edge of the bearing top plate**, see Figure 3.4.

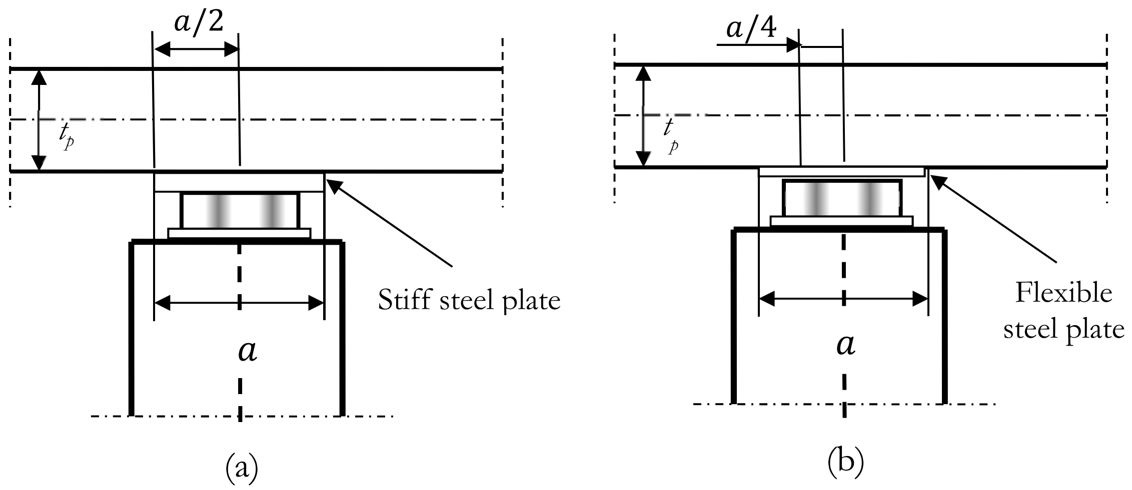


Figure 3.4 Result section for bending moments, for a bearing support modelled in a single point or along a discrete line (typically beam and shell models), (a) rigid bearing top plate and (b) flexible (or no) bearing top plate.

3.2. Result sections for shear forces

The location of result sections for shear forces at supports depend on where failure-critical shear cracks may occur. A critical shear crack will develop where it will transfer the largest possible shear force across the inclined shear crack. This means that a critical shear crack will occur no closer to a support than with its lower end at the edge of the support, see Figure 3.5. Note that, if we assume the potential shear crack to be moved further towards the centre of the support, part of the load acting in the span of the slab would be transferred directly down to the support without passing the shear crack. Consequently, the shear force transferred over the shear crack would decrease. This observation is valid independently of the distribution of support pressure.

The shear force in a slab section is caused by the part of the vertical load that is transferred towards the support across this section. Consequently, the shear force obtained from the slab analysis in the section at a distance $z \cot \theta$, where z is the internal lever arm, is the shear force that needs to be transferred across the critical shear crack (see Figure 3.5). Any load that is acting on the slab top surface closer to the support than this will be transferred directly to the support. The self-weight of the slab can be treated as a load acting on the top of the slab.

We can conclude that **the critical result section in a slab with respect to shear forces are not located closer to the support edge than $\boldsymbol{z \cot \theta}$** This is independent of the design and stiffness of the slab-support connection. For slabs without shear reinforcement, the critical shear crack can generally be assumed to have an inclination θ not steeper than 45 degrees (CEN, 2004) In this case $z \cot \theta = z \approx d$.

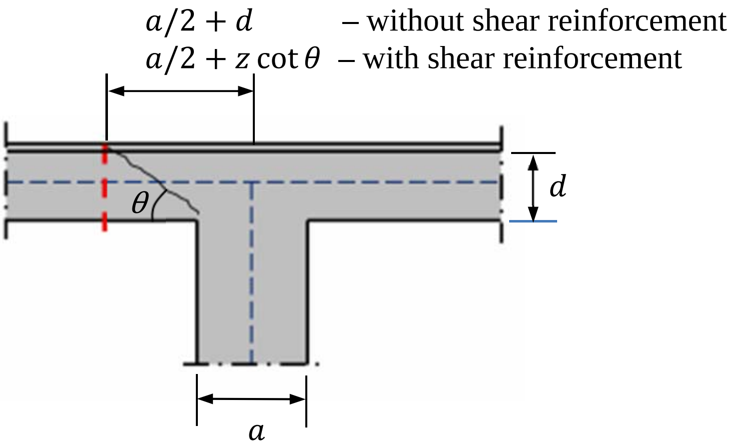


Figure 3.5 Critical section for shear force (independent of the design and stiffness of the slab-support connection).

Notes:

1. For the case of moving loads (as for instance traffic loads) choosing $\cot \theta = 1.5$ will produce shear reinforcement amounts that will automatically comply with the provisions of Eurocode 2 (SS-EN 1992-1-1:2005, Section 6.2.3(8))
2. In slabs with shear reinforcement, the risk for shear compression failure must also be checked. For this case, the entire shear force at the support edge must be accounted for.

4. Redistribution of sectional forces and moments

At the beginning of this chapter it must be reiterated that the present recommendations apply to concrete slab and shell structures subjected primarily to bending effects with limited membrane forces, i.e. slab and shell structures subjected to loading in a direction normal to the plane of the structure. In this case, the membrane forces are normally generated by temperature or shrinkage effects or (locally) by the geometrical effect of capitals or drops. The recommendations do not apply to plates and shells subjected to in-plane loading (e.g. walls and deep beams) or to pre-stressed concrete slabs for load effects in the direction of pre-stressing.

4.1. General

4.1.1. Effects of simplifications in the modelling

As already pointed out, unrealistic concentrations of cross-sectional moments and shear forces will generally occur in linear FE analyses due to simplifications in the modelling. The origin of these stress concentrations can be traced back to simplified assumptions concerning the geometrical modelling of the structure at hand or to the modelling of the mechanical properties of the materials. Geometrical simplifications are typically simplified modelling of supports and connections between structural elements, or simplified modelling of concentrated loads e.g. wheel pressures on bridge slabs. The material simplifications are mainly related to the assumption that reinforced concrete behaves like a linear elastic and isotropic material. In reality however, reinforced concrete has a highly non-linear behaviour involving both cracking and crushing of concrete and yielding of reinforcement.

Simplifications made in the geometrical modelling often lead to very high concentrations of moments and shear forces at least locally. This occurs, for example, when a slab modelled with shell elements is supported by columns modelled with beam elements or by bearings modelled with boundary conditions applied at a single node. It also occurs when a distributed wheel pressure is modelled by a concentrated point load. In such situations a singularity is introduced in the solution. As a consequence the bending moments and shear forces in the slab will tend to infinity when the element mesh is being refined.

Nevertheless, in Section 2.2 it is recommended to model the supports in discrete points or lines. The reason is that in most situations the results at the support nodes are not of practical interest for the design of the slab. The singularities that may occur are local disturbances of the moment and force fields, and do not influence the cross-sectional moments and shear forces a short distance from the support point where the singularity appears. This has been illustrated by e.g. Davidson (2003) and Rombach (2004). Instead, it is the moments and forces in the adjacent critical sections of the slab that are needed for the design, Sustainable Bridges (2007). In order to get sufficiently accurate results in the critical sections, the finite element mesh needs to be sufficiently dense in the support region, see Section 2.2. In conclusion, **as long as the results in the critical sections are used and the finite element mesh is dense enough, modelling of support conditions in discrete points or lines does not influence the designing cross-sectional moments and shear forces**.

However, not even the high stresses obtained in the critical sections do normally exist in reality. The concrete will crack already for service loads, leading to redistribution of moments and forces. In the ultimate limit state, the reinforcement will start to yield, leading to even larger redistributions. The material simplification introduced through the assumption of linear elastic response will lead to higher cross-sectional moments than in reality, e.g. around a column or a concentrated support, since cracking and subsequent yielding in the reinforcement is not included in the model.

4.1.2. Sectional forces and moments for reinforcement design

A 3D slab analysis will give a moment field consisting of bending and torsional moments. In an ultimate limit state, the forces in each main reinforcement layer times its inner level arm will result in a bending **reinforcement moment** resistance. These reinforcement moment resistances must balance the complete linear moment field, including the torsional moment.

Methods to determine the sectional forces and moments for design of slab reinforcement are treated in e.g. Eibl (1995), CEB-FIP (2008) or Blaauwendraad (2010). The reinforcement moments m_{rx} and m_{ry} for design of reinforcement in two perpendicular directions x and y can be defined according to equations (4.1) and (4.2):

$$m_{rx,pos(neg)} = m_x \pm \mu|m_{xy}| \tag{4.1}$$

$$m_{ry,pos(neg)} = m_y \pm \frac{1}{\mu}|m_{xy}| \tag{4.2}$$

Where: m_x and m_y are the linear bending moments in the x and y directions, i.e. moments generated by the normal stresses acting in x and y directions, respectively (and leading to reinforcement in the x and y directions, respectively). Furthermore, m_{xy} is the torsional moment and μ is a factor that can be chosen with respect to practical considerations, usually close to 1. In the above equations, the indices pos and neg refer to the top and bottom of the slab, respectively with the positive z direction pointing to the top.

In addition to the reinforcement moments, associated membrane forces can be evaluated as:

$$n_{rx,pos(neg)} = n_x \pm \mu|n_{xy}| \tag{4.3}$$

$$n_{ry,pos(neg)} = n_y \pm \frac{1}{\mu}|n_{xy}| \tag{4.4}$$

and included in the computation of the reinforcement areas. In equations (4.3) and (4.4) n_x, n_y and n_{xy} are the membrane forces at the mid-surface of the slab. The indices pos and neg refer in this case to tension and compression, respectively.

If the reinforcement directions x and y are not orthogonal, equations (4.1) and (4.2) are replaced by equations (4.5) and (4.6).

$$m_{rx,pos(neg)} = \frac{1}{sin^2\psi}\begin{bmatrix} m_1\sin^2(\psi-\delta) + m_2\cos^2(\psi-\delta) \pm \\ \pm|m_1 \sin\delta \sin(\psi-\delta) - m_2 \cos\delta \cos(\psi-\delta)| \end{bmatrix} \tag{4.5}$$

$$m_{ry,pos(neg)} = \frac{1}{sin^2\psi}\begin{bmatrix} m_1\sin^2\delta + m_2\cos^2\delta \pm \\ \pm|m_1 \sin\delta \sin(\psi-\delta) - m_2 \cos\delta \cos(\psi-\delta)| \end{bmatrix} \tag{4.6}$$

In equations (4.5) and (4.6) m_1 and m_2 denote the principal moments at the considered location and the angles δ and ψ are defined in Figure 4.1.

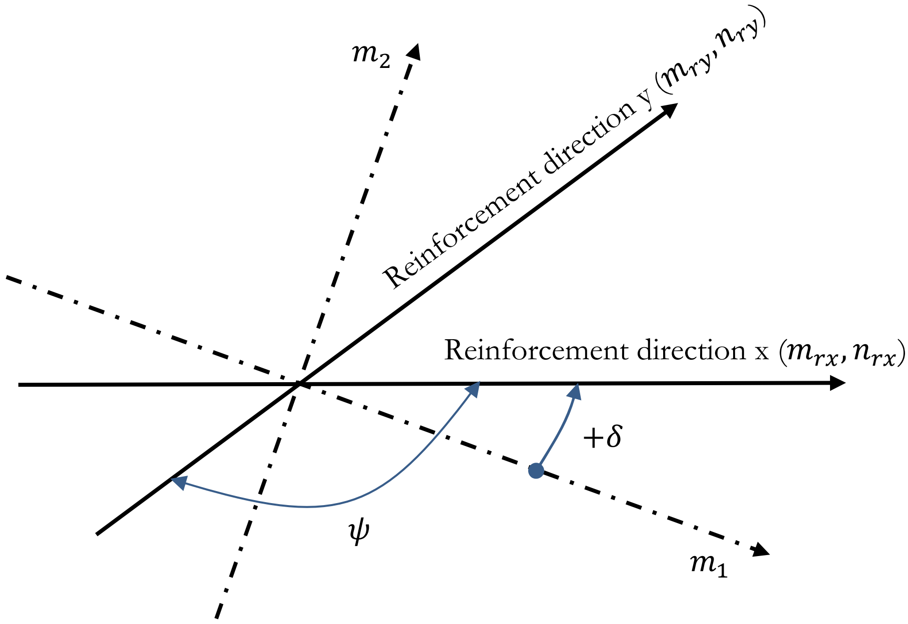


Figure 4.1 Direction definition for skew reinforcement.

Equations (4.3) and (4.4) are modified in the same manner giving:

$$n_{rx,pos(neg)} = \frac{1}{sin^2\psi}\begin{bmatrix} n_1\sin^2(\psi-\delta)+n_2\cos^2(\psi-\delta)\pm \\ \pm|n_1\sin\delta\sin(\psi-\delta)-n_2\cos\delta\cos(\psi-\delta)| \end{bmatrix} \tag{4.7}$$

$$n_{ry,pos(neg)} = \frac{1}{sin^2\psi}\begin{bmatrix} n_1\sin^2\delta+n_2\cos^2\delta\pm \\ \pm|n_1\sin\delta\sin(\psi-\delta)-n_2\cos\delta\cos(\psi-\delta)| \end{bmatrix} \tag{4.8}$$

In equations (4.7) and (4.8) n_1 and n_2 denote the principal membrane forces at the considered location and the angles δ and ψ have the same significance as defined in Figure 4.1 (i.e. δ is the angle between x and the direction of n_1 and ψ is the angle between x and y).

In addition to the moments, the FE analysis of the slab will also provide shear forces in two directions. Any necessary shear reinforcement area should be computed for the resultant shear force defined as:

$$v_0 = \sqrt{v_x^2+v_y^2} \tag{4.9}$$

4.1.3. Redistribution of reinforcement moments

Owing to the capacity of plastic redistributions in concrete structures, the **reinforcement moments** (as well as the shear forces) can be redistributed over a certain width, here denoted ***w***. The average value of the moment m_{av} can then be used to compute the necessary reinforcement which is normally placed within the distribution width.
The procedure can be illustrated for the simple example depicted in Figure 4.2. Consider a slab supported by four columns monolithically connected to the slab. The diagram at the bottom left shows the variation of the reinforcement moment m_{rx} along line L_1 in a direction parallel to the moment's direction (in this case the x direction). The diagram at the right side shows the

distribution of m_{rx} along line L_2 (of length w) in a direction orthogonal to the moment direction (in this case the y direction). The distribution of the moment m_{rx} along line L_2 is replaced by a constant distribution with the average value $m_{rx,av}$ computed according to the equation in Figure 4.2. In this equation the integral is nothing else than the total moment over a strip of width w. The averaging procedure aims then to design the reinforcement in the slab strip of width w for the total moment within the strip and distributing the reinforcement uniformly over the width of the strip. **Note that the averaging procedure described above always takes place in a direction normal to the direction of the moment.**

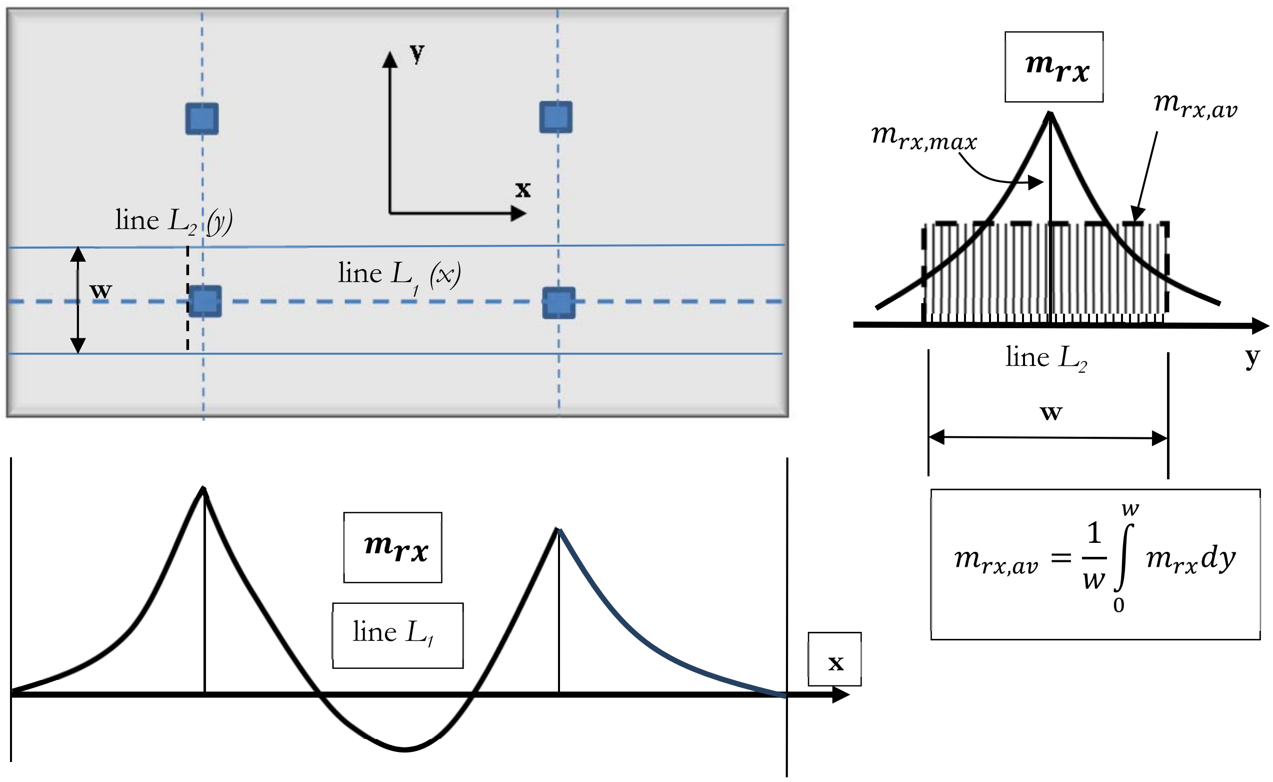


Figure 4.2 Redistribution of the reinforcement moment m_{xt} over a width w.

As an alternative, the averaging over the strip width can instead be made after calculating the corresponding required reinforcement (as continuous fields over the slab), thus giving:

$$A_{sx,av} = \frac{1}{w}\int_0^w A_{sx} dy \tag{4.10}$$

Which approach that is preferred is a question of what is most convenient, for example depending on which approach that is implemented in the software used for the structural analysis.

As a general remark it should be noted that the distribution width (strip width) used for reinforcement design is, at least for ultimate limit states, limited to the width over which yielding of the reinforcement can distribute without exceeding the rotational capacity in the point with the largest rotation. Consequently, what limits the distribution width is the rotational capacity of the slab.

In Sustainable Bridges (2007), the effect of three different design methods with different distribution (strip) widths was compared through non-linear FE analyses of a slab bridge. It was found that the choice of distribution widths had only a minor influence on the response in the ultimate limit state. In fact, the slab design with the widest strips (i.e. strips widths adapted to traffic lane widths rather than the support geometry of the slab) was found to give the smallest plastic rotations of the cases compared. The crack widths for the design load in serviceability limit state were almost the same for the three designs. This implies that distribution widths used for design of slabs can be chosen according to common design guidelines found in codes handbooks, like e.g. Eurocode 2 (SS-EN 1992-1-1:2005, SS-EN 1992-2:2005) and ACI, see McGregor (1992).

4.1.4. One-way and two-way spanning slabs

The recommendations given in this report for the choice of distribution widths for concrete slabs differentiate between two main cases: one-way and two-way spanning slabs. The distinction between these cases is based on the provisions of Eurocode 2 (SS-EN 1992-1-1:2005) which distinguishes between slabs, usually assumed to be supported by line supports, and flat slabs supported by columns. Flat slabs are usually assumed to have a column layout giving span lengths of the same magnitude in two main directions and can, consequently, be regarded as two-way spanning slabs. For slabs, Eurocode 2 distinguishes between one-way and two-way spanning slabs, where one-way spanning slabs are defined as:

"A slab subjected to dominantly uniformly distributed loads may be considered to be one-way spanning if either:
- *it possesses two free (unsupported) and sensibly parallel edges, or*
- *it is the central part of a sensibly regular slab supported on four edges with a ratio of the longer to shorter span greater than 2."*

The definitions are not always directly applicable, particularly not for slab bridges. Bridge slabs are often supported by a combination of line and point supports, or the point supports (like columns and bearings) are arranged in rows giving considerably larger spans in one direction compared to the other. Consequently, we have here chosen to distinguish between three categories with respect to the support arrangement for slabs: two-way, one-way and predominantly one-way spanning slabs.

Typical examples of these categories, applicable to bridges, are shown in Figure 4.3. The slab at the top of the figure (A) is supported by two rows of columns at support lines S_4 and S_5 and by bearings at lines S_1, S_2 and S_3. It has an irregular support arrangement with spans of approximately the same magnitude in the different directions. Consequently, it is considered to be a two-way spanning slab.

The slab at the middle of the figure (B) is continuously supported at lines S_1, S_2, and S_3 and it possesses two free (unsupported) and parallel edges E_1 and E_2. Thus, according to the definition for slabs in Eurocode, it is a one-way spanning slab.

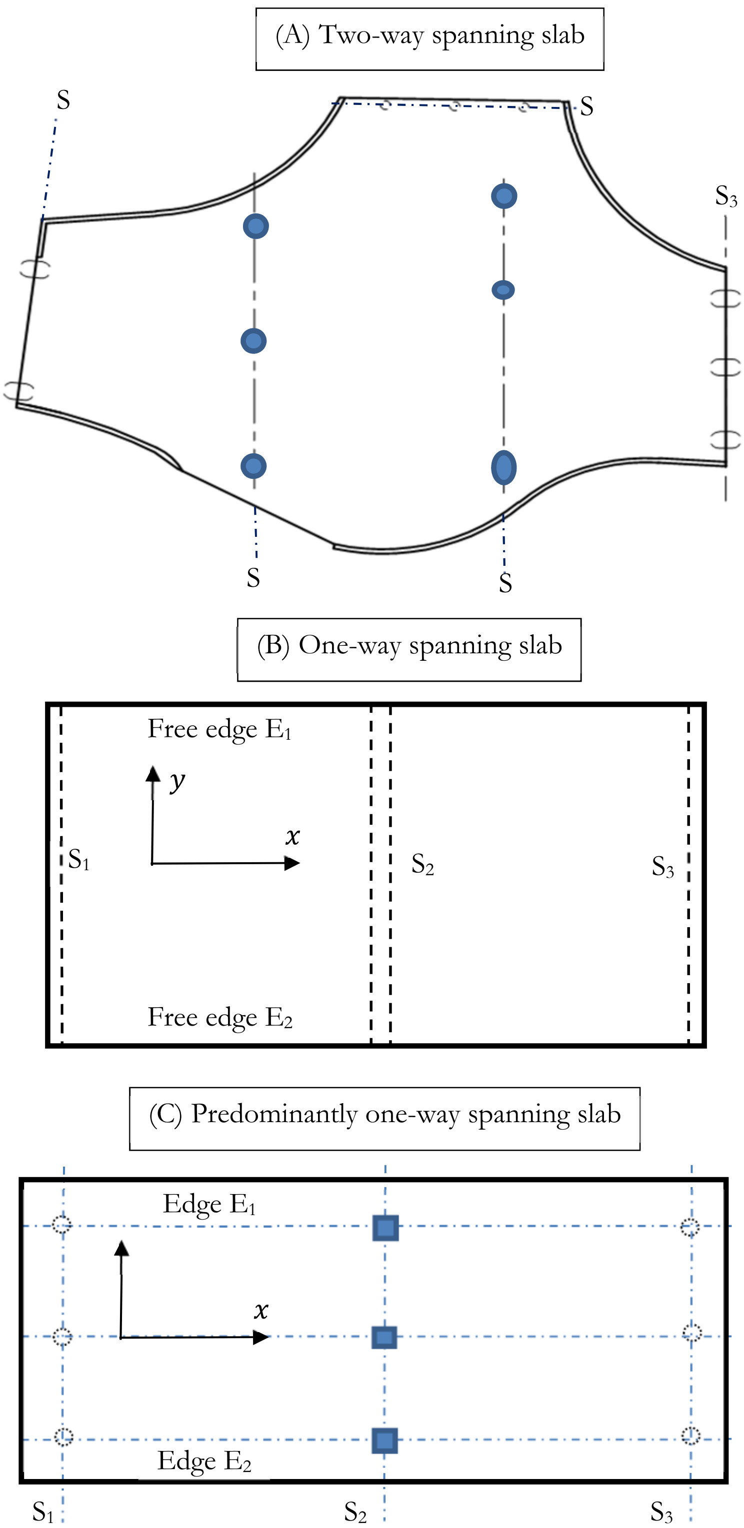


Figure 4.3 *Examples of two-way (top) one-way (middle) and predominantly one-way spanning slabs.*

The slab at the bottom of the figure (C) has a geometry that is typical for many bridge slabs; it has point supports arranged along support lines with considerably smaller distances between the point supports here. These kinds of slabs are here called predominantly one-way spanning slabs. With this geometry the situation is somewhat in between the two limiting cases (A) and (B). For instance, if point loads are placed in-between the columns of line S_2 these will be transferred to the nearest columns in y direction. However, for distributed loads and most point load positions, the slab transfers the applied loads predominantly in one direction, the x direction. On the other hand, with modified geometrical configurations – i.e larger width to length ratio, sparser placement of point supports, shorter distance between support lines, etc. – the slab can be assumed to be a two-way spanning slab.

The cantilever slab is frequent for bridge structures. Here, the loads are mainly carried in the transversal direction of the bridge, to the main load carrying bridge girders. However, the distribution of the load in the other direction is essential for the response. These slabs are therefore treated separately in Section 4.4.

4.2. Distribution widths for reinforcement moments

The recommendations given here are applicable for slabs supported on concentrated supports, like columns or bearings, or loaded with large concentrated forces, like traffic loads. For slabs with distributed loads and line supports, more liberal distribution widths can be used.

The recommendations given in this section apply to reinforcement moments and associated membrane effects. They are based on the provisions given in Eurocode 2 (SS-EN 1992-1-1:2005) but, since no specific guidelines are given for redistribution of moments and forces from linear FE analysis, the more detailed advices are based on what has been found in literature and on practical considerations from engineering practice.

In literature, it is not possible to find detailed, scientifically based advices for moment redistributions from linear FE analysis and choice of distribution widths. The recommendations found are generally based on the assumption that reinforced concrete slabs have good capabilities for plastic redistributions in ultimate limit state, but that the reinforcement need to be concentrated to regions with concentrated supports with respect to the response in service state. For flat slabs, the reinforcement is typically arranged in support strips over the columns with a middle strip in between, in the two main directions. These recommendations are quite liberal compared to the recommendations given here.

For bridges in Sweden, more rigorous demands have been applied for redistribution of moments and forces from linear FE analysis, Davidson (2003), Bro 2004 (2004). Bridges are often more heavily reinforced than flat slabs in buildings and more rigorous demands may be motivated here. Nevertheless, in Sustainable Bridges (2007) it was found that the distribution widths had only a minor influence on the response in the ultimate limit state and that they could be chosen according to common design guidelines. For service loads, no correlation between distribution widths and crack widths were found. The recommendations given here are based on these findings, but the lack of a solid knowledge foundation has motivated a certain caution.

The recommendations given here are believed to be conservative. This implicates that there is a potential to improve them and to find more liberal provision based on improved knowledge on the response of concrete slabs.

4.2.1. Ultimate limit states

The distribution widths at a support (column or bearing) can be chosen according to the recommendations below:

$$w = \min(3h, \frac{L_c}{10}) \quad \text{for } \frac{x_u}{d} = 0.45 \text{ (0.35 for concrete strength classes} \geq \text{C55/67)} \tag{4.11}$$

$$w = \min\left(5h, \frac{L_c}{5}\right) \quad \text{for } \frac{x_u}{d} = 0.30 \text{ (0.23 for concrete strength classes} \geq \text{C55/67)} \tag{4.12}$$

$$w = \frac{L_c}{4} \quad \text{for } \frac{x_u}{d} = 0.25 \text{ (0.15 for concrete strength classes} \geq \text{C55/67)} \tag{4.13}$$

$$w = \frac{L_c}{2} \quad \text{for } \frac{x_u}{d} = 0.15 \text{ (0.10 for concrete strength classes} \geq \text{C55/67)} \tag{4.14}$$

$$w = \min\left(5h, \frac{L_c}{5}\right) \quad \text{for } \frac{x_u}{d} = 0.0 \tag{4.15}$$

In the above equations, h is the height of the section, x_u is the depth of the neutral axis at the ultimate limit state after redistribution and d is the effective depth of the section. L_c is the characteristic span width, determined differently for different categories of slabs in the following sections. For values of $\frac{x_u}{d}$ in between the limits above w can be determined by linear interpolation. Some comments concerning the above limits and deformation capacity requirements can be found in Appendix B.

Regardless of the ductility requirements or any other of the limitations defined in the reminder of this report the value of the distribution width should never be taken less than $2h + a$, i.e. $w \geq w_{min} = 2h + a$, where a is the dimension of the support in the considered direction.

The following limitations apply for the distribution width determined using equations (4.11) to (4.15):

1. The ratio of the averaged and maximum reinforcement moments (see also Figure 4.2) should be restricted to $\delta = \frac{m_{rx,av}}{m_{rx,max}} \geq 0.6$.
2. If the column has a capital (or a drop panel) the distributions width should be chosen as shown in Figure 4.4. In addition, before redistribution, the reinforcement moments and associated membrane forces must be transformed so that they are defined with respect to the same reference line.
3. If the capital (drop panel) extends continuously over a line of columns or bearings it can be dimensioned as a beam. The beam forces (normal forces, bending moments and shear forces) can be obtained by integration from the shell (slab) results.

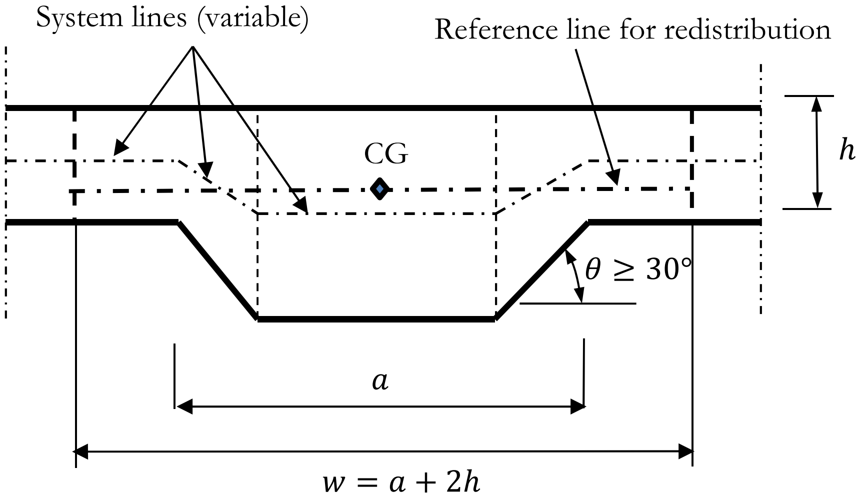
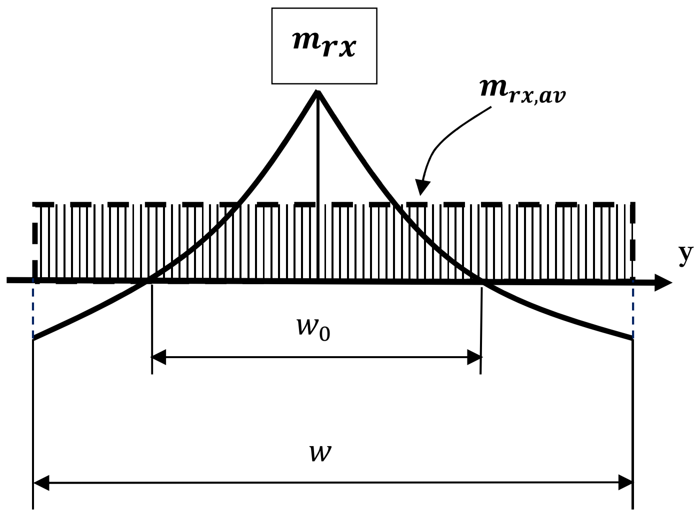


Figure 4.4 Distribution widths for capitals (or drop panels).

4. If the distribution width exceeds the distance between points of zero moment w_0 in the direction normal to the direction of the considered moment (i.e. the direction of redistribution) then the average value should be computed according to the principle illustrated in Figure 4.5. This principle is illustrated for the reinforcement moment in the x direction, i.e. m_{rx} but the same applies for m_{ry}.



$$m_{rx,av} = \frac{1}{w}\int_{0}^{w_0} m_{rx} dy$$

Figure 4.5 Definition of the average value for cases where the distribution width exceeds the distance between points of zero moment.

5. For supports placed near the edge of the slab, the distribution width should be evaluated according to the principle illustrated in Figure 4.6. This amounts to choosing a w according to equations (4.11) to (4.15) and then evaluating an effective width w_{eff} as indicated in the figure. This effective value should further be used in evaluating the averaged moment values.

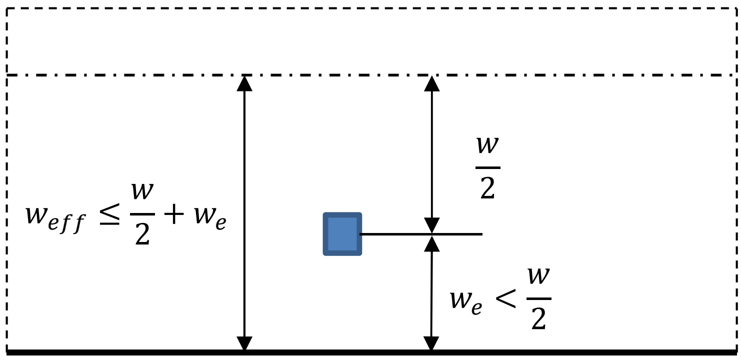


Figure 4.6 Support near the edge of the slab.

These distribution widths can also be used for averaging the associated membrane forces as defined in equations (4.3) and (4.4) (or alternatively equations (4.7) and (4.8) for the case of non-orthogonal reinforcement directions).

4.2.1.1. Distribution widths for two-way spanning slabs

For two-way spanning slabs the characteristic span width L_c is determined according to:

$$L_c = \frac{L_{min,1} + L_{min,2}}{2} \tag{4.16}$$

where $L_{min,1}$ and $L_{min,2}$ are the distances from the considered column to adjacent columns in the directions with the shortest column distances. Thus, referring to Figure 4.7, the characteristic span width L_c is determined by examining the distances L_1 to L_7 and selecting the two shortest ones.

The distribution widths determined in the way described above are valid in both x and y directions. The distribution width for points between columns, for instance in between P_1 and P_2, can be chosen at intermediate values between the values at the two columns.

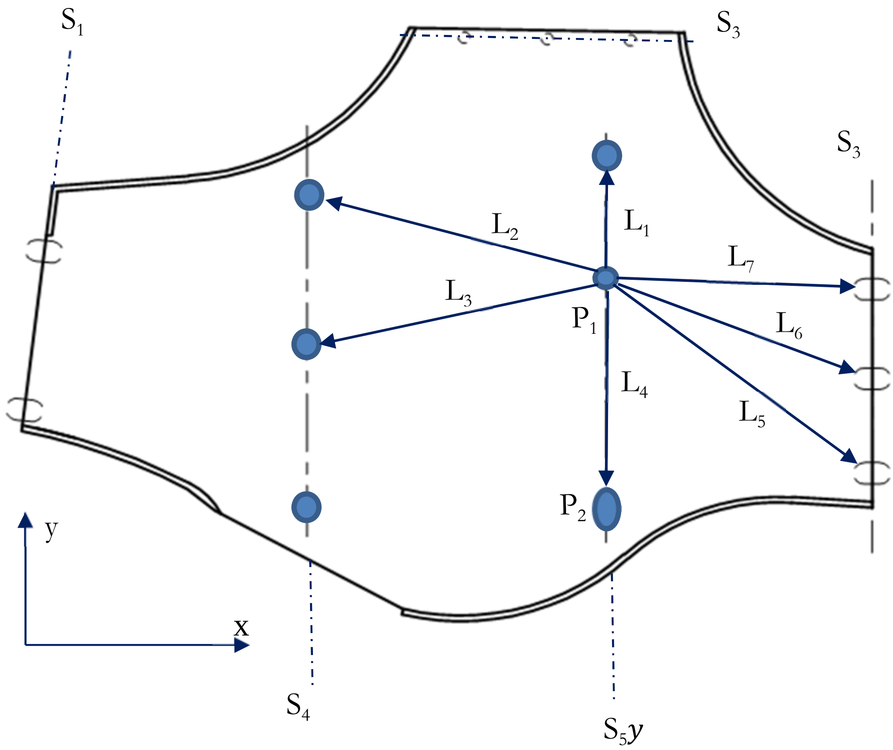


Figure 4.7 Choice of distribution widths for two-way slabs – characteristic span width L_c.

4.2.1.2. Distribution widths for predominantly one-way spanning slabs

For predominantly one-way spanning slabs, the equations (4.11) to (4.15) in Section 4.2.1 are applied differently depending on whether the averaging procedure refers to effects in the longitudinal, main load carrying direction of the structure (*x* direction) or to effects in the transversal direction (*y* direction), see Figure 4.8.

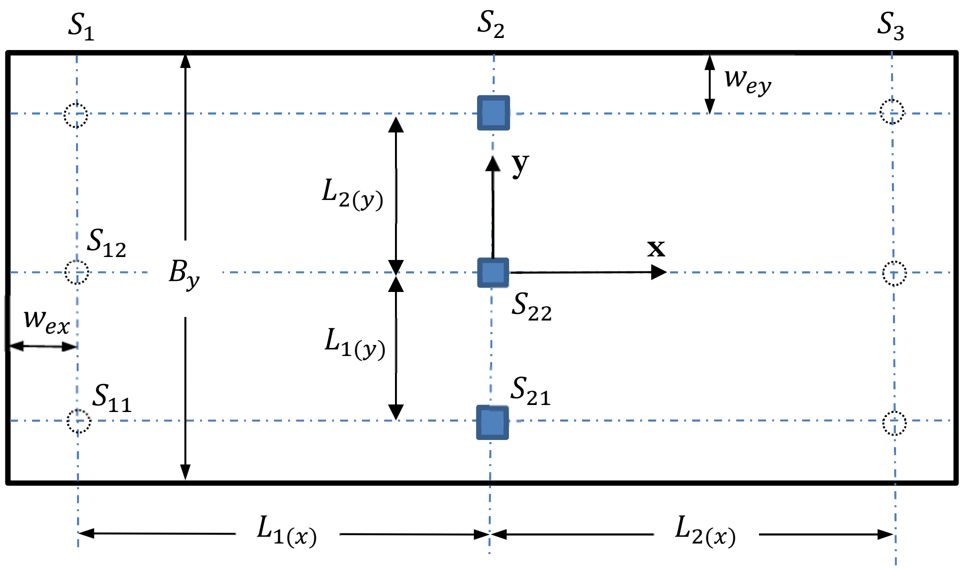


Figure 4.8 Predominantly one-way spanning slab bridge.

Distribution width for the moment in longitudinal direction

The distribution width w_y for the reinforcement moment m_{rx} (and associated membrane force n_{rx}) in equations (a) to (e) can be determined with the characteristic span width L_c computed according to:

- For the supports in line S_1: $L_c = L_{1(x)}$
- For the supports in line S_2: $L_c = \frac{L_{1(x)}+L_{2(x)}}{2}$

In addition to the general restrictions defined in equations (4.11) to (4.15), in Section 4.2.1, the distribution width should also respect the condition:

$$w_y \leq B_y/2 \quad (4.17)$$

Distribution width for the moment in transversal direction

The distribution width w_x for the reinforcement moment m_{ry} (and associated membrane force n_{ry}) in equations (4.11) to (4.15) can be determined with the characteristic span width L_c computed according to:

- For support S_{11} in line S_1: $L_c = \frac{L_{1(x)}+L_{1(y)}}{2}$
- For support S_{12} in line S_1: $L_c = \frac{L_{1(x)}+L_{m(y)}}{2}$; $L_{m(y)} = \frac{L_{1(y)}+L_{2(y)}}{2}$
- For support S_{21} in line S_2: $L_c = \frac{L_{m(x)}+L_{1(y)}}{2}$; $L_{m(x)} = \frac{L_{1(x)}+L_{2(x)}}{2}$
- For support S_{22} in line S_2: $L_c = \frac{L_{m(x)}+L_{m(y)}}{2}$; with $L_{m(x)}$ and $L_{m(y)}$ as above.

In addition to the general restrictions defined in Section 4.2.1 (points 1 to 5), the distribution width should also respect the condition:

$$w_x \leq \frac{L_{1(y)} + L_{2(y)}}{2} \quad (4.18)$$

4.2.1.3. Distribution widths for one-way spanning slabs

For one-way spanning slabs supported on line supports, for example single span integral bridges, see Figure 4.9, the distribution widths w_y and w_x are determined with the characteristic span width L_c computed according to:

Distribution width for the moment in longitudinal direction

For the distribution width w_y for moment in longitudinal *x* direction m_{rx}:

$$L_c = L_x \tag{4.19}$$

In addition to the general restrictions defined in equations (4.11) to (4.15), in Section 4.2.1, equations (4.17) and (4.18) also applies.

Distribution width for the moment in transversal direction

For the distribution width w_x for moment in transversal *y* direction m_{ry}:

$$L_c = B_y \tag{4.20}$$

In addition to the general restrictions defined in equations (4.11) to (4.15), in Section 4.2.1, the distribution width is limited to:

$$w_x \leq \frac{L_x}{2} \tag{4.21}$$

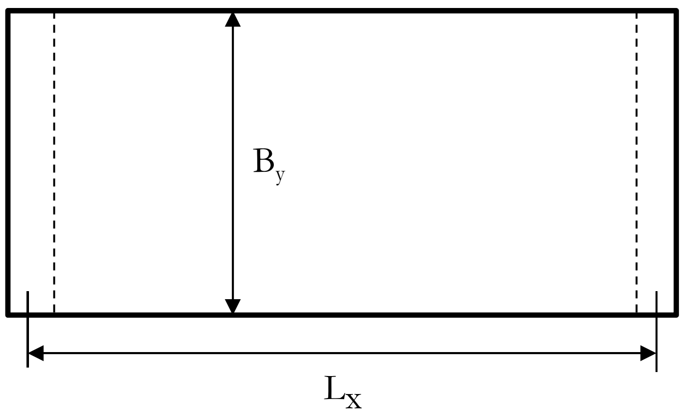


Figure 4.9 One-way spanning bridge (here, a single span integral bridge).

4.2.2. Serviceability limit states

The choice of an appropriate distribution width for serviceability limit states is by far more intricate than for ultimate limit states and there are very few (if any) recommendations in the literature. This is mainly due to the fact that for serviceability limit states it is very difficult to determine the degree to which moment redistribution will take place. When a slab starts to crack, moments will redistribute from cracked areas to un-cracked areas (from support to field sections or vice versa). When the whole slab is cracked, the stiffer parts of the slab will attract larger moments. This means that the parts with larger maximum moments will contain a higher amount of reinforcement and hence become relatively stiffer after cracking. Consequently these parts will attract larger parts of the total moment after cracking than before.

In Eurocode 2 (SS-EN 1992-1-1:2005), it is pointed out that the reinforcement distribution should reflect the behaviour of the slab under working conditions, with a concentration of moments over the column. Unless rigorous checks are made for serviceability, half of the total top reinforcement should be concentrated into a column strip with the width:

$$w = \frac{l_1}{8} + \frac{l_2}{8} \tag{4.22}$$

where l_1 and l_2 are the distances from the column of the strip to the adjacent columns, in the direction perpendicular to the reinforcement. This leads generally to a larger concentration of reinforcement to the column strip than what is given by a linear analysis.

Given the above reasons the distribution width for serviceability limit states should be chosen more conservative than for ultimate limit states. Thus, for serviceability limit states the distribution width should be chosen between the limits given by equations (4.16) and (4.17) in Section 4.2.1:

$$\min(3h, \frac{L_c}{10}) \leq w \leq \min(5h, \frac{L_c}{5}) \tag{4.23}$$

4.3. Distribution widths for shear forces

Similarly to reinforcement moments and associated membrane effects, no specific guidelines are given for redistribution of shear forces from linear FE analysis in Eurocode 2 (SS-EN 1992-1-1:2005). Furthermore, very little scientifically based information has been found in literature on the subject. The recommendations found are generally based on the assumption that reinforced concrete structures have good capabilities for plastic redistributions in ultimate limit state and that the distribution widths for shear forces can be chosen similarly to the distribution widths for reinforcement moments. For bridges in Sweden, somewhat more rigorous demands have been used, Davidson (2003), Bro 2004 (2004).

The recommendations given here are based on what was found in literature, combined with engineering judgement and considerations from engineering practice. The recommendations are believed to be conservative, implicating a potential for improvement based on increased knowledge on the response and distribution of shear in concrete slabs and how this is reflected by linear FE analysis. In this context it must be emphasized once again that there may also be other approaches that can be shown to be equally reasonable as the ones presented here. Such alternatives may also be preferred, depending on the software used for structural analysis.

The recommendations for redistribution refer in this case to the resultant shear force v_0 as defined by equation (4.9) in Section 4.1.2. It should also be noted that the redistribution is performed in a direction which is orthogonal to the direction of the resultant shear force, see Figure 4.10.

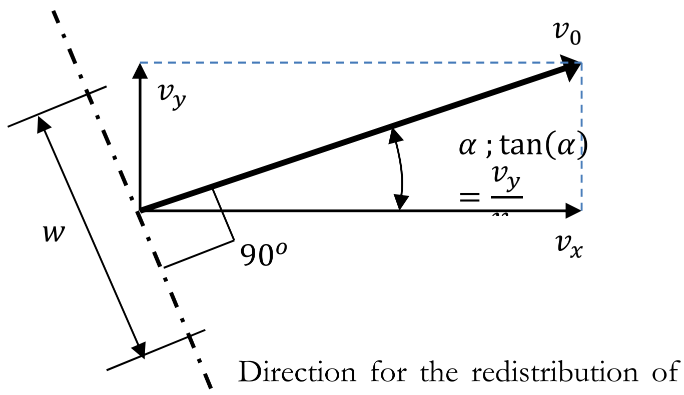


Figure 4.10 Redistribution of the resultant shear force.

The distribution width for shear forces can in principle be chosen as equal to that used for the reinforcement moments. Apart from the limiting equations (4.11) to (4.15), in Section 4.2.1, two more conditions should be observed:

1. The distribution width for the shear force should not exceed $5h$ where h is the thickness of the slab at the considered section.

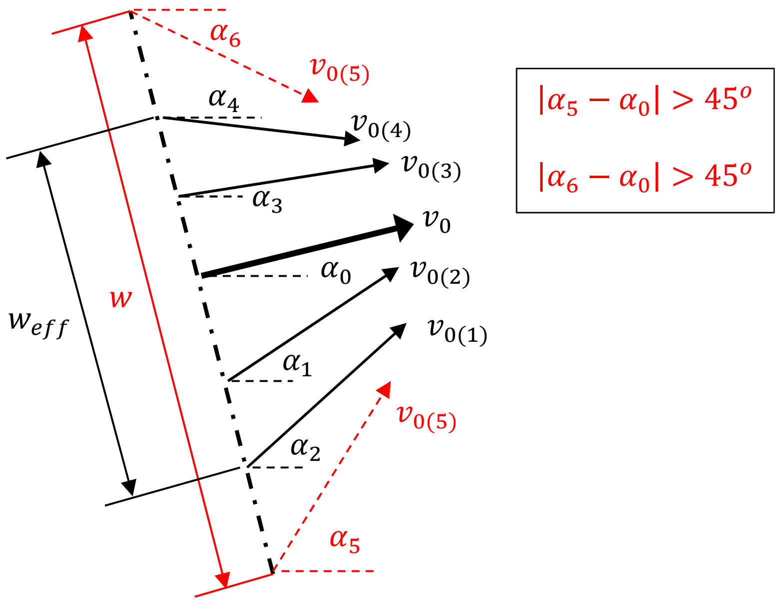


Figure 4.11 Variation of the direction of the resultant shear force within a distribution width.

2. The distribution width should be restricted to a variation of the angle α of less than 45°. Mathematically this can be expressed as $|\alpha_i - \alpha_0 | \leq 45^o$. Thus, considering the example in Figure 4.11, the distribution width w is restricted to w_{eff} since the above condition is not fulfilled for the points outside w_{eff}.

Note: The mean value of the resultant shear force after redistribution $v_{0,av}$ shall be used to without any further reduction for the effect of forces near supports as defined in Eurocode 2 (SS-EN_1992-1-1, Section 6.2.2 (6)).

From a FE model it may be somewhat cumbersome to determine the shear forces along a line perpendicular to an arbitrary direction. Hence, as an alternative to use the direction of the resultant force v_0 the following method, based on global directions, is proposed. In Figure 4.12 a schematic case with global coordinates x and y is shown. Using shear force components in the global x and y direction the resultant shear force $v_{0,i}$ is determined in each section point i as

$$v_{0,i} = \sqrt{v_{x,i}^2 + v_{y,i}^2} \tag{4.24}$$

and its angle α_i to the x axis is determined as:

$$\alpha_i = \arctan\left(\frac{v_{y,i}}{v_{x,i}}\right) \tag{4.25}$$

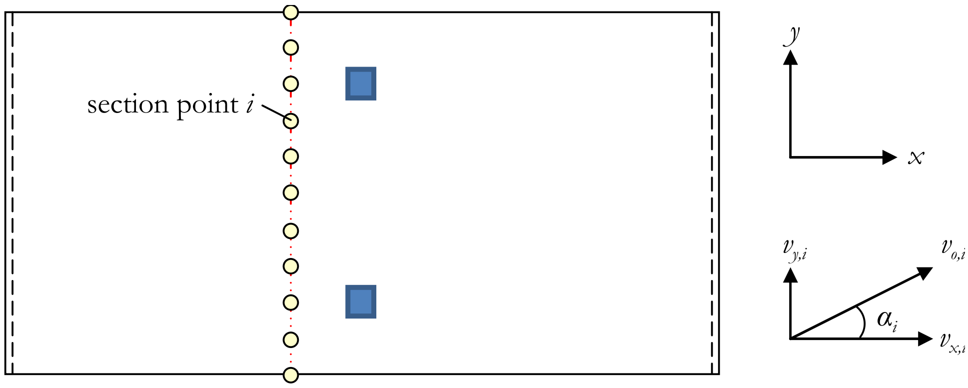


Figure 4.12 Variation of the direction of the resultant shear force within a distribution width.

The capacity control is made in the global x and y directions. However, design shear forces v_{dx} and v_{dy}, based on the resultant shear force v_0, are used for each section point i studied. Following a proposal from Marti (2000) and also Davidson (2003), if the angle of the resultant α_i is less than or equal to 45° the designing shear force v_d is approximated to act in the x direction and if α_i is larger than 45° it is assumed to act in the y direction; i.e.:

$$\alpha_i = \leq 45^\circ \rightarrow \begin{cases} v_{dx,i} = v_{0,i} \\ v_{dy,i} = 0 \end{cases} \tag{4.26}$$

$$\alpha_i => 45^\circ \rightarrow \begin{cases} v_{dx,i} = 0 \\ v_{dy,i} = v_{0,i} \end{cases} \tag{4.27}$$

How the resultant shear force $v_{0,i}$ is determined depends on how different load positions are treated in the FE analysis. However, often an envelope of several load cases is used for a certain section point and in such a case two resultant shear forces $v_{01,i}$ and $v_{02,i}$ may be determined as

$$v_{01,i} = \sqrt{{v_{x,max,i}}^2 + {v_{y,cor,i}}^2} \tag{4.28}$$

$$v_{02,i} = \sqrt{{v_{x,cor,i}}^2 + {v_{y,max,i}}^2} \tag{4.29}$$

and their corresponding angles a_i as:

$$\alpha_{1,i} = \arctan\left(\frac{v_{y,cor,i}}{v_{x,max,i}}\right) \tag{4.30}$$

$$\alpha_{2,i} = \arctan\left(\frac{v_{y,max,i}}{v_{x,cor,i}}\right) \tag{4.31}$$

Here the index max indicates the maximum value while the index cor indicates its corresponding value, i.e. to the load case that causes the maximum value. From this, four different cases can be identified from which the design shear forces v_{dx} and v_{dy} are determined, see Table 4.1.

Table 4.1 Determination of design shear forces $v_{dx,i}$ and $v_{dy,i}$ in section point i as a function of the resultant angles $\alpha_{1,i}$ and $\alpha_{2,i}$.

Case	$\alpha_{1,i}$	$\alpha_{2,i}$	$v_{dx,i}$	$v_{dy,i}$
(1)	$\leq 45°$	$\leq 45°$	$\max(v_{01,i}, v_{02,i})$	0
(2)	$\leq 45°$	$> 45°$	$v_{01,i}$	$v_{02,i}$
(3)	$> 45°$	$\leq 45°$	$v_{02,i}$	$v_{01,i}$
(4)	$> 45°$	$> 45°$	0	$\max(v_{01,i}, v_{02,i})$

It is possible to distribute the design shear forces v_{dx} and v_{dy}. Such a distribution is then made in a direction perpendicular to the acting shear force; i.e. in the y direction for v_{dx} and in the x-direction for v_{dy}. However, such a distribution is only allowed into a region in which the design shear force has the same direction as the force being distributed. Relating this to the schematic case shown in Figure 4.13 this means that the distribution of the force v_{dx} will be limited to those section points along the marked distribution line in which $v_{dx,i} \neq 0$. Further, the limitations of distribution widths given above are still valid.

Hence, the distribution width in the y direction for the design shear force v_{dx} will be limited by a section point in which $v_{dx,i} = 0$ as schematically shown in Figure 4.13.

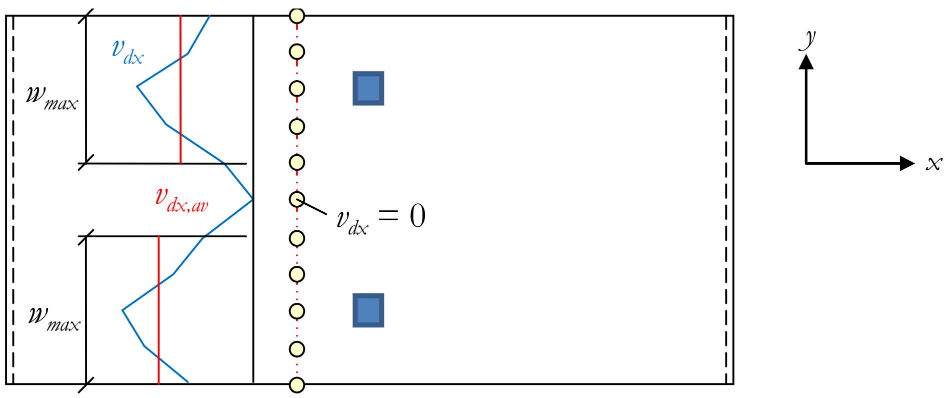


Figure 4.13 Schematic illustration of distribution width limitation due to $v_{dx,i} = 0$ in a section point.

4.4. Cantilever slabs

This chapter deals with the analysis and dimensioning of cantilever slabs, which is a problem relevant to composite bridges or concrete beam bridges. Some typical examples of cantilever slabs in bridge decks are shown in Figure 4.14.

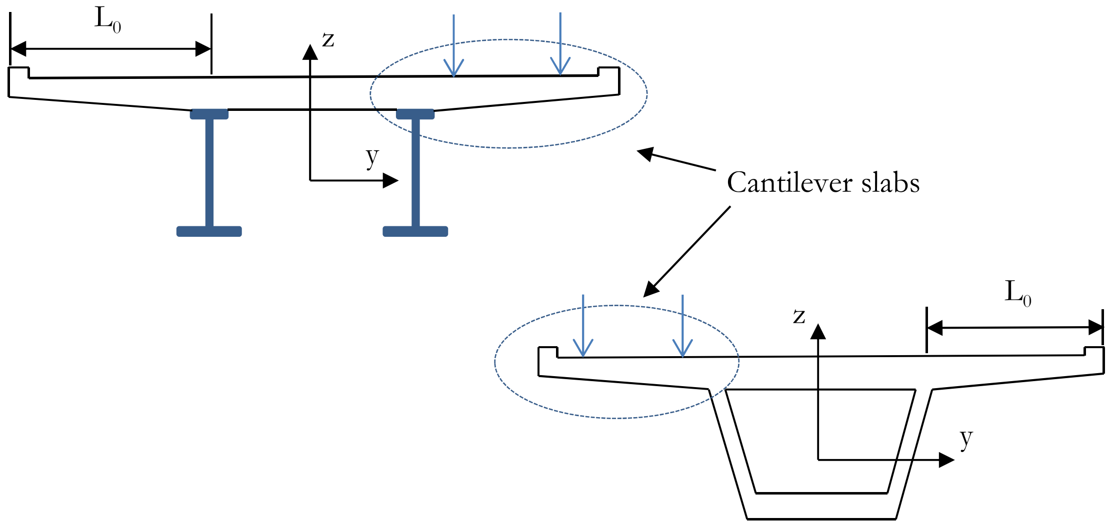


Figure 4.14 Cantilever slab – load transfer in transversal direction.

In literature, specific recommendations were found for cantilever slabs subjected to concentrated loads, like traffic point loads, which motivated their treatment in a separate section. The recommendations given here are mainly based on recommendations found in Swedish handbooks, like BBK 04 (2004) and Davidson (2003). These are originally based on tests performed by Hedman and Losberg (1976). The recommendations on the distribution of the shear force were also compared to a test performed by Vaz Rodrigues *et al.* (2007) by Shams Hakimi (2012), which for this case indicated their reasonableness. The recommendations given are not believed to be conservative in the same way as the recommendations in Sections 4.2 and 4.3. However, increased knowledge may still bring forth improved recommendations.

The issues addressed in this section refer only to the load transfer in the transversal *y* direction (i.e. m_{ry}, v_0) of the cantilever slab. For the purpose of computing the reinforcement area in transversal *y* direction a simplified model can be used. Such a model is shown in Figure 4.15. The model consists of a slab which is assumed as clamped at one end and free at the other. The width of the slab is equal to L_0 (see Figure 4.14) while the length of the slab L_x is equal to the length of the bridge. If the cantilever has an edge beam, the edge beam can also be included in the model. The axle loads can be modelled either as concentrated or distributed loads. The loading is applied at the centre of the model as shown Figure 4.15.

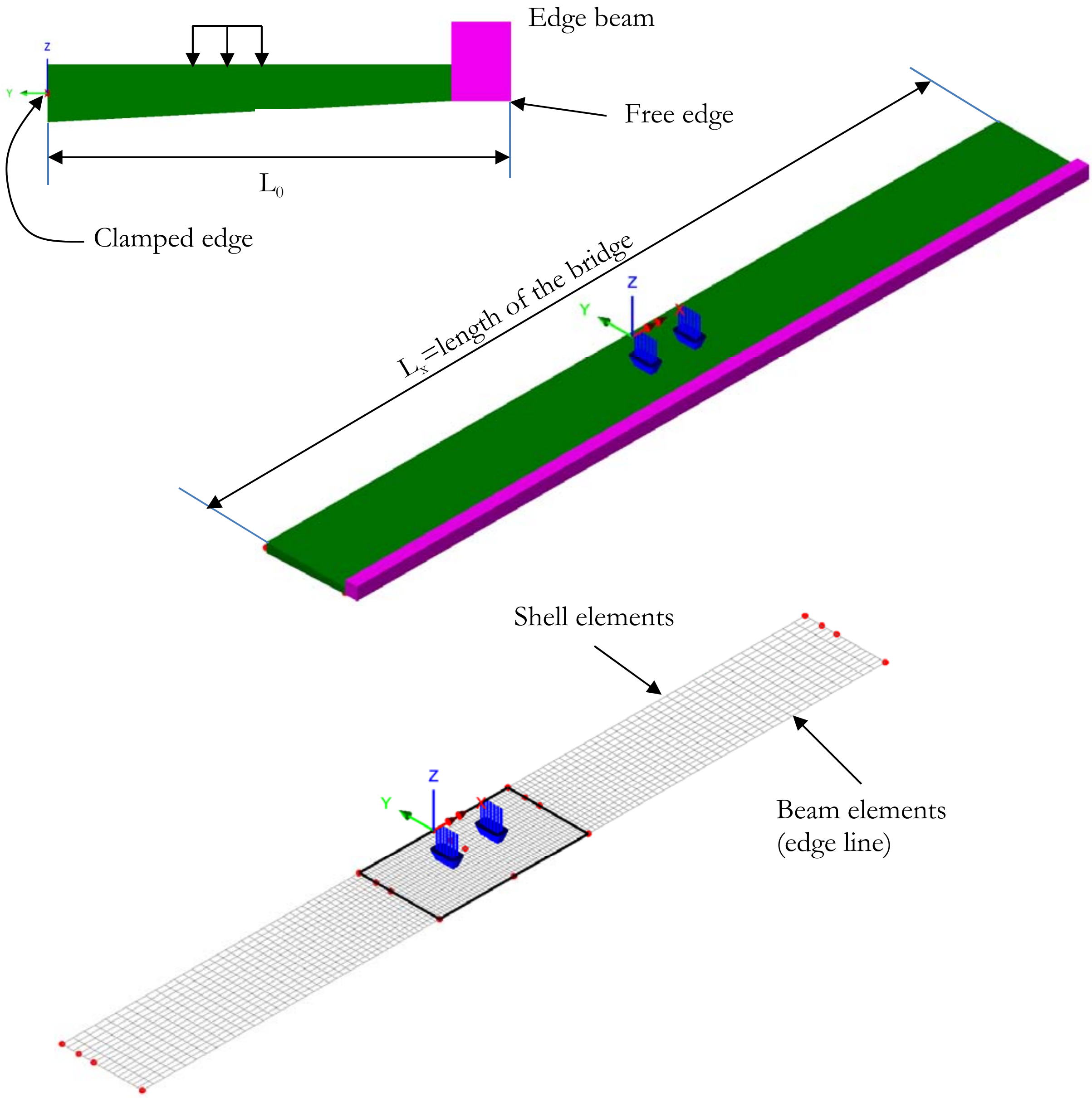


Figure 4.15 Simplified model for the analysis of cantilever slabs – effects in transverse direction.

For the design of the cantilever slab, the result sections can be chosen in accordance with Chapter 3. Consequently, the position of the result sections depends on the stiffness provided by the supporting main girders. For cantilever slabs supported on steel beams, the flange may provide a very low support stiffness for the slab compared to the web; in such cases a conservative approach is to choose the section over the web as the clamped-in section for the slab as shown in Figure 4.14.

4.4.1. Distribution width for moments in transversal direction

The redistribution refers to the reinforcement moment in transversal direction m_{ry}, as defined in equation (4.2) in Section 4.1 and is performed in the longitudinal x direction.

For a single load F, see Figure 4.16, the distribution width w_x for ultimate limit states is given by:

$$w_x = \min\begin{cases} 7d + b + t \\ 10d + 1.3y_{cs} \end{cases} \tag{4.32}$$

for $0.25 \geq \frac{x_u}{d} \geq 0.15$ ($0.15 \geq \frac{x_u}{d} \geq 0.10$ for concrete classes ≥ C55/67).

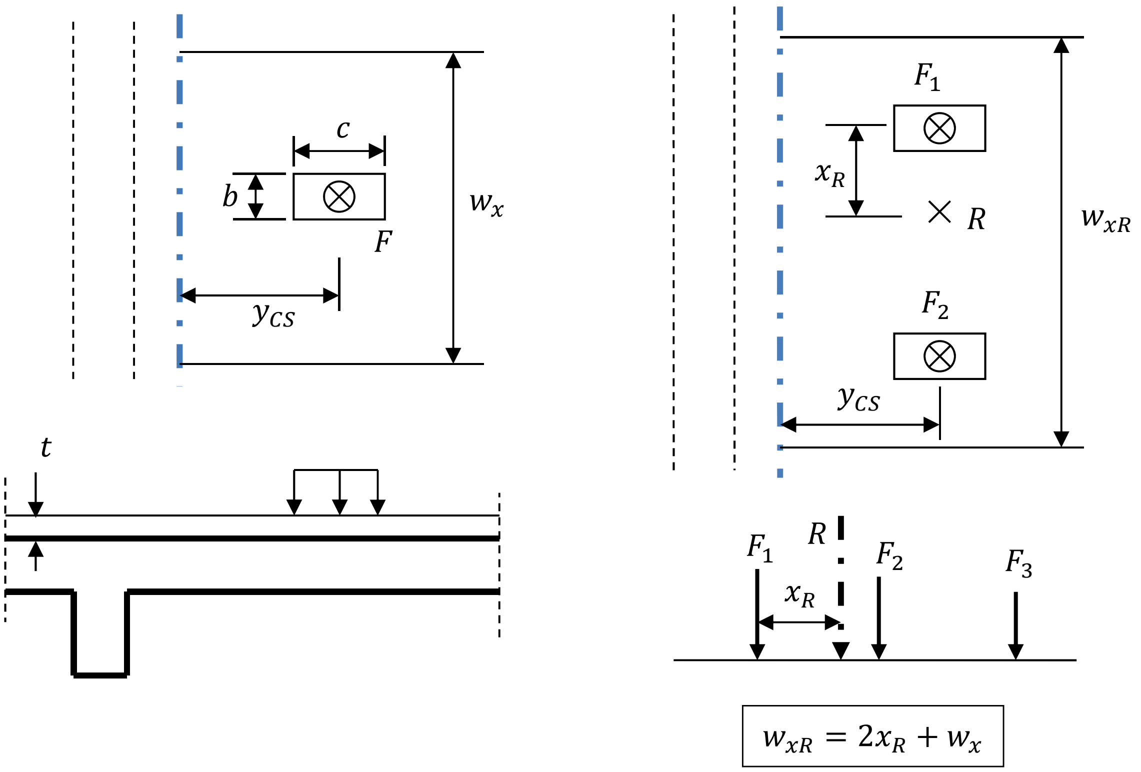


Figure 4.16 Distribution width for the reinforcement moment m_{ry} – one force (left); two or several forces (right).

For values of $\frac{x_u}{d}$ outside the above limits the distribution width will be taken as:

$$w_x = 2h + b + t \tag{4.33}$$

The distribution width for serviceability limit states is also given by equation (4.33).

In the above equations:

- x_u = the depth of the neutral axis at the ultimate limit state after redistribution. x_u should be evaluated for the section with the highest reinforcement ratio.
- d = the effective depth at the critical cross section
- h = the height of the cantilever at the critical cross section
- b = the width of the load (see Figure 4.16)
- t = the thickness of the surfacing (see Figure 4.16)
- y_{CS} = the distance from the centre of the load to the critical cross section.

For loading situations involving several forces situated close enough so that the individual distributions widths overlap, a distribution width w_{xR} for the whole group of forces can be computed according to the principle shown in Figure 4.16.

4.4.2. Distribution width for shear forces

The redistribution refers to the resultant shear force v_0 as defined in equation (4.9) in Section 4.1.2 and is performed in the longitudinal x direction. For a single force or a group of forces situated in the same row, the critical cross section for shear forces is always placed at a distance $y_{CS} = \frac{c+d}{2}$ as shown in Figure 4.17. For two rows of forces, the critical cross section is chosen according to the principle illustrated in this figure.

Depending on the position y of the critical cross section on the console, the distribution width $w_{x(v_0)}$ can be determined through linear interpolation between the max and min values shown below:

$$w_{max} = \max\begin{cases} 7d + b + t \\ 10d + 1.3y_{CS} \end{cases} \quad \text{for } y = 0 \tag{4.34}$$

- $$w_{min} = \min\begin{cases} 7d + b + t \\ 10d + 1.3y_{CS} \end{cases} \quad \text{for } y = y_{max} \tag{4.35}$$

where y_{max} is defined according to Figure 4.18.

For loading situations involving several forces situated close enough so that the individual distributions widths overlap, a distribution width $w_{xR(v_0)}$ for the whole group of forces can be computed according to the same principle illustrated in Figure 4.16.

The limitation 2 in Section 4.3 applies in this case also. However, for this case the limiting condition is defined according to Figure 4.19.

Note: The mean value of the resultant shear force after redistribution $v_{0,av}$ shall be used to dimension the necessary reinforcement area without any further reduction for the effect of forces near supports.

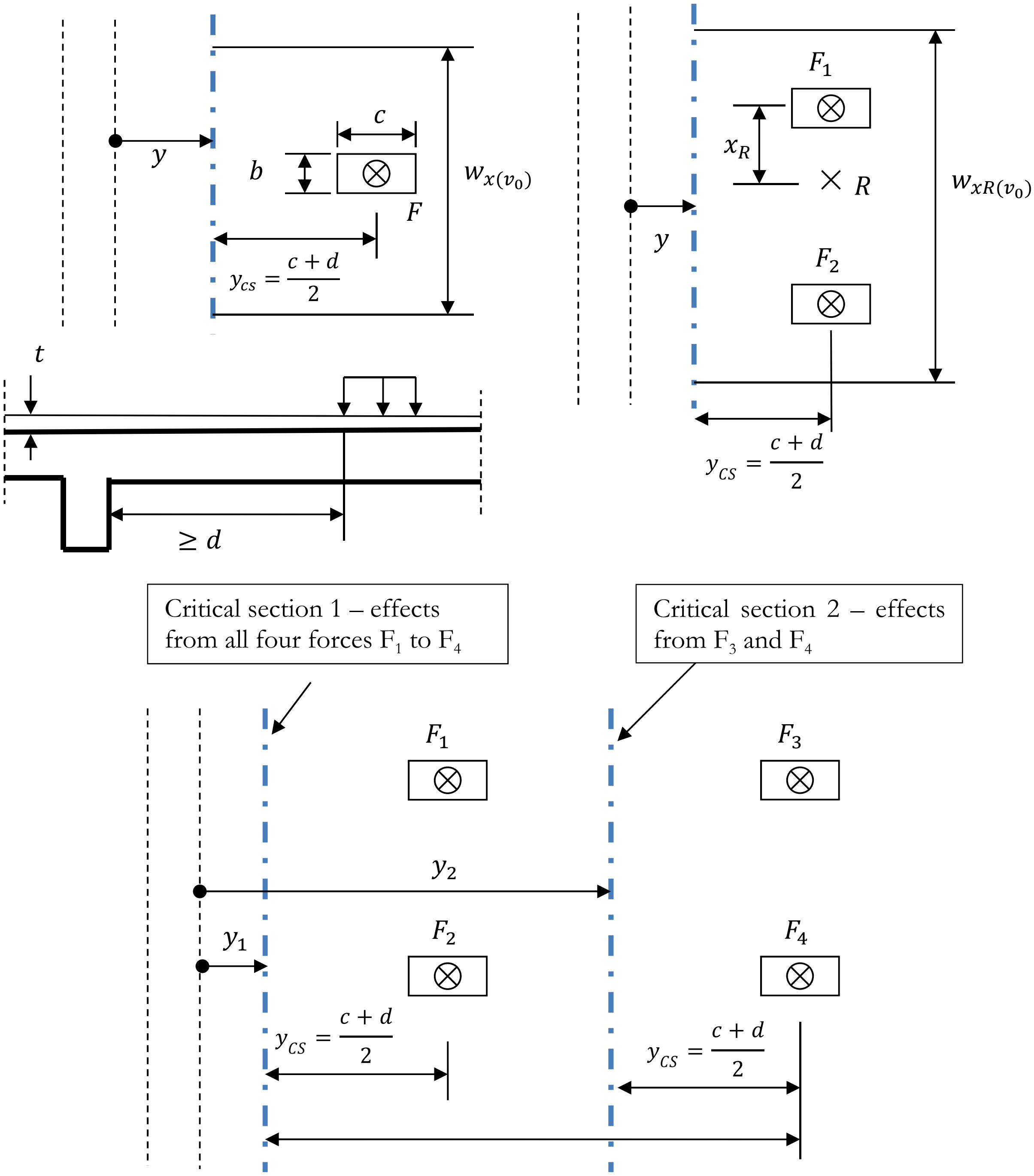


Figure 4.17 Critical sections for shear forces – a single force (upper left), several forces on the same row (upper right) and a group of four forces (lower centre).

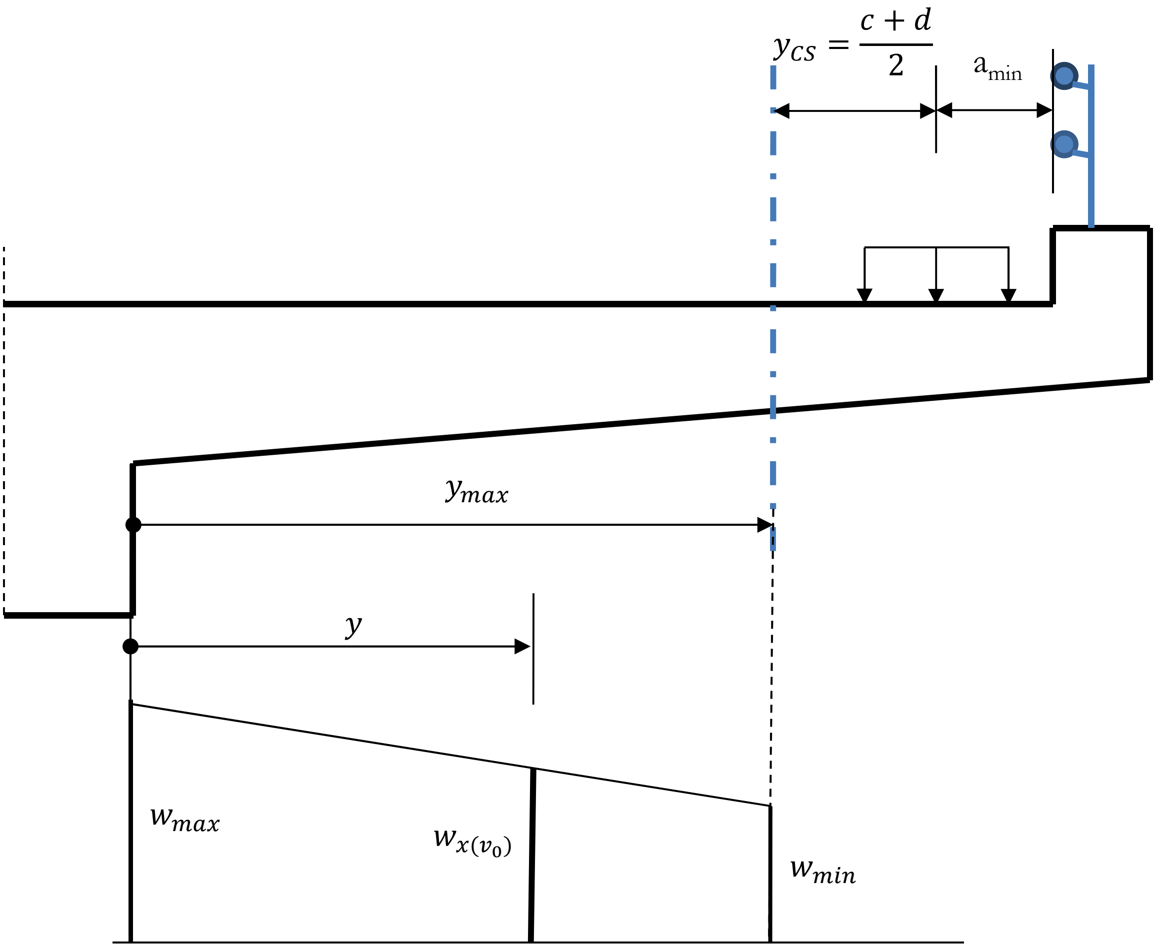
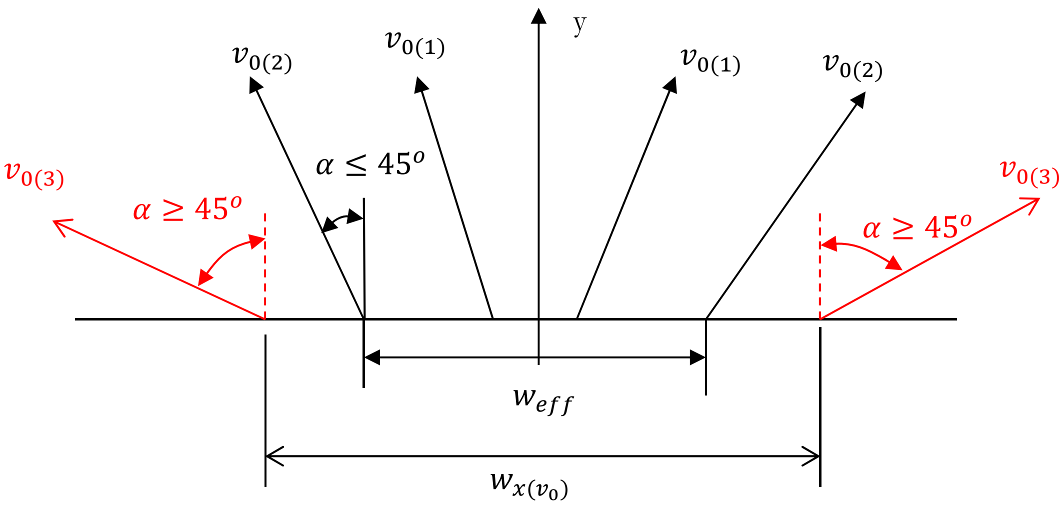


Figure 4.18 Distribution widths for shear forces – single load.



$\tan(\alpha) = \frac{v_x}{v_y{'}}$ where $v_y{'}$ also includes the effects of permanent loads (self-weight of concrete and surfacing)

Figure 4.19 Limiting condition for the distribution width $w_{x(V_0)}$.

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Appendix A Moment distributions for different assumptions regarding support pressure distribution

In this appendix the moment distribution in a continuous slab strip (or beam) with an equally distributed load, q, and equal span lengths, L, are studied for different assumptions regarding the support pressure distribution. In Figure A.1 the moment distributions for discrete supports are compared to the case with equally distributed support pressures over a support width, a, determined both with detailed calculation and with the approximation according to Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.3.2.2), for $a = L/5$.

For the strip supported by discrete supports, the bending moment $M(x)$ is

$$M(x) = M_s - \frac{Rx}{2} + \frac{qx^2}{2} = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} \tag{A.1}$$

Here, x is the coordinate along the beam, M_s is the support moment and R is the support reaction.

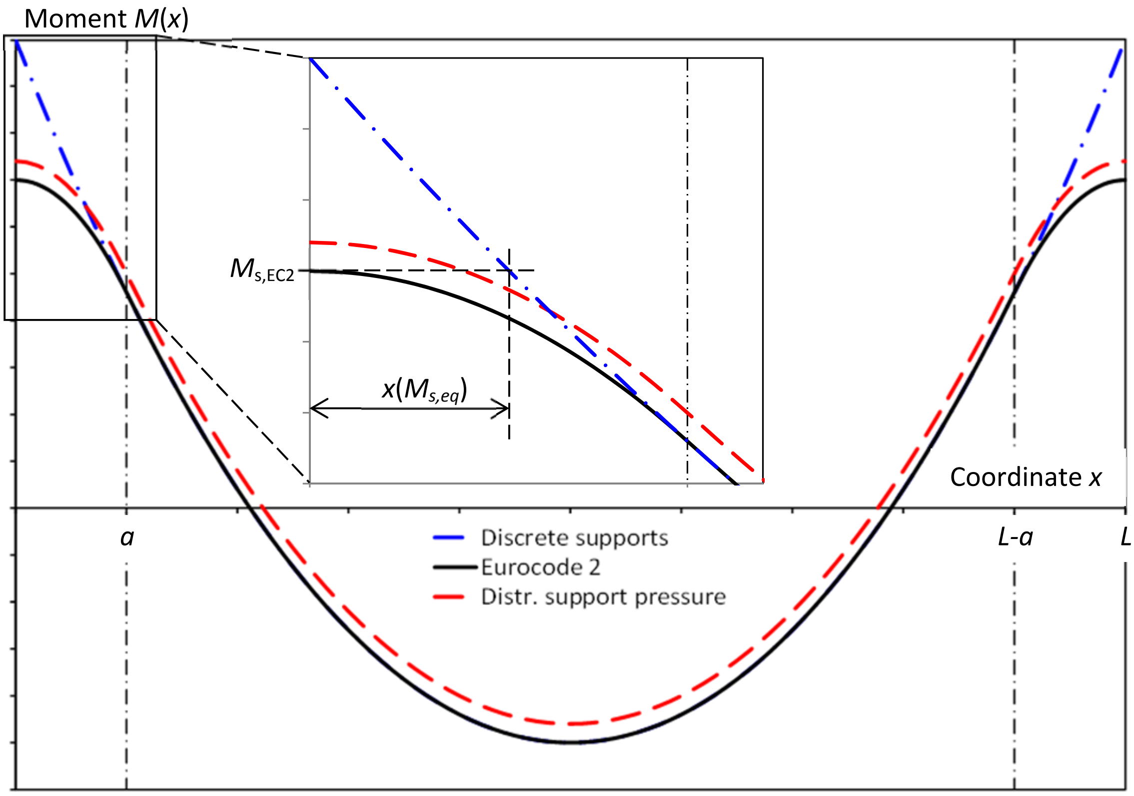


Figure A.1 Moment distributions for different assumptions regarding the support pressure (for a = L/5): distributed support pressure.

According to Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.3.2.2), the design support moment, calculated with centre-to-centre distance between support points, can be reduced with $\Delta M = Ra/8$. This means that the moment from a distributed support pressure balancing the discrete support reaction is superimposed to the moment distribution for discrete supports according to equation (A.1). The resulting moment, expressed as a function of x, over the support becomes

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} - \Delta M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} - \frac{qL}{2a}\left(\frac{a}{2} - x\right)^2 \quad \text{for } x \leq \frac{a}{2} \qquad \text{(A.2)}$$

If, instead, the moment is derived for a strip with uniform support pressures over support widths a, the following expressions are obtained:

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} + \frac{qa^2}{12} \qquad \text{for } \frac{a}{2} \leq x \leq L - \frac{a}{2} \qquad \text{(A.3)}$$

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} + \frac{qa^2}{12} - \frac{qL}{2a}\left(\frac{a}{2} - x\right)^2 \quad \text{for } x \leq \frac{a}{2} \qquad \text{(A.4)}$$

Comparing equation (A.1) with (A.3) and equation (A.2) with (A.4), it can be seen that the difference between the assumption of uniform support pressures and the Eurocode recommendation is equal to $qa^2/12$. With the Eurocode method this shift of the entire moment curve, from span to support moment, is disregarded, or rather considered to be accommodated through plastic redistributions.

As seen above, if a structural model with discrete supports is used for overall analysis, it is sufficient to design for the support moment according to the Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.3.2.2):

$$M_{s,EC2} = \frac{qL^2}{12} - \frac{qLa}{8} \qquad \text{(A.5)}$$

The equivalent theoretical moment for the strip with discrete supports is found at a distance $x = x(M_{s,eq})$ from the support, see Figure A.1. From equations (A.1) and (A.4) the distance relative to the support width, x/a, can be derived as a function of support width to span ratio, a/L:

$$\frac{x}{a} = \frac{1 - \sqrt{1 + {}^{a}/_{L}}}{2 \cdot {}^{a}/_{L}} \qquad \text{(A.6)}$$

This relation is shown in Figure A.2. It can be seen that the relative position of the equivalent moment are found no closer to the theoretical support than 0.25a, for all support width to span ratios. From a practical point of view, this means that the support moment to be designed for can be taken from the theoretical moment distribution for discrete supports, at half the distance from the theoretical support point to the support edge.

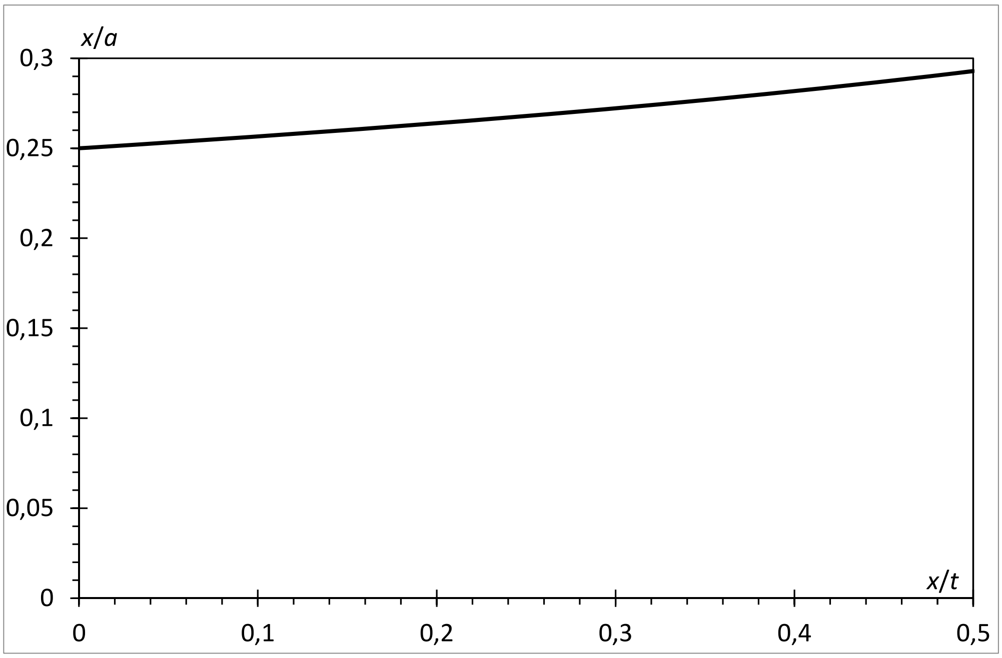


Figure A.2 *Relative distance, x/a, to the section where the theoretical moment equals the reduced moment according to Eurocode 2, as a function of support width to span ratio, a/L.*

In Figure A.3, the moment distribution for discrete supports is compared to a case where the support pressure is represented by support resultants at the edges of the support, determined both with detailed calculation and with an approximation similar to the Eurocode 2 approximation for distributed support pressure, for $a = L/5$.

Similarly to Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.3.2.2), the design support moment, calculated with centre-to-centre distance between support points, can be reduced with $\Delta M = Ra/4$ if the support pressure is assumed to be represented by support resultants at the edges of the support. The resulting moment, expressed as a function of x, over the support becomes in this case

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} - \Delta M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} - \frac{qL}{2}\left(\frac{a}{2} - x\right) \quad \text{for } x \leq \frac{a}{2} \tag{A.7}$$

If, instead, the moment is derived for a strip with resultants at the edges of the supports with the support widths a, the following expressions are obtained:

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} + \frac{qa^2}{8} \quad \text{for } \frac{a}{2} \leq x \leq L - \frac{a}{2} \tag{A.8}$$

$$M(x) = \frac{qL^2}{12} - \frac{qLx}{2} + \frac{qx^2}{2} + \frac{qa^2}{8} - \frac{qL}{2}\left(\frac{a}{2} - x\right) \quad \text{for } x \leq \frac{a}{2} \tag{A.9}$$

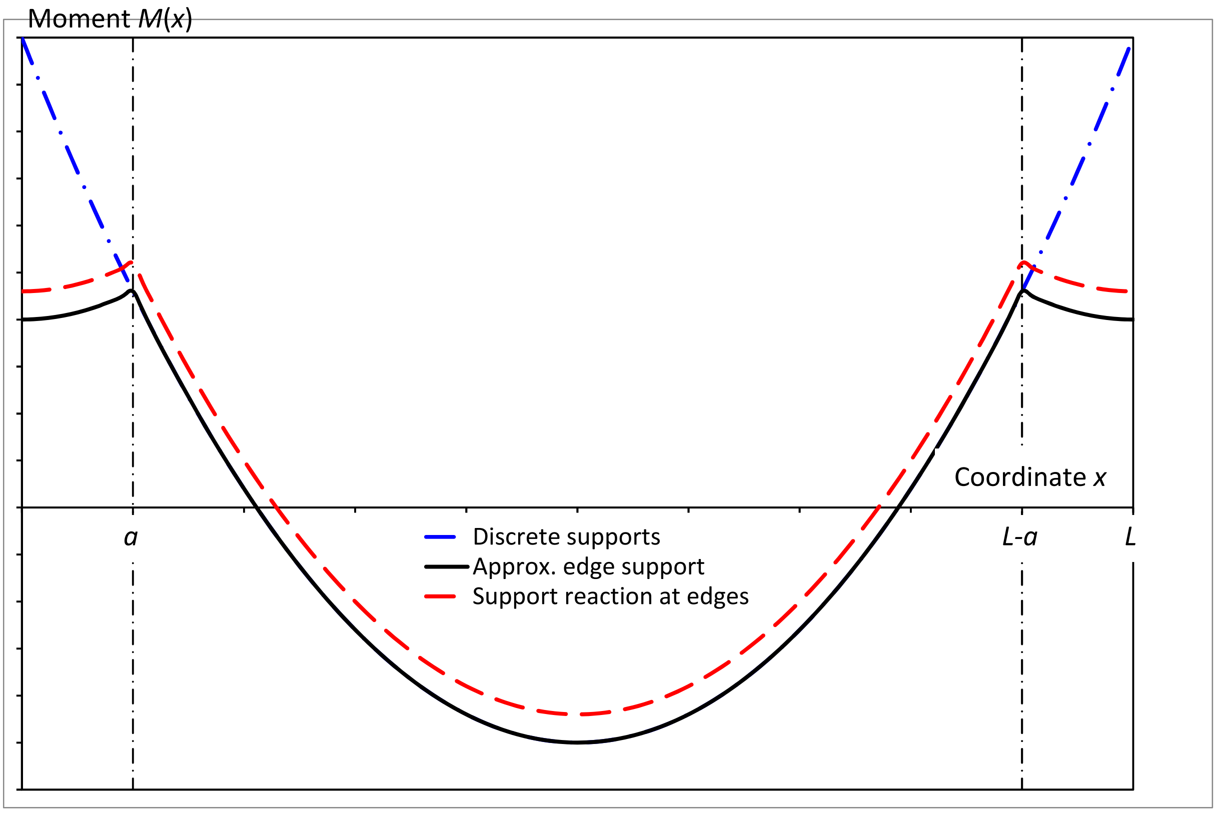


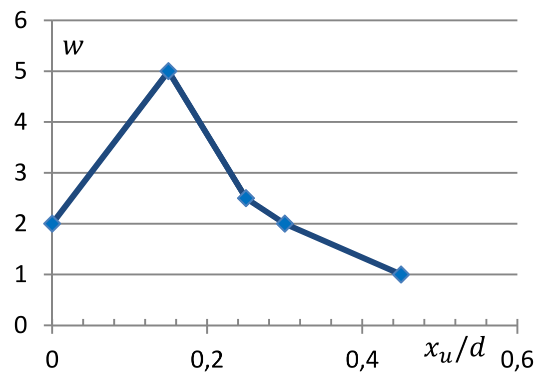
Figure A.3 Moment distributions for different assumptions regarding the support pressure (for a = L/5): support resultants at the edges of the supports.

Similarly to the case with equally distributed support pressure, it can be seen by comparing equations (A.1) and (A.7) to (A.9) that the difference between the detailed calculation and the approximation similar to the Eurocode recommendation is equal to $qa^2/8$. The approximation corresponds to that the shift of the entire moment curve, from span to support moment, is considered to be accommodated through plastic redistributions. From a practical point of view, the approximation means that the support moment to be designed for can be taken from the theoretical moment distribution for discrete supports at the position of the support edges.

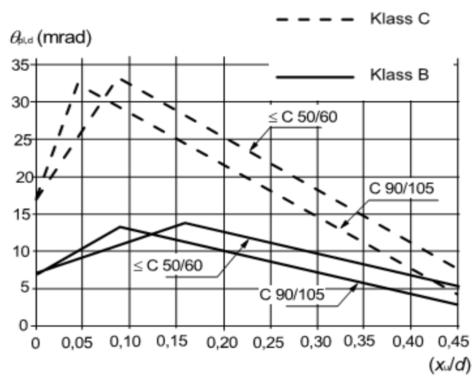
Appendix B Some comments regarding the values of the distribution widths and associated ductility requirements

The recommendations concerning the distribution widths presented in equations (4.11) to (4.15), in Section 4.2.1, and the associated ductility requirements where chosen based on the following considerations

a) The dependency of the distribution width w on $\frac{x_u}{d}$ follows the dependency of the rotation capacity of the cross-section on the same parameter as illustrated in Figure B.1



(a)



(b)

Figure B.1 (a) Variation of the distribution width with x_u/d, (b) variation of the rotation capacity with x_u/d.

b) $w = \min(3h, \frac{L_c}{10})$ for $\frac{x_u}{d} = 0.45$ (0.35 for concrete strength classes $\geq$ C55/67).
This is the value recommended in the old Swedish bridge code (Bro 2004). This value is very restrictive and can be used with a minimum concern for ductility requirements. Thus, the associated ductility requirement corresponds to the minimum value recommended in Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.6.3 (2))

c) $w = \min\left(5h, \frac{L_c}{5}\right)$ for $\frac{x_u}{d} = 0.30$ (0.23 for concrete strength classes $\geq$ C55/67)
With a minor modification, this value is the one recommended in the Swedish code for concrete structures (BBK 04 (2004)). The value $5h$ (instead of $6h$ as in BBK 04) was adopted based on the definition of a slab given in Eurocode 2 (SS-EN 1992-1-1, Section 5.3):
"A slab is a member for which the minimum panel dimension is not less than 5 times the overall slab thickness"
The associated ductility requirement corresponds to the minimum value recommended in Eurocode 2 (SS-EN 1992-2:2005, Section 5.6.3 (102)) which is more conservative than the one in SS-EN 1992-1-1:2005.

d) $w = \frac{L_c}{4}$ for $\frac{x_u}{d} = 0.25$ (0.15 for concrete strength classes $\geq$ C55/67)

This value is recommended in the literature (MacGregor 1992) and is more liberal than the values at points a and b above. For this reasons the associated ductility requirements are chosen in accordance with Eurocode 2 (SS-EN 1992-1-1:2005, Section 5.6.2 (2)).

e) $w = \frac{L_c}{2}$ for $\frac{x_u}{d} = 0.15$ (0.10 for concrete strength classes $\geq$ C55/67)
This is the maximum value that can be chosen for the distribution width. The associated ductility requirements are the most restrictive ones, chosen in accordance with Eurocode 2 (SS-EN 1992-2:2005, Section 5.6.2 (102)).