



# ANALYSIS OF FOOTBRIDGE COMFORT VIBRATIONS A Parametric Study

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Structural Mechanics

Master's Dissertation

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# Abstract

The aim of this master's thesis was to investigate and increase the knowledge of how different parameters affect the dynamic properties of steel beam bridges used for pedestrian traffic.

Steel footbridges are commonly used due to their high strength and low weight. Due to the lower static loads caused by pedestrians, the dynamic loads are usually decisive. For lightweight bridges with a low ratio of stiffness to mass, the natural frequencies can coincide with the walking frequency of pedestrians. The dynamic properties of a bridge are usually more difficult to determine at an early design phase than for the static case. To be on the safe side, this generally results in a bridge with a low utilization ratio, and oversized dimensions. This comes at a great cost, both environmentally and financially.

There are different guides and methods on determining the dynamic properties of a bridge. The one used in this thesis, the Sétra technical guide on footbridges, is commonly used in the industry. It is based on different bridge classes depending on crowd density and use. Depending on the natural frequency of the bridge, different dynamic load cases are achieved.

This guide was used to evaluate a large number of footbridges for their dynamic properties. The different bridges varied both in geometry, stiffness, and mass. This was to see if there were any thresholds for where the dynamic criteria could be met reliably, and to compare the dynamic and static criteria. The analysis was achieved by writing and using a Python script to create and analyze each numerical model in the finite element analysis tool Abaqus. This allowed for a study of the different parameters.

The analysis concluded that the choice of bridge class had the individually largest impact on the results according to the Sétra guide. It also concluded that for bridges with a relatively small bearing area of less than  $140 \text{ m}^2$ , it is not possible to achieve a high utilization ratio. The natural frequencies had to be close to threshold values for the acceleration criteria to be met.

# Sammanfattning

Syftet med detta examensarbete var att undersöka och öka kunskapen om hur olika parametrar påverkar de dynamiska egenskaperna hos balkbroar i stål som används för gångtrafik.

Stålbroar används ofta på grund av sin höga hållfasthet och låga vikt. På grund av de lägre statiska lasterna som orsakas av fotgängare är de dynamiska lasterna vanligtvis dimensionerande. För lätta broar med ett lågt förhållande mellan styvhet och massa kan de naturliga frekvenserna sammanfalla med fotgängares gångfrekvens. De dynamiska egenskaperna hos en bro är vanligtvis svårare att fastställa i projekteringsfasen än för det statiska fallet. För att vara på den säkra sidan resulterar detta väldigt ofta i en bro med mycket låg utnyttjandegrad, och för stora dimensioner. Detta resulterar i en stor kostnad, både miljömässigt och ekonomiskt.

Det finns olika guider och metoder för att bestämma en bros dynamiska egenskaper. Den som används i detta examensarbete, Sétras tekniska guide "Footbridges - Assessment of vibrational behaviour of footbridges under pedestrian loading" används ofta i branschen. Den är baserad på olika broklasser beroende på fotgängardensitet och användning. Beroende på brons egenfrekvens uppnås olika dynamiska lastfall.

Denna guide användes för att utvärdera ett stort antal gångbroar för deras dynamiska egenskaper. De olika broarna varierade både i geometri, styvhet och massa. Detta var för att se om det fanns några trösklar för var de dynamiska kriterierna kunde uppfyllas på ett tillförlitligt sätt, och för att jämföra de dynamiska och statiska kriterierna. Analysen åstadkoms genom att skriva och använda ett Python-skript för att skapa och analysera varje numerisk modell i datorverktyget Abaqus med finita element. Detta gjorde det möjligt att studera de olika parametrarna.

Analysen kom fram till att valet av broklass hade den individuellt största inverkan på resultaten enligt Sétra-guiden. Slutsatsen kunde dras att för broar med en relativt liten användingsarea på mindre än 140 m<sup>2</sup> är det inte möjligt att uppnå en hög utnyttjandegrad. De naturliga frekvenserna måste ligga nära tröskelvärdena för att accelerationskriterierna skulle kunna uppfyllas.

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# Notations and Symbols

## Abbreviations

- FFT Fast Fourier Transform
- DOF Degree Of Freedom

SDOF Single Degree Of Freedom

- MDOF Multi Degree Of Freedom
- ULS Ultimate Limit State
- SLS Serviceability Limit State

## Latin letters

$A_V$	shear area
E	modulus of elasticity
Ι	moment of inertia
$f_y$	steel yield strength
$M_{c,Rd}$	design bending resistance of a cross-section
$M_{Ed}$	design value of bending moment
$M_{pl,Rd}$	plastic moment design resistance of a cross-section
$V_{c,Rd}$	design shear resistance of a cross-section
$V_{Ed}$	design value of shear force
$V_{pl,Rd}$	plastic shear force design resistance of a cross-section
$W_{pl}$	plastic section modulus
$f_n$	natural frequency

## Greek letters

$\gamma_{M0}$	global partial resistance coefficient regardless of cross-section class
ν	Poisson's ratio
ξ	critical damping ratio
${oldsymbol{\phi}}$	displacement shape vector
$\psi$	reduction factor for likelihood of resonance
$\omega_n$	natural angular frequency

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# 1 Introduction

# 1.1 Background

A large number of footbridges in Sweden and Europe are built using steel for the main load-bearing structure. For every tonne of cold-rolled steel produced in Europe, 106 tonnes of  $CO_2$  equivalents are released [1]. The steel industry accounts for one third of the total carbon dioxide emissions in Sweden [2].

Footbridges have much lower design loads than their vehicle-bearing counterparts. This means that when designed in the ultimate limit state very slender dimensions can be obtained. Slender structures are vulnerable to vibrations excited by pedestrian loads, because of their lower resonance frequencies coinciding with pedestrian walking frequency.

These vibrations do not pose a risk to the structural integrity of the bridge but rather the comfort of the pedestrians. This is in other words a problem within the comfort criteria of the bridge.

The resonance frequency of a lighter structure tends to be close to the walking frequency of the pedestrians and more likely to cause larger accelerations than in a heavier structure. These dynamic problems can be hard to anticipate during the design phase.

During this phase, the rudimentary bridge geometry is set in cooperation with the project owner. These geometrical proportions are difficult to alter after the fact. This often means that the only way to fulfill the dynamic comfort criteria is to increase the stiffness by increasing beam dimensions. This is an inefficient material use and counteracts the goal of minimising built in  $CO_2$ . To increase the sustainability of a bridge, increasing material efficiency while maintaining an acceptable comfort level is one of the priorities.

It is desirable to be able to determine at an early stage of a project whether a certain bridge will need to be designed specifically for these dynamic loads. There are several factors that may be decisive for the vibration criteria in footbridges, e.g. span, width, support conditions, material choice, etc. In this thesis, a number of these factors and their influences will be examined. Such knowledge is valuable at an early stage to guide the proceeding of a construction project.

The bridges referenced in this thesis are steel beam footbridges. These consist of a number of main beams running lengthwise and crossbeams running across the bridge. A typical design of this type can be seen in figure 1.1.



Figure 1.1: A typical steel beam footbridge.[3]

# 1.2 Aim and Objective

The aim of this master's thesis is to increase the understanding of how several parameters affect the dynamic properties of footbridges. In the long term, this may reduce the amount of material required to ensure the vibration criteria in the serviceability limit state, and thereby increase the utilization ratio of the structures. To achieve the aims the following questions need to be answered.

- Is it possible to achieve the comfort vibration criteria below the 5 Hz natural frequency?
- How does the choice of dynamic bridge class affect the achievement of the dynamic comfort criteria?
- How does the utilization ratio compare to the dynamic comfort of the bridge?

## 1.3 Method

To be able to determine the characteristics of and influences on the bridges, the finite element analysis tool Abaqus [4] was used. A number of steel beam bridges were modelled and from a study of parameters their dynamic behaviors were examined. This was achieved by a script in Python that made it possible to vary the different parameters between set ranges and intervals. To evaluate the vibration criteria, the Eurocodes and the technical guide on Footbridges from Sétra were used.

# 1.4 Limitations

There were many factors not examined in this thesis. A lot of the bridge model was simplified to get as general a result as possible. This includes the stiffness and structure of hand rails, superstructure and other parts of the bridge. Damping is also one of the factors that is difficult to realistically determine. Hence minimal damping was assumed, to provide the most adverse case for the results. The bridge model was connected as if though it was made from one piece. This is not always the case with welded steel. The probabilistic steps of pedestrians were simplified to be a uniformly distributed dynamic load. All the structural members were made of steel.

# 2 Theory

## 2.1 The Finite Element Method

Many physical phenomena are described by complicated differential equations. These equations are usually very difficult to solve using traditional analytical methods. The finite element method (FEM) is an approach to approximating solutions for these differential equations.

The differential equations are assumed to be valid for a certain region either in one, two, or three dimensions. The finite element method aims not to seek approximations that are valid for the entire region, instead it divides the region into a number of smaller elements that are finite. The approximations are then performed over each element. This collection of finite elements is called the finite element mesh.

The differential equations go from a continuum with an infinite number of unknowns to a discrete system of a finite number of unknowns.

This concept is visualised in figure 2.1.



Figure 2.1: Approximating the function.

## 2.2 Dynamics of Structures

#### 2.2.1 The Equation of Motions

In a large dynamic system a Multi Degree Of Freedom (MDOF) system can be used to describe the loading response along the entire model. MDOF systems are formed by n degrees of freedom (DOF), with n masses connected by springs and typically viscous dampers. The equations 2.1-2.3 describe the equation of motions for such a system [5].

For these equations,  $m_i$  is the mass,  $c_i$  is the damping,  $k_i$  is the spring stiffness, and t is time. The variables  $\dot{u}$  and  $\ddot{u}$  are the first and second time derivatives of the displacement u, respectively. These being velocity and acceleration. Variable  $p_i$  is the force exerted on the system. The system for two degrees of freedom forms equations which are described as [6]:



Figure 2.2: MDOF system. Illustration based on Sétra [5].

$$\begin{cases} m_1 \ddot{u}_1(t) + (c_1 + c_2)\dot{u}_1(t) - c_2 \dot{u}_2(t) + (k_1 + k_2)u_1(t) - k_2 u_2(t) = p_1(t) \\ m_2 \ddot{u}_2(t) + (c_2 + c_3)\dot{u}_2(t) - c_2 \dot{u}_1(t) + (k_2 + k_3)u_2(t) - k_2 u_1(t) = p_2(t) \end{cases}$$
(2.1)

Which can easily be written in matrix form as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix} \begin{bmatrix} \dot{u}_1(t) \\ \dot{u}_2(t) \end{bmatrix} + \\ + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix}$$
(2.2)

These matrices can be written as:

$$\boldsymbol{m}\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\boldsymbol{u}(t) = \boldsymbol{p}(t)$$
(2.3)

The expression 2.3 is the general equation of motion for any amount of degrees of freedom.

#### 2.2.2 Natural Modes and Frequencies

To determine the dynamic properties of a structure, the natural frequencies and modes need to be obtained. Proven by Fourier, all wave motions can be expressed as harmonic functions which means that the deflection  $\boldsymbol{u}(t)$  is given by equation 2.4 where  $\boldsymbol{\phi}_n$  is the displacement shape vector of mode n and  $\omega_n$  are the natural angular frequencies [6].

$$\boldsymbol{u}(t) = \boldsymbol{\phi}_n(A_n \cos\omega_n t + B_n \sin\omega_n t) \tag{2.4}$$

Inserting the expression for u(t) in equation 2.3 and assuming there is no influence of external forces and damping on the system, makes it possible to rewrite equation 2.3 as equation 2.5.

$$(\boldsymbol{k} - \omega_n^2 \boldsymbol{m})\boldsymbol{\phi}_n(A_n \cos\omega_n t + B_n \sin\omega_n t) = 0$$
(2.5)

Now as the deflections at the resonance frequencies are non-zero the harmonic terms can be removed which turns equation 2.5 into equation 2.6.

$$(\boldsymbol{k} - \omega_n^2 \boldsymbol{m})\boldsymbol{\phi}_{\boldsymbol{n}} = 0 \tag{2.6}$$

For a non-trivial solution to exist for  $\phi_n$ , then equation 2.7 needs to be zero.

$$\det(\boldsymbol{k} - \omega_n^2 \boldsymbol{m}) = 0 \tag{2.7}$$

It becomes an eigenvalue problem where the eigenvalues, the natural angular frequencies, are obtained. These frequencies can then be inserted in equation 2.6 to determine the displacement vectors for each resonance frequency.

The natural angular frequency  $\omega_n$  can be used to obtain the natural frequency  $f_n$ 

$$f_n = \frac{\omega_n}{2\pi} \tag{2.8}$$

#### 2.2.3 Steady State Modal Dynamics

The accelerations for each frequency are of interest and needs to be determined. A steady state analysis is therefore required. As stated previously, the equation of motion can be described as

$$\boldsymbol{m}\ddot{\boldsymbol{u}}(t) + \boldsymbol{c}\dot{\boldsymbol{u}}(t) + \boldsymbol{k}\boldsymbol{u}(t) = \boldsymbol{p}(t)$$
(2.9)

With the natural modes as basis, one can represent the displacement vector  $\boldsymbol{u}$  [6]. This modal expansion of the displacement vector has the following form:

$$\boldsymbol{u}(t) = \sum_{r=1}^{N} \boldsymbol{\phi}_r q_r(t) = \boldsymbol{\phi} \boldsymbol{q}(t)$$
(2.10)

where  $q_r$  are scalar multipliers between the mode shapes and displacements, and  $\boldsymbol{q}(t) = \begin{bmatrix} q_1(t) & q_2(t) & \cdots & q_n(t) \end{bmatrix}^T$ . If equation 2.10 is inserted in equation 2.9, it gives

$$\sum_{r=1}^{N} \boldsymbol{m} \boldsymbol{\phi}_{r} \ddot{q}_{r}(t) + \sum_{r=1}^{N} \boldsymbol{c} \boldsymbol{\phi}_{r} \dot{q}_{r}(t) + \sum_{r=1}^{N} \boldsymbol{k} \boldsymbol{\phi}_{r} q_{r}(t) = \boldsymbol{p}(t)$$
(2.11)

Premultiplying each term by  $\boldsymbol{\phi}_n^T$  gives

$$\sum_{r=1}^{N} \boldsymbol{\phi}_{n}^{T} \boldsymbol{m} \boldsymbol{\phi}_{r} \ddot{q}_{r}(t) + \sum_{r=1}^{N} \boldsymbol{\phi}_{n}^{T} \boldsymbol{c} \boldsymbol{\phi}_{r} \dot{q}_{r}(t) + \sum_{r=1}^{N} \boldsymbol{\phi}_{n}^{T} \boldsymbol{k} \boldsymbol{\phi}_{r} q_{r}(t) = \boldsymbol{\phi}_{n}^{T} \boldsymbol{p}(t)$$
(2.12)

All terms in the summations that are not r = n vanish because of modal orthogonality, except for the damping, giving

$$(\boldsymbol{\phi}_n^T \boldsymbol{m} \boldsymbol{\phi}_n) \ddot{q}_n(t) + \sum_{r=1}^N (\boldsymbol{\phi}_n^T \boldsymbol{c} \boldsymbol{\phi}_n) \dot{q}_n(t) + (\boldsymbol{\phi}_n^T \boldsymbol{k} \boldsymbol{\phi}_n) q_n(t) = \boldsymbol{\phi}_n^T \boldsymbol{p}(t)$$
(2.13)

or

$$M_n \ddot{q}_n(t) + \sum_{r=1}^N C_{nr} \dot{q}_n(t) + K_n q_n(t) = P_n(t)$$
(2.14)

where

$$M_n = \boldsymbol{\phi}_n^T \boldsymbol{m} \boldsymbol{\phi}_n; \qquad K_n = \boldsymbol{\phi}_n^T \boldsymbol{k} \boldsymbol{\phi}_n; \qquad P_n(t) = \boldsymbol{\phi}_n^T \boldsymbol{p}(t); \qquad (2.15)$$

and

$$C_{nr} = \boldsymbol{\phi}_n^T \boldsymbol{c} \boldsymbol{\phi}_r \tag{2.16}$$

For classical damping,  $C_{nr} = 0$  if  $n \neq r$  reducing the equations to

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + K_n q_n(t) = P_n(t)$$
(2.17)

where

$$C_n = \boldsymbol{\phi}_n^T \boldsymbol{c} \boldsymbol{\phi}_n \tag{2.18}$$

Dividing by  $M_n$  and using that  $K_n = \omega_n^2 M_n$  because of modal orthogonality, equation 2.17 can be rewritten as

$$\ddot{q}_n(t) + 2\zeta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{P_n(t)}{M_n}$$
(2.19)

where

$$\zeta_n = \frac{C_n}{2M_n\omega_n} \tag{2.20}$$

For a single degree of freedom system, p in equation 2.9 can be described as

$$p^* = p_0 e^{i\omega_n t} \tag{2.21}$$

and u as

$$u^* = u_0 e^{i\omega_n t} \tag{2.22}$$

where  $u_0$  is the amplitude of deflection. If differentiated in regards of time, this becomes an expression of velocity

$$\dot{u}^* = i\omega_n u_0 e^{i\omega_n t} \tag{2.23}$$

if differentiated again, this becomes an expression of acceleration

$$\ddot{u}^* = -\omega_n^2 u_0 e^{i\omega_n t} \tag{2.24}$$

The product  $-\omega^2 u_0$  can be described as the amplitude of acceleration  $a_0$  giving

$$\ddot{u}^* = a_0 e^{i\omega_n t} \tag{2.25}$$

The deflection amplitude  $u_0$  in resonance is

$$u_0 = \frac{p_0}{\omega_n c} \tag{2.26}$$

This means that the acceleration amplitude, i.e. the maximum acceleration, can be described as

$$|a_0| = \omega_n^2 u_0 = \omega_n^2 \frac{p_0}{\omega_n c} = \frac{\omega_n p_0}{c}$$
(2.27)

where the damping c can be described as

$$c = 2\xi m\omega_n \tag{2.28}$$

giving

$$|a_0| = \frac{\omega_n p_0}{c} = \frac{\omega_n p_0}{2\xi m \omega_n} = \frac{p_0}{2\xi m}$$
(2.29)

The steady state deformation of the system due to harmonic force can be written as

$$u(t) = u_0 \sin(\omega t - \phi) = (u_{st})_0 R_d \sin(\omega t - \phi)$$
(2.30)

where  $\phi$  is the phase lag, not to be confused with the mode shape,  $(u_{st})_0$  is the static displacement, and  $R_d$  is the dynamic deflection response factor. This response factor can be solved for

$$R_d = \frac{u_0}{(u_{st})_0} = \frac{1}{\sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta(\omega/\omega_n))^2}}$$
(2.31)

The natural angular frequency of a generalized SDOF system can be described as

$$\omega_n = \sqrt{\frac{\tilde{k}}{\tilde{m}}} \tag{2.32}$$

#### 2.2.4 Fast Fourier Transform (FFT)

The fast Fourier transform (FFT) is a highly efficient algorithm of computing the discrete Fourier transform and its inverse. This Fourier transform converts the total response of the structure from the time domain to the frequency domain over a certain time period. This is used to evaluate steady state dynamics from equation 2.19 in the frequency domain [6].

## 2.2.5 Modal Truncation

The equation 2.10 can be approximated for a number of modes J < N which gives

$$\boldsymbol{u}(t) = \sum_{r=1}^{J} \boldsymbol{\phi}_r q_r(t) \tag{2.33}$$

Including only J modes produces an error, depending on the force distribution. If the force distribution is adequately described for J modes, the error will be very small [6].

# **3** Footbridge Design Principles

The static design of the beam bridges were done according to the Eurocode standards, and these standards are also followed in the dynamic design case. However, the Eurocodes do not indicate a method for analysing the resulting accelerations for a bridge with natural frequencies below the 5 Hz limit. The technical guide on footbridge dynamic design from Sétra was therefore considered in the dynamic case [5]. This guide presents a way of evaluating the dynamic behaviour for bridges with natural frequencies under the 5 Hz limit corresponding to risk of resonance.

## 3.1 Ultimate limit state in static loading

The ultimate limit state design was performed to compare the static utilization ratio with the achievement of the dynamic comfort criteria.

#### 3.1.1 Actions

For static loads in Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges [7], there are two different and separate load groups that correspond to different limit states. These are denoted gr1 and gr2. Group gr1 is the ultimate limit state design case and contains the vertical uniformly distributed load  $q_{fk}$  and the horizontal longitudinal force  $Q_{flk}$ .

The uniformly distributed load  $q_{fk}$  is defined according to Load Model 4 (crowd loading) corresponding to  $q_{fk} = 5 \text{ kN/m}^2$ .

A concentrated load is also to be taken into account. It consists of a load  $Q_{fwk}$  and should be taken as equal to 10 kN acting on a square surface with a side of 0.10 m. This is mostly for analysing local effects and less relevant to the entire structure. It will therefore be neglected in this study.

The horizontal longitudinal force  $Q_{flk}$  is defined as 10% of the uniformly distributed load  $q_{fk}$  in gr1. This horizontal force will not be considered in this study as it will have a negligible effect on the primary bearing structure.

These actions will be used as the applied loading on the finite element analysis, whereby all other section forces and moments will be calculated.

All loads and self-weights were multiplied with safety factors according to Eurocode: Basis of structural design [8]. These are shown in table 3.1.

Table 3.1: Partial coefficients for the differe	at loads.
---	-----------

Load	$\gamma_F$
Self-weight steel, $g_s$	1.2
Self-weight pavement, $g_p$	1.2
Uniformly distributed load, $q_{fk}$	1.5

#### 3.1.2 Resistances

Resistances to be used for the bridge is detailed in Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings [9].

#### Bending and Shear

As the bridge is affected both by bending and shear, both bending moments and shear forces need to be considered. The resistances for these are described in Eurocode 3. Since all of the cross-sections examined were in cross-section class 1, this meant that buckling could be neglected.

The plastic moment design resistance of a cross-section is described as

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl}f_y}{\gamma_{M0}} \tag{3.1}$$

where  $W_{pl}$  is the plastic section modulus, described as

$$W_{pl} = bt(h-t) + d(h-2\cdot t)^2/4$$
(3.2)

 $f_y$  is the yield strength of the steel, and  $\gamma_{M0}$  is the global partial coefficient for the resistance regardless of cross-section class.

The design value of the bending moment  $M_{Ed}$  at each cross-section needs to fulfill:

$$\frac{M_{Ed}}{M_{c,Rd}} \le 1.0\tag{3.3}$$

The plastic shear force design resistance of a cross-section is described as

$$V_{c,Rd} = V_{pl,Rd} = \frac{A_V(f_y/\sqrt{3})}{\gamma_{M0}}$$
 (3.4)

where  $A_V$  is the shear area.

The design value of the shear force  $V_{Ed}$  at each cross-section needs to fulfill:

$$\frac{V_{Ed}}{V_{c,Rd}} \le 1.0\tag{3.5}$$

Shear forces have a negative effect on the bending moment resistance of the crosssection. If the shear force is less than half of the plastic shear resistance, its effect on the bending moment resistance can be neglected. Otherwise the reduced moment resistance should be taken as the design moment resistance calculated using the reduced yield strength of  $(1 - \rho)f_y$ , and where

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,Rd}} - 1\right)^2 \tag{3.6}$$

This means that the reduced plastic bending moment design resistance becomes:

$$M_{pl,Rd} = \frac{W_{pl}(1-\rho)f_y}{\gamma_{M0}}$$
(3.7)

This reduced plastic bending moment design resistance must also fulfill equation 3.3

# 3.2 Comfort vibration criteria according to Eurocode

There are multiple serviceability requirements for pedestrian comfort described in Eurocode - Basis of structural design.[8] The recommended maximum acceptable acceleration of any part of the deck are:

- i)  $0.7 \text{ m/s}^2$  for vertical vibrations
- ii)  $0.2 \text{ m/s}^2$  for horizontal vibrations due to normal use
- iii) 0.4 m/s<sup>2</sup> for exceptional crowd loading

This verification needs to be done if the lowest natural frequency is below 5 Hz for vertical vibrations [8].

Excessive deflection should be prevented for the static design. The Swedish Transport Administration (Trafikverket) defines the maximum allowable deflection as L/400. This is also to be taken into account as a criteria.

# 3.3 Assessment of comfort vibration criteria according to Sétra

The dynamic loads affecting the bridge is detailed in the Sétra technical guide. The guide describes five stages to determining the appropriate loads [5]. These can be seen in the figure below. Stage 5 is irrelevant as the bridge would not be modified after calculations. The methodology in the Sétra technical guide can be seen in figure 3.1.



Figure 3.1: Methodology organization chart. Illustration based on Sétra [5].

#### 3.3.1 Stage 1: Determination of Footbridge Class

A footbridge class must first be determined. The classes describe describe different pedestrian densities for different areas. The classes determine how the loading should be calculated in the dynamic comfort criteria. There are four different classes where class I is the most heavily trafficked [5]:

- **Class IV:** Seldom used footbridge, built to link sparsely populated areas or to ensure continuity of the pedestrian footpath in motorway or express lane areas.
- **Class III:** Footbridge for standard use, that may occasionally be crossed by large groups of people but that will never be loaded throughout its deck area.
- **Class II:** Urban footbridge linking up populated areas, subjected to heavy traffic and that may occasionally be loaded throughout its deck area.
- **Class I:** Urban footbridge linking up high pedestrian density areas (for instance, nearby presence of a rail or underground station) or that is frequently used by dense crowds (demonstrations, tourists, etc.), subjected to very heavy traffic.

Two of these footbridge classes were evaluated, Class I and Class III, as this would give an opportunity to evaluate both a heavily used urban footbridge and a more standard footbridge. That would provide the study with results for footbridge dynamic design in the more adverse loading case, as well as the less demanding but more oftenly used footbridge class.

## 3.3.2 Stage 2: Choice of Comfort Level by the Owner

The owner of the bridge can choose between three acceptable comfort levels which corresponds to different maximum accelerations. The comfort levels as stated in Sétra [5] are:

- Range 1, maximum comfort: Accelerations undergone by the structure are practically imperceptible to the users.
- Range 2, average comfort: Accelerations undergone by the structure are merely perceptible to the users.
- Range 3, minimum comfort: Under loading configurations that seldom occur, accelerations undergone by the structure are perceived by the users, but do not become intolerable.
- Range 4, unacceptable comfort: Accelerations undergone by the structure are unacceptable.

As stated, the comfort levels correspond to different acceleration ranges, as seen in figure 3.2. There are different ranges for vertical and horizontal accelerations.

**Table 3.2:** Accelerations corresponding to comfort ranges, in  $m/s^2$ . Table based on Sétra[5].

Acc. $[m/s^2]$ (	) 0.	.5 1	2	.5
Range 1	Max.			
Range 2		Avg.		
Range 3			Min.	
Range 4				Unacceptable

### 3.3.3 Stage 3: Determination of Frequencies

If the footbridge class is I-III, determination of the natural frequencies is required. These natural frequencies concern vibrations in each of three directions: vertical, transverse horizontal, and longitudinal horizontal.

They are determined for two different masses. One where the bridge is empty, and one where its deck area is covered with 70 kg pedestrian per square meter. In both cases the self-weight of the bridge is accounted for.

There are four ranges of frequencies corresponding to risk of resonance from pedestrians as can be seen in table 3.3. These are:

**Range 1:** Maximum risk of resonance,  $1.7 \text{ Hz} \le f_n \le 2.1 \text{ Hz}$ 

**Range 2:** Medium risk of resonance,  $1 \text{ Hz} \le f_n \le 1.7 \text{ Hz}$  and  $2.1 \text{ Hz} \le f_n \le 2.6 \text{ Hz}$ 

**Range 3:** Low risk of resonance for standard loading situations, 2.6 Hz  $\leq f_n \leq 5$  Hz

**Range 4:** Negligible risk of resonance ,  $0 \text{ Hz} \le f_n \le 1 \text{ Hz}$  and  $f_n \ge 5 \text{ Hz}$ 

These ranges can be seen in table 3.3.

**Table 3.3:** Frequencies corresponding to resonance risk ranges, in Hz. Table based onSétra [5].

Frequency [Hz]	0 1	l 1.	7 2.	1 2.	6 5	)
Range 1						
Range 2						
Range 3						
Range 4						

There are three different load cases for different frequency ranges and class. These are shown in table 3.4.

Class	Donsity	Natural frequency range					
Class	Density	$1.7 \le f_n \le 2.1$	$1 \le f_n < 1.7,  2.1 < f_n \le 2.6$	$2.6 < f_n \le 5$			
Ι	1.0	Case 2	Case 2	Case 3			
II	0.8	Case 1	Case 1	Case 3			
III	0.5	Case 1					
IV			-				

Table 3.4: Load cases for different classes and frequency ranges. Table based on Sétra [5].

#### 3.3.4 Stage 4: Calculation with Dynamic Load Cases

The load cases are made considering crowd density d as pedestrians/m<sup>2</sup>. These densities are shown in table 3.5. It is important to note that a bridge with a smaller area will have a larger dynamic load than a larger bridge.

 Table 3.5:
 Crowd densities for different footbridge classes.

Class	Density $d$ of the crowd
Ι	$1.0 \text{ pedestrians/m}^2$
II	$0.8 \text{ pedestrians/m}^2$
III	$0.5 \text{ pedestrians/m}^2$

#### Case 1

This case is considered for class II and III bridges according to table 3.4. The vertical load per m<sup>2</sup> where d is the crowd density according to table 3.4,  $\xi$  is the critical damping ratio of the material, n is the total number of people on the bridge, given by  $n = d \cdot A_{Deck}$ , and  $\psi$  is a reduction factor describing likelihood of resonance for the crowd at a given frequency, is described as:

$$q_v = d \cdot 280 \cdot \cos(2\pi f_n t) \cdot 10.8 \cdot \sqrt{\xi/n} \cdot \psi \tag{3.8}$$

The factor  $\psi$  can be seen in figure 3.2.



Figure 3.2: Factor  $\psi$  for vertical vibrations in Case 1 and 2. Figure based on Sétra [5].

#### Case 2

This case is considered for class I bridges according to table 3.4. The vertical load per  $m^2$  is described as:

$$q_v = 1.0 \cdot 280 \cdot \cos(2\pi f_n t) \cdot 1.85 \cdot \sqrt{1/n \cdot \psi}$$
(3.9)

The reduction factor  $\psi$  for Case 2 is the same as for Case 1 as described in figure 3.2.

#### Case 3

This case is considered for class I and II bridges according to table 3.4. The vertical load per  $m^2$  for class I is described as:

$$q_v = 1.0 \cdot 70 \cdot \cos(2\pi f_n t) \cdot 1.85 \cdot \sqrt{(1/n)} \cdot \psi$$
 (3.10)

And for class II as:

$$q_v = 0.8 \cdot 70 \cdot \cos(2\pi f_n t) \cdot 10.8 \cdot \sqrt{(\xi/n)} \cdot \psi \tag{3.11}$$

The factor  $\psi$  for this case is shown in figure 3.3.



Figure 3.3: Factor  $\psi$  for vertical vibrations in Case 3. Figure based on Sétra [5].

#### Load Graphs from Sétra Classes

All of these stages describe the loads for different frequencies and deck areas. A smaller deck area will yield a larger load on that area. This can be shown for Class I-III. In figures 3.4-3.6, the Sétra loads from the different classes are shown in a visual way.



Figure 3.4: Distributed load from Sétra bridge Class I.

As can be seen for Class I in figure 3.4, the  $\psi$  factor causes the loads to be very coarse along the frequency axis. This causes a lot of the accelerations to become zero. It is also obvious that the loads tend to infinity when the deck area approaches zero. The surface begins at 8 m<sup>2</sup>, the smallest deck area reviewed in this thesis.

For class II in figure 3.5, the shape is similar, but with reduced loads.



Figure 3.5: Distributed load from Sétra bridge Class II.



Figure 3.6: Distributed load from Sétra bridge Class III.

Class III in figure 3.6 has a very narrow sliver where the loads are nonzero. This is the frequency range with the highest risk of resonance in table 3.3.

#### Critical damping ratios

The critical damping ratios  $\xi$  must also be known for the materials. The guide from Sétra describes this in table 3.6 [5].

Table 3.6: Critical damping ratios of different materials.

Material	ξ
Reinforced concrete	1.3%
Pre-stressed concrete	1%
Mixed	0.6%
Steel	0.4%
Timber	1%

These values are used to determine the modal damping for each frequency.

## 3.4 Dynamic Behaviour

As described in the chapter Dynamics of Structures, a mode shape is given by  $\phi_n$  for each mode N. A requirement in the Sétra technical guide is that the loading occurs in the corresponding mode shape so that excitation can occur. The loads are in other words applied in the same directions as the accelerations of the mode, as seen in figure 3.7.



Figure 3.7: Concept of the load positioning. The dotted line is the mode shape, and the arrows the distributed loads.

In practice, this means that a evenly distributed load over the entire bridge area can only trigger the first bending mode. To trigger the second bending mode or the first torsion mode, the loading has to be in the correct shape. In the case of the first torsion mode, splitting the deck area in two along the width. This concept can be seen in figure 3.8, where the large rectangular outlines are the corresponding loading shapes, and the meshed surface is the bridge mode shape.



Figure 3.8: Different mode shapes and their loading shape. The red shapes are the positive distributed load area and the blue shapes are the negative distributed load area.

# 3.5 Calculation Process

The calculations that were made to achieve the results can be summarized as follows:

- 1. Determine the natural frequencies.
- 2. Use the steady state dynamic modal analysis to determine acceleration amplitudes for frequencies between 0.1 and 6 Hz for a uniformly distributed load of  $1 \text{ N/m}^2$  in the different mode shapes shown in figure 3.8.
- 3. Multiply the accelerations from the steady state analysis with the load cases in Sétra.
- 4. Determine maximum section forces and moments for the loads according to the Eurocodes.
- 5. Compare the static and dynamic results with requirements.
# 4 Method

## 4.1 Bridge Model

### 4.1.1 Geometry Model

The bridge model used in the study was a steel beam bridge with at least two main beams. By using Python scripting, a program was created that could generate models and run analyses in Abaqus. The program was constructed to be able to linearly vary each variable between different ranges. The number of main beams, the bridge width, the bridge length and dimensions of the main beams were all made to be varied through this script. These variables corresponding to different properties like stiffness and distributed mass.

#### Bridge Geometry

The model consisted of a plate with a number of evenly spaced main beams along the width of the bridge. The crossbeams were placed evenly every meter along the length of the bridge.

The welds were placed with the crossbeams and the plate welded to the main beams, and each other. The plate was set to be 0.025 m thick. Although this can be considered to be thicker than what is normally expected, it was chosen so that it would not be the lowest ULS design case. The height, width, and other dimensions of the crossbeams were 70% of the corresponding dimension for the main beams.

The nodes of the beams were placed in the top of the cross-section. The nodes of the plate were placed in the bottom surface. This was done to have the elements share node location.

The parts described are shown in figure 4.1.

#### Material parameters

The material chosen for the bridge was steel with quality S355, both for the beams and plate. Material parameters have been chosen as according to Isaksson and Mårtensson [10]. Values for the parameters are shown in table 4.1.

To avoid having natural frequencies calculated for the crossbeams, their mass was set to zero. Their inertia was then added as point masses on the intersections between the crossbeams and main beams to not affect the analysis.



Figure 4.1: Parts of the bridge.

Parameter	Value
Density $\rho$	$7850 \text{ kg/m}^3$
Modulus of Elasticity $E$	210 GPa
Yield strength $f_y$	355 MPa
Poisson's ratio $\nu$	0.3

 Table 4.1: Different material parameters of steel.

#### 4.1.2 Finite Element Model

#### Finite Element mesh

The element type for the plate was chosen to be rectangular shell elements of type S4R in Abaqus with a length and width of 0.2 m. The element type for the main beams were beam elements of type B31 in Abaqus with the same length as the shell elements.

The mesh size was chosen after a convergence analysis was performed and both the static and dynamic models provided adequate results, balancing accuracy and computation time. The mesh can be seen in figure 4.2

#### Inertias

Some parts with little to negligible stiffness were not modelled. Their mass were however included as inertias. These include the handrails, surfacing on top of the bridge, e.g. asphalt, and the mass of the crossbeams as mentioned above. In the dynamic modeling, the mass of a crowd consisting of 70 kg/m<sup>2</sup> as per the Sétra guide [5] was added as a distributed non-structural mass .



Figure 4.2: The Finite element mesh of the bridge, as seen from above.

#### Handrails

The self-weight of the handrails was chosen to be 100 kg/m. This was to get a rough approximate of the inertia it would contribute. This weight has a small effect on the global mass, but it will affect the mass distribution. These were placed as point masses on the vertices between the crossbeams and the outer main beams, as shown in figure 4.3. Their influence width was  $L/n_{spans}$ .



Figure 4.3: Placement of handrails in green.

#### Asphalt

The asphalt was taken as a distributed non-structural mass which was to be equal to a self-weight of  $24 \text{ kN/m}^3$ . The thickness was assumed to be 50 mm meaning the mass corresponded to  $120 \text{ kg/m}^2$ . As the bridge is relatively lightweight, this mass will have a large effect on the overall mass. It was placed over the entire deck area of the bridge as a non-structural distributed mass.

#### Constraints

The welds between the different parts were modelled as ties, meaning there was no relative motion between them.

#### Boundary conditions

The bridge was assumed to be simply supported. Supports were located underneath the main beams. One of the edge supports was fixed in all directions to prevent rigid body motion. The other supports along this end was fixed in the longitudinal and vertical directions.

On the other end, the edge support opposite the fixed support was fixed in the transverse and vertical directions. All other supports were only prevented to move in the vertical direction, as can be seen figure 4.4. These boundary conditions made for a bridge that could expand freely within its plane.



Figure 4.4: Boundary conditions of the bridge, as seen from above. The orange arrows show the directions in which movement was restricted for a point.

#### Load models

All of the load models are described in chapter 3 Footbridge Design Principles.

### 4.1.3 Determination of Natural Frequencies

The Frequency step in Abaqus was used to determine the natural frequencies in 2.7 and the corresponding mode shapes in 2.6. This step uses the Lanczos algorithm to solve the eigenvalue problem. To limit the computation time while still providing adequate accuracy, the 20 first eigenmodes were solved for.

#### 4.1.4 Steady State Dynamics

The Steady State Dynamics, modal step in Abaqus was used to determine the steady state response of the bridge structure. This step computes the displacements, velocities, and accelerations of the structure from a periodic excitation. These are then converted from the time domain to the frequency domain.

Values were extracted from 0.1 Hz to 6 Hz, with 10 points between each natural frequency. Although Eurocode and Sétra only require frequencies between 1 Hz and 5  $\,$ 

Hz, it was important to see the values just out of the boundary aswell.

The accelerations were calculated with a unit load of  $1 \text{ N/m}^2$ , to get the frequency response function. These accelerations could then be multiplied with the corresponding loads to get the real accelerations.

Direct modal damping was applied, using the critical damping ratio of steel for each mode.

## 4.2 Convergence Analysis

A convergence study was made by varying the mesh size of the bridge. This was to see when the results converged to a specific value. Both the dynamic and static results had to converge to get adequate accuracy when comparing the two. This was achieved by viewing the values of the three first natural frequencies in the dynamic and the maximum deflection in the static. If these converged, an appropriate approximation had been achieved.



Figure 4.5: Dynamic convergence.

The approximations for the first three natural frequencies can be seen converging for a mesh element size of approximately 400 mm in figure 4.5. It is rough in shape for the largest element sizes. The reason for this is that the mode shapes are difficult to describe with such a coarse mesh.



Figure 4.6: Static convergence.

The approximation for the static displacements can be seen converging for a mesh element size of approximately 500 mm in figure 4.6. From these results it can be seen that the approximations converge to one value for a decreasing element size.

# 5 Parameter Study

The influence of a number of parameters on the comfort vibration levels were to be studied. One parametric study evaluated the connection between the bridge stiffness and the natural frequencies in the bridge. The other parametric study was broader and evaluated all the chosen parameters.

## 5.1 Parameters Affecting Bridge Dynamics

A study of the relevant literature was conducted to find the parameters that affect bridge dynamics. It was concluded in consultation with the supervisors that the influence of six interesting parameters should be examined.

- 1. Length: The length of the bridge will affect its stiffness, both torsional and bending , and its mass.
- 2. Width: A very important factor for torsional rigidity is the bridge width. A wider bridge will have a lower torsional stiffness in proportion to its mass.
- 3. Number of main beams: The number of main beams will affect where the stiffness and mass is distributed. Especially the torsional stiffness will be reduced with more beams with the same total mass.
- 4. Weight per length: The mass of the bridge will directly change its natural frequencies, as can be seen in equation 2.7.
- 5. Beam profiles: An important parameter for the dynamic design of the bridge is the dimensions of the beams. These contribute both most of the mass and stiffness to the bridge. It is obvious that if the beam dimension is increased, both its mass and stiffness is also increased. These profiles can be seen in table 5.1
- 6. Stiffness: This was mainly studied to see how the steel quality and its elasticity could affect the results.

## 5.2 Beam Profile Evaluation

The first parametric study evaluated, as stated before, how the bridge stiffness was connected to the natural frequencies, the accelerations of the bridge. This was done by picking a representative bridge geometry and varying the profiles of the main beams along with the number of main beams. In this case a bridge model was chosen that was 16 m long, 3 m wide. The beam dimensions were chosen according to HEA profiles, as these are commonly used. The dimensions of the beams are shown in table 5.1 [10]. However, any beam profile with a similar cross-section and stiffness would give a similar result.

## 5.3 Geometrical Multiparametric Study

The broader study was conducted to cover all the permutations of changing parameters. This was done by simulating a large number of bridge models where the parameters were varied within their below specified spans. The different increments of the parameters are described below.

- 1. Length: To examine how the length of the bridge would affect its dynamic properties, it was varied from 4 m to 20 m. The increment between each length was set to 1 m to provide sufficient accuracy while reducing computation time.
- 2. Width: A To examine how the width of the bridge would affect its dynamic properties, it was varied in width between 2, 3, 4 and 7 m.
- **3. Number of main beams:** The number of main beams mostly affects where the stiffness and mass is distributed. It was varied between 2, 3 and 7 main beams.
- 4. Beam profiles: When studied together with the geometric parameters the beam profile was varied between HEA200, HEA400 and HEA800 to decrease the amount of simulations needed. This will obviously also change the weight and stiffness of the bridge.

Dimension	$h  [\rm{mm}]$	b [mm]	$t  [\rm{mm}]$	$d  [\mathrm{mm}]$
HEA100	96	100	8.0	5.0
HEA120	114	120	8.0	5.0
HEA140	133	140	8.5	5.5
HEA160	152	160	9.0	6.0
HEA180	171	180	9.5	6.0
HEA200	190	200	10.0	6.5
HEA220	210	220	11.0	7.0
HEA240	230	240	12.0	7.5
HEA260	250	260	12.5	7.5
HEA280	270	280	13.0	8.0
HEA300	290	300	14.0	8.5
HEA320	310	300	15.5	9.0
HEA340	330	300	16.5	9.5
HEA360	350	300	17.5	10.0
HEA400	390	300	19.0	11.0
HEA450	440	300	21.0	11.5
HEA500	490	300	23.0	12.0
HEA550	540	300	24.0	12.5
HEA600	590	300	25.0	13.0
HEA650	640	300	26.0	13.5
HEA700	690	300	27.0	14.5
HEA800	790	300	28.0	15.0
HEA900	890	300	30.0	16.0
HEA1000	990	300	31.0	16.5

 Table 5.1: Different dimensions of HEA profiles.

## 5.4 Evaluation of Material Properties

There were two studies conducted for the material properties. The first was to study only the influence of the stiffness. This study was achieved by varying the modulus of elasticity E between 50 GPa and 300 GPa, to study the influence of only the stiffness EI.

The second was to study the influence of the weight per length. This was studied by varying the density of the steel. This was meant to represent additional weight from welds, bolts and other parts not modelled. The density was for this reason varied from  $7850 \text{kg/m}^3$  to  $9000 \text{kg/m}^3$ . The upper limit was 115% of the steel's usual density. This analysis was not to be quantitatively exact but rather to see how the dynamic properties changed in a qualitative sense.

It is important to note that these are only fictitious densities and elasticities. For real steel, the parameters are within a much smaller span. This is only to represent an addition of mass or stiffness.

# 6 Results

In this chapter follows the results of the different parametric studies. These being the parametric beam profile study, the geometrical multiparametric study and lastly an evaluation of the stiffness. The tables in the multiparametric study are put in the appendix.

## 6.1 Parametric Beam Profile Study

In figure 6.1, the number of main beams and beam dimensions were varied. The resulting bridge stiffnesses were then plotted against the three first natural frequencies. The green dots represent accelerations for a mode that are below  $2.5 \text{ m/s}^2$  and the red dots represent the ones that are not.

It can be seen that the plots become flatter for more beams. In practice this means that a bridge with fewer beams will achieve a higher natural frequency, with the same amount of material. It is also apparent that none of the acceleration requirements are met below 4 Hz in natural frequency.



Figure 6.1: Natural frequencies for different stiffnesses and amount of beams. The different coloured areas correspond to the natural frequency ranges for risk of resonance in table 3.3.

## 6.2 Geometrical Multiparametric Study

Below follows the results for the geometric parametric study. The tables in the appendix contain how natural frequencies, accelerations, ULS utilization ratio and SLS deflection ratio depend on bridge length, bridge width and number of main beams. In these studies, the ULS utilization ratio will be considered as the more critical limit as opposed to the maximum SLS deflection.

This multiparametric study will be presented with tables in the appendix with a set number of main beams with set beam profiles where the length is varied between each column from 4 to 20 m. Going from a width of 2 m to 7 m the results for the three first natural frequencies are shown in the each row followed by the accelerations corresponding to the different modes. Then, the ULS utilization ratio is presented and lastly, the SLS deflection ratio.

Each of the permutations is a different bridge model created in Abaqus. The footbridges are first examined against the dynamic criteria for dynamic class I footbridges and then dynamic class III. The description of the dynamic classes can be found in section 3.3.1.

### 6.2.1 Results For Class I Footbridges

These results are for class I footbridges, meaning footbridges bearing a very dense crowd.

In table A.1 the results can be found for a dynamic class I bridge with two HEA200 main beams. The table concludes that no bridge in this test achieves the dynamic comfort criteria when longer than 7 m. The ULS utilization rate remains under 50% in all the cases that achieve the dynamic comfort criteria, however the SLS criteria get breached at lower lengths. The wider bridges never achieve the SLS criteria.

In table A.2 the results can be found for a dynamic class I bridge with two HEA400 main beams. Three out of the four bridges achieve the dynamic comfort criteria up until reaching a bridge length around 11 m, except in the case with the 4 m wide bridge that has considerable more issues, already at 7 m length. This is an outlier as the wider bridge with 7 m width achieves less adverse dynamic response at those lengths. The ULS utilization rate remains low in all the cases that achieve the dynamic comfort criteria. The SLS utilization rate varies arbitrarily but is generally also close to the deflection limit when the dynamic criteria is breached.

In table A.3 the results can be found for a bridge with two HEA800 main beams. The bridge achieves the dynamic comfort criteria in all the bridges until the length of around 18 m. The ULS utilization ratio remains under 40% in all the cases that achieve the dynamic comfort criteria, lesser for the narrower bridges. The bridge succeeded the SLS criteria in all the cases where the dynamic criteria was achieved.

In table A.4 the results can be found for a dynamic class I bridge with three HEA200 main beams. The bridge achieves the dynamic comfort criteria up until around the length of 7 m, with better results with the narrower bridges. The ULS utilization rate remains well under 50% in most of the cases that achieve the dynamic comfort criteria. The widest bridge with the length 6 m is the only that reaches above 60% in ULS utilization rate. The deflection criteria gets breached before any of the bridges reaches the dynamic limit.

In table A.5 the results can be found for a dynamic class I bridge with three HEA400 main beams. The bridges all achieve the dynamic comfort criteria up until the length of 11 m where the bridges achieve more the narrower they are. The widest bridge with the length 11 m is the only that reaches around 40% in ULS utilization rate where the other bridges that achieve the dynamic comfort criteria all lie well under 30%. The SLS criteria gets breached at about the same dimensions as the dynamic criteria.

In table A.6 the results can be found for a dynamic class I bridge with three HEA800 main beams. The bridges achieve the dynamic comfort criteria in all but the largest widths and lengths. The ULS utilization rate remains under 40% in all the cases that achieve the dynamic comfort criteria, where the rate fast is decreasing for the narrower

bridges. The bridges also remains under the deflection limit in all the cases where the dynamic criteria is achieved.

In table A.7 the results can be found for a bridge with seven HEA200 main beams. The bridge achieves the dynamic comfort criteria up until the length of 8 m where the two narrower bridges manages until 9 m. The ULS utilization rate remains under 25% in most of the cases that achieve the dynamic comfort criteria. The widest bridge with the length 8 m is the only that reaches 40% in ULS utilization rate. However, the bridges reaches the deflection limit just before the dynamic criteria gets breached.

In table A.8 the results can be found for a bridge with seven HEA400 main beams. The bridges all achieve the dynamic comfort criteria up until the length of 14 m where the bridges achieve a little more the narrower they are. All the bridges that achieve the dynamic comfort criteria all lie well under 30% in ULS utilization rate. The deflection criteria gets breached at about the same dimensions as the dynamic criteria, or just after.

In table A.9 the results can be found for a dynamic class I bridge with seven HEA800 main beams. The bridges achieve the dynamic comfort criteria in all examined widths and lengths. The natural frequency is above 4.6 Hz in all the examined bridges. The ULS utilization rate remains under 21% in all the bridges, where the rate is fast decreasing for the narrower bridges. The SLS deflection ratio is under 61% in all the examined cases.

### 6.2.2 Results For Class III Footbridges

These results are for class III footbridges. The static criteria are the same for both class I and class III, which means that the conclusions regarding these results will be the same. A bridge that fails the static criteria in class I will fail the same criteria in class III.

In table A.10 the results can be found for a dynamic class III bridge with two HEA200 main beams. In this table the conclusion can be found that the bridges fails the servicability limit criteria before the bridge reaches the dynamic limit or the ULS limit. However, the bridge accelerations varies largely along the different bridge dimensions. The bridges are generally achieving the criteria in both shorter and longer lengths but can inbetween these dimensions fail the dynamic criteria, often in the first torsional mode or the second bending mode.

In table A.11 the results can be found for a dynamic class III bridge with two HEA400 main beams. In this table the conclusion can be found, just as with the previous bridge, that the bridges fails the serviceability limit criteria before the bridge reaches the dynamic limit or the ULS limit. However, the bridge accelerations varies largely along the different bridge dimensions. The bridges are generally achieving the criteria in both shorter and longer lengths but can in-between these dimensions fail the dynamic criteria, often in the first torsional mode or the second bending mode.

In table A.12 the results can be found for a dynamic class III bridge with two HEA800  $\,$ 

main beams. The bridge achieves the dynamic comfort criteria in all examined bridges. The ULS utilization ratio remains under 55%, lesser for the narrower bridges. The limiting factor in this case is the SLS criteria. The widest bridge reaches the deflection limit at 17 m length and the 4 m wide bridge at 20 m length.

In table A.13 the results can be found for a dynamic class III bridge with three HEA200 main beams. The bridges breaches the dynamic criteria at around the same dimensions as the ULS criteria. The limiting factor however in this case is the SLS deflection limit. This limit gets reached already at a bridge length of 7 m for the 3 m wide bridge, and less than that for the rest of the bridges. At this point the ULS utilization rate does not reach more than 40% before the bridges fail the SLS criteria.

In table A.14 the results can be found for a dynamic class III bridge with three HEA400 main beams. The bridges breaches the dynamic criteria at 20 m, where the widest bridge fails at 18 m. The widest bridge with the length 18 m is the only that reaches 90% in ULS utilization rate where the other bridges that achieve the dynamic comfort criteria reaches 60% or less in utilization rate. The SLS deflection limit is however the critical factor and the bridges reach this at bridge length 15 m for the narrowest bridge, and less when increasing the bridge width.

In table A.15 the results can be found for a dynamic class III bridge with three HEA800 main beams. The bridges achieve the dynamic criteria in all lengths with 39% utilization ratio at the most in the widest and longest bridge. This bridge dimension is also the only one that fails the SLS criteria.

In table A.16 the results can be found for a dynamic class III bridge with seven HEA200 main beams. The SLS criteria is the critical factor in this simulation where it fails at around 8 m. The dynamic criteria also gets breached before the ULS criteria, at 13 m bridge length for the widest bridge, and longer for the narrower bridges.

In table A.17 the results can be found for a dynamic class III bridge with seven HEA400 main beams. The SLS criteria is the critical factor in this simulation where it fails at around 16 m bridge length, at longer lengths for narrower bridges and short for the wider. The bridges all achieve with margin the ULS and dynamic criteria.

In table A.18 the results can be found for a dynamic class III bridge with seven HEA800 main beams. The bridges in any of the examined dimensions achieve all three criteria. The natural frequencies are all higher than 4.6 Hz and the utilization ratio never reaches more than 21%.

### 6.2.3 Comparison of Acceleration Results

In equation 2.29, the acceleration amplitude is described for a SDOF system. This can be used to calculate the maximum accelerations for a theoretical SDOF system for the entire bridge. If the load  $p_0$  in equation 2.29 is the total loads described by Sétra, the acceleration  $|a_0|$  can be shown in the same manner as figure 3.4, for different beam profiles. The theoretical accelerations for 2 main beams and beam profile HEA200 is shown in figure 6.2.



Figure 6.2: Accelerations for different frequencies and deck areas for 2 main beams and HEA200.

The accelerations for 2 main beams and beam profile HEA400 is shown in figure 6.3.



Figure 6.3: Accelerations for different frequencies and deck areas for 2 main beams and HEA400.

The accelerations for 2 main beams and beam profile HEA800 is shown in figure 6.4.



Figure 6.4: Accelerations for different frequencies and deck areas for 2 main beams and HEA800.

These can be compared to the results in tables A.1-A.9, which are the accelerations from the models. These tables are plotted in figure 6.5 for the first natural mode.



Figure 6.5: Accelerations for the first natural mode. The green dots are the accelerations below  $2.5 \text{ m/s}^2$  and the red dots are the accelerations above.

The accelerations for the second natural mode can be seen in figure 6.6.

The accelerations for the third natural mode can be seen in figure 6.7.



Figure 6.6: Accelerations for the second natural mode. The green dots are the accelerations below  $2.5 \text{ m/s}^2$  and the red dots are the accelerations above.



Figure 6.7: Accelerations for the third natural mode. The green dots are the accelerations below  $2.5 \text{ m/s}^2$  and the red dots are the accelerations above.

### 6.2.4 Weight per Length

The 24 different bridges with separate densities were evaluated and the results of the natural frequencies can be seen in figure 6.8.



Figure 6.8: Natural frequencies for densities in a span of 7850-9000 kg/m<sup>3</sup> with a bridge length of 16 m, width of 4 m, 3 main beams, and beam dimension HEA200.

All of the natural frequencies seem to vary linearly with the density in this span. However, it can be seen varying with  $\sqrt{1/\rho}$  in figure 6.9 for a wider span.



Figure 6.9: Natural frequencies for densities in a span of 5000-13000 kg/m<sup>3</sup> with a bridge length of 16 m, width of 4 m, 3 main beams, and beam dimension HEA200.

In figure 6.10, the accelerations for the different densities within the same span as in figure 6.8 can be seen.

The accelerations varied linearly for this span as well. Interesting to note is that the acceleration for the second bending mode varied most, followed by the first bending mode, and the first torsion mode last.



Figure 6.10: Accelerations for densities in a span of 7850-9000 kg/m<sup>3</sup> with a bridge length of 16 m, width of 4 m, 3 main beams, and beam dimension HEA200.

If a wider span is chosen as in figure 6.11, the accelerations can also be seen varying more, and the thresholds can be seen.



Figure 6.11: Accelerations for densities in a span of 5000-13000 kg/m<sup>3</sup> with a bridge length of 16 m, width of 4 m, 3 main beams, and beam dimension HEA200.

## 6.3 Stiffness

The stiffness was varied in the same manner as for the density, giving the graph visible in figure 6.12. It varies with  $\sqrt{E}$ .



Figure 6.12: Natural frequencies for stiffnesses in a span of  $50 \cdot 10^9 - 300 \cdot 10^9$  Pa with a bridge length of 16 m, width of 4 m, 3 main beams, and beam dimension HEA200.

# 7 Discussion

# 7.1 Bridge Model

The model was made to be as general a beam bridge as possible. This meant that a lot of parts were simplified or even neglected. These factors that were not taken into account would most likely have decreased the resulting accelerations. Damping is one of the factors that is difficult to realistically determine. Hence minimal damping was assumed, to provide a design case.

Another aspect to consider is that the model was an approximation of a real bridge structure. This also introduces a large number of simplifications and uncertainties that will affect the results. One of the most important and obvious simplifications are the boundary conditions. The prescribed boundary conditions were assumed to be infinitely stiff and that there was no stiffness in other directions. In reality all supports have some finite stiffness due to friction, either greater or smaller, in almost all directions.

The simplification of the boundary conditions will have a large effect on both the structural stiffness and damping. This in turn will affect the natural frequencies and accelerations. These results will be on the safe side, however not entirely accurate.

The approximation converged before the chosen mesh size of 0.2 m, both for the dynamic and static results. This meant that an even finer mesh size would not improve the result accuracy in any substantial way.

The modal truncation gave a high accuracy already at a few modes, meaning that the 20 modes used in this thesis might have been excessive.

## 7.2 Bridge Class I

If the bridge is Class I and has a small deck area, it is better to design it for the 5 Hz lowest natural frequency criteria than for accelerations. The distributed dynamic loads increased with decreasing deck areas. Since the dynamic loads are much greater at the smaller bridges, those will usually not achieve the acceleration criteria below 5 Hz. It is possible for it to achieve the acceleration criteria if the natural frequency is higher than circa 4.5 Hz.

It would seem that the threshold for when the acceleration criteria is surpassed in the span between 20% and 50% in static utilization ratio. This means that a bridge in class I can achieve 20% in static utilization ratio at a minimum. To be noted is that even if the static ULS utilization ratio is very low in these cases, the bridge fails in the

static SLS criteria. Bridges designed after the static SLS criteria are more optimised.

It is interesting to note that in table A.4 for the 17 and 18 meter long bridges with HEA200 beam profiles, 3 main beams and 7 m wide, the dynamic criteria is fulfilled. This is because the first two natural frequencies are low enough and the third natural frequency is close to 2.6 Hz, where the load is zero. However, these two bridges will have more complicated higher modes that must also be taken into account. These can be the third bending mode, or the second torsion mode. The static criteria are far exceeded as well for this bridge.

### 7.3 Bridge Class II

The results for Class II were not calculated, as this is easily achieved by multiplying the results for Class I by constants for the different cases. This is shown in equations 3.8 and 3.11. The bridges that have zero accelerations in Class I will also have zero acceleration in Class II.

### 7.4 Bridge Class III

Only the dynamic criteria are different in class III. This means, as stated before, that the results from the examinations in the static criteria are the same. For class III, the easiest way to achieve the dynamic criteria is to be outside of the natural frequency span between 1.7 Hz and 2.1 Hz, as can be seen in 3.6. For this class, a majority of bridges achieve the dynamic criteria.

It is duly noted that the bridges achieve the dynamic criteria both above and below the frequency range with the maximum risk of resonance. Hence, a very high utilization ratio is achieved in this class. The SLS criteria are however often the critical factor in this case. The bridges with slender beam dimensions generally breaches this deflection limit at very low lengths and widths. In comparison to class I, the dynamic criteria is much easier to fulfill in this dynamic class. That means that the deflection criteria that were about as critical as the dynamic criteria in class I, are now significantly the designing factor.

## 7.5 Weight per Length

The weight per length was varied by increasing the density of the steel. The natural frequencies and accelerations seemed linearly dependent on the density within the chosen span. When a larger span was chosen, the frequencies and accelerations could be seen varying by  $\sqrt{1/\rho}$ . This corresponds to the expression of the natural frequency shown in equation 2.32.

## 7.6 Stiffness

The effect of the stiffness was studied by varying the modulus of elasticity E. The natural frequencies varied by  $\sqrt{E}$ . This corresponds to the expression of the natural frequency shown in equation 2.32.

## 7.7 Sétra Technical Guide

The results obtained for the accelerations were quite binary: either large and unacceptable, or they were zero. This is probably a result of the Sétra guide's description of the dynamic loads, as can be seen in figures 3.4-3.6. For bridges with deck areas below 200 m<sup>2</sup>, the dynamic loads will be great. As the bridges analysed in this thesis are all below 200 m<sup>2</sup> in deck area, the loads and accelerations will be very large or zero.

It is important to note that a very small shift in natural frequency can have a great impact on the accelerations. This means that while it is possible to interpolate between the natural frequencies, the accelerations are more difficult to predict.

As can be seen between the results of Class I and Class III, the choice of bridge class will have the largest impact. Class I is suited for large cities with dense crowds. As this class is the most adverse, it will also be used for bridges in the center of cities with very large populations, e.g. Paris or London. Therefore it might not be descriptive for conditions of a typical railway crossing in a Swedish city.

## 7.8 Comparison of Accelerations

The accelerations seen in figures 6.2-6.7 showed a similarity in results. The accelerations from the numerical model was a bit lower than the generalized SDOF. However, it shows that this simple calculation can yield quite realistic results for bridges of this type.

# 8 Conclusions

The aim of the thesis was to research if there is a way of reducing the amount of material used to secure the vibration criteria, and if there is a way of predicting these criteria in the design phase. The conclusions that could be drawn have been listed below.

- It is difficult to reduce the amount of material used for bridges in class I without exceeding the vibration criteria. For class II and III however, it is more reasonable.
- The Sétra technical guide is possibly not the best tool to evaluate bridges with smaller deck areas.
- The bridge classes used in the Sétra technical guide will have the largest impact on if the acceleration criteria are met or not.
- For the bridges that had a greater length than width, the first bending mode was always first, first torsion mode second, and second bending mode third.
- A simple generalized SDOF system is quite apt at predicting the accelerations with a large safety margin.
- Class I bridges evaluated in this thesis did not meet the acceleration criteria below 4.5 Hz in the first natural frequency. Therefore similar bridges below 4.5 Hz are not worth investigating for acceleration requirements.
- Class II bridges were not evaluated specifically in this thesis, but their acceleration criteria will be met somewhere between the class I and class III frequencies.
- Class III bridges needed only to be outside of the frequency range 1.7-2.1 Hz with the highest risk of resonance to achieve the dynamic criteria. This means that it is wiser to avoid this frequency range rather than trying to achieve a lower acceleration within it.
- For smaller footbridges as the ones in this thesis, the deflection requirements in SLS will usually be transgressed before the ULS utilization ratios. This deflection requirement is, however, dependent on the country and norm in question.

## 8.1 Future Work

In this section, possible related topics to be worked on in the future is presented.

 Conduct a similar study of parameters for longer footbridges with multiple spans and how the accelerations are affected.  Conduct a similar study of parameters with a different guide than Sétra to see if it is better suited for smaller footbridges.

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# Appendix A

# **Bridge Tables**

## A.1 Class I tables

**Table A.1:** Natural frequencies and accelerations for class I bridges with HEA200 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	16.30	10.87	7.69	5.70	4.39	3.48	2.83	2.34	1.97	1.68	1.45	1.26	1.11	0.98	0.88	0.79	0.71
$f_{n_2}$ [Hz]	23.01	15.24	10.89	8.25	6.49	5.29	4.41	3.77	3.26	2.88	2.55	2.31	2.09	1.92	1.76	1.64	1.52
$f_{n_3}$ [Hz]	39.36	32.78	25.97	20.55	16.37	13.25	10.90	9.10	7.70	6.60	5.71	4.99	4.39	3.90	3.48	3.13	2.83
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	7.22	8.93	2.39	16.83	30.83	28.69	18.26	10.31	4.18	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	5.89	7.58	5.98	2.47	2.46	14.90	25.13	24.37	23.69	21.15	16.62
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	5.08	6.51	6.32	4.06	1.70
UR	0.13	0.18	0.25	0.33	0.42	0.51	0.61	0.72	0.85	0.98	1.12	1.27	1.43	1.60	1.78	1.97	2.17
u/L/400	0.31	0.54	0.88	1.36	2.00	2.83	3.86	5.12	6.63	8.42	10.50	12.90	15.65	18.76	22.26	26.17	30.52
b=3 m																	
$f_{n_1}$ [Hz]	12.54	8.93	6.49	4.88	3.78	3.01	2.45	2.03	1.71	1.46	1.26	1.10	0.97	0.86	0.76	0.69	0.62
$f_{n_2}$ [Hz]	20.35	13.49	9.59	7.21	5.62	4.54	3.75	3.18	2.72	2.38	2.09	1.88	1.68	1.53	1.40	1.29	1.19
$f_{n_3}$ [Hz]	21.96	19.78	17.36	14.97	12.64	10.64	8.97	7.61	6.51	5.62	4.89	4.29	3.79	3.37	3.01	2.71	2.45
$a_{B1} [m/s^2]$	0.00	0.00	0.00	1.44	8.63	4.14	9.13	28.92	27.62	17.40	9.48	3.46	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	4.67	7.77	5.35	1.08	11.91	26.26	25.37	23.89	18.21	13.08	9.36	5.88
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92	5.66	6.11	5.66	2.95	0.77	6.36
UR	0.18	0.26	0.35	0.46	0.58	0.71	0.85	1.01	1.18	1.36	1.56	1.76	1.99	2.22	2.47	2.73	3.00
u/L/400	0.61	0.89	1.34	1.99	2.86	3.99	5.40	7.14	9.21	11.67	14.54	17.85	21.63	25.91	30.73	36.12	42.11
b=4 m																	
$f_{n_1}$ [Hz]	9.31	7.24	5.51	4.24	3.33	2.67	2.18	1.81	1.53	1.31	1.13	0.98	0.87	0.77	0.69	0.62	0.56
$f_{n_2}$ [Hz]	13.86	12.20	8.69	6.52	5.07	4.08	3.35	2.82	2.41	2.09	1.83	1.63	1.46	1.32	1.20	1.10	1.01
$f_{n_3}$ [Hz]	18.17	12.68	11.61	10.58	9.45	8.35	7.29	6.35	5.54	4.84	4.25	3.76	3.34	2.98	2.67	2.41	2.18
$a_{B1} [m/s^2]$	0.00	0.00	0.00	8.59	7.37	0.63	23.56	26.53	19.09	10.59	4.28	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	7.86	7.02	1.98	10.46	26.15	25.20	21.99	15.38	10.49	6.24	3.12	0.26
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.34	5.94	6.02	5.28	2.60	0.48	7.77	16.52
UR	0.25	0.34	0.45	0.58	0.71	0.83	0.97	1.13	1.32	1.51	1.73	1.96	2.22	2.48	2.77	3.07	3.43
u/L/400	1.21	1.50	2.02	2.80	3.88	5.29	7.07	9.25	11.88	14.99	18.63	22.82	27.62	33.06	39.19	46.02	53.63
b=7 m																	
$f_{n_1}$ [Hz]	3.84	3.56	3.19	2.75	2.32	1.94	1.63	1.38	1.18	1.02	0.89	0.78	0.68	0.61	0.54	0.49	0.44
$f_{n_2}$ [Hz]	5.28	4.86	4.54	4.32	4.06	3.26	2.68	2.24	1.90	1.64	1.43	1.26	1.12	1.01	0.91	0.83	0.75
$f_{n_3}$ [Hz]	9.60	7.41	6.21	5.18	4.11	3.92	3.72	3.50	3.27	3.03	2.79	2.56	2.34	2.14	1.96	1.79	1.64
$a_{B1} [\mathrm{m/s^2}]$	0.00	0.00	0.00	1.59	17.00	27.14	22.43	12.55	5.57	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	8.76	5.67	0.62	17.88	23.84	21.03	13.56	8.00	3.55	0.20	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	2.23	6.26	8.05	8.40	7.85	7.46	7.13	5.66	3.45	1.42	1.90	11.17	18.61	19.19	18.27	16.03
UR	0.25	0.34	0.45	0.58	0.71	0.83	0.97	1.13	1.32	1.51	1.73	1.96	2.22	2.48	2.77	3.07	3.43
u/L/400	6.79	6.51	6.82	7.68	9.12	11.14	13.81	17.16	21.26	26.16	31.93	38.62	46.32	55.04	64.90	75.92	88.20

**Table A.2:** Natural frequencies and accelerations for class I bridges with HEA400 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	42.35	29.26	21.00	15.76	12.21	9.73	7.92	6.58	5.54	4.73	4.09	3.57	3.14	2.78	2.49	2.23	2.02
$f_{n_2}$ [Hz]	55.40	37.56	27.02	20.43	15.98	12.93	10.65	9.00	7.68	6.70	5.86	5.24	4.66	4.25	3.83	3.54	3.23
$f_{n_3}$ [Hz]	81.31	76.46	62.92	52.49	42.50	35.24	29.24	24.73	21.04	18.15	15.77	13.84	12.22	10.88	9.73	8.77	7.93
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.02	5.85	5.66	3.69	1.22	4.75	14.80	19.60
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.81	3.92	4.05	3.94	3.03
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.04	0.06	0.07	0.09	0.12	0.14	0.17	0.20	0.23	0.27	0.30	0.34	0.38	0.42	0.47	0.52	0.57
u/L/400	0.05	0.07	0.11	0.16	0.23	0.33	0.44	0.59	0.76	0.96	1.19	1.47	1.77	2.12	2.52	2.96	3.45
b=3 m																	
$f_{n_1}$ [Hz]	33.45	24.51	18.06	13.76	10.74	8.60	7.03	5.85	4.94	4.22	3.65	3.19	2.80	2.49	2.22	2.00	1.80
$f_{n_2}$ [Hz]	49.47	33.87	24.37	18.39	14.34	11.53	9.46	7.94	6.73	5.83	5.07	4.49	3.97	3.58	3.21	2.94	2.67
$f_{n_3}$ [Hz]	51.91	49.96	44.35	39.59	33.65	28.94	24.54	21.10	18.14	15.79	13.79	12.16	10.77	9.61	8.62	7.77	7.04
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	5.45	5.38	3.80	1.28	4.39	14.36	18.45	17.99
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.92	4.40	4.27	3.18	1.71	0.34
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.06	0.08	0.10	0.13	0.16	0.19	0.23	0.27	0.31	0.36	0.41	0.46	0.51	0.57	0.63	0.69	0.76
u/L/400	0.08	0.11	0.16	0.23	0.32	0.45	0.60	0.79	1.01	1.28	1.59	1.94	2.35	2.81	3.33	3.91	4.56
b=4 m																	
$f_{n_1}$ [Hz]	9.31	7.24	5.51	4.24	3.33	2.67	2.18	1.81	1.53	1.31	1.13	0.98	0.87	0.77	0.69	0.62	0.56
$f_{n_2}$ [Hz]	13.86	12.20	8.69	6.52	5.07	4.08	3.35	2.82	2.41	2.09	1.83	1.63	1.46	1.32	1.20	1.10	1.01
$f_{n_3}$ [Hz]	18.17	12.68	11.61	10.58	9.45	8.35	7.29	6.35	5.54	4.84	4.25	3.76	3.34	2.98	2.67	2.41	2.18
$a_{B1} [m/s^2]$	0.00	0.00	0.00	8.59	7.37	0.63	23.56	26.53	19.09	10.59	4.28	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	7.86	7.02	1.98	10.46	26.15	25.20	21.99	15.38	10.49	6.24	3.12	0.26
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.34	5.94	6.02	5.28	2.60	0.48	7.77	16.52
UR	0.07	0.10	0.13	0.17	0.20	0.25	0.29	0.34	0.39	0.45	0.51	0.57	0.64	0.71	0.79	0.87	0.95
u/L/400	0.15	0.19	0.24	0.33	0.44	0.59	0.78	1.01	1.28	1.61	1.99	2.44	2.94	3.51	4.16	4.88	5.67
b=7 m																	
$f_{n_1}$ [Hz]	11.09	10.27	9.09	7.86	6.65	5.62	4.75	4.04	3.47	3.00	2.62	2.30	2.04	1.81	1.62	1.46	1.32
$f_{n_2}$ [Hz]	13.94	13.01	12.26	11.82	10.77	8.68	7.13	5.96	5.05	4.34	3.76	3.30	2.91	2.60	2.32	2.10	1.90
$f_{n_3}$ [Hz]	19.33	16.52	14.77	13.73	11.29	10.87	10.29	9.78	9.13	8.54	7.88	7.29	6.69	6.16	5.65	5.20	4.78
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	1.77	5.13	4.77	2.26	0.10	9.76	15.69	15.14	13.05	9.39	6.38
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.58	4.16	3.51	1.52	0.07	8.08	14.24	13.91
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.09
UR	0.12	0.17	0.22	0.28	0.34	0.41	0.48	0.56	0.64	0.73	0.83	0.93	1.04	1.15	1.27	1.39	1.52
u/L/400	0.85	0.81	0.82	0.92	1.06	1.28	1.55	1.90	2.32	2.82	3.41	4.10	4.89	5.78	6.79	7.92	9.18

**Table A.3:** Natural frequencies and accelerations for class I bridges with HEA800 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	75.36	56.53	42.01	32.48	25.56	20.66	16.97	14.19	12.02	10.31	8.94	7.82	6.90	6.13	5.48	4.93	4.45
$f_{n_2}$ [Hz]	88.77	64.59	48.26	37.37	29.67	24.19	20.05	16.98	14.51	12.64	11.05	9.83	8.75	7.92	7.14	6.55	5.98
$f_{n_3}$ [Hz]	104.85	102.14	85.65	72.73	61.98	53.26	45.95	39.95	34.90	30.72	27.16	24.19	21.63	19.48	17.60	16.00	14.58
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	2.73
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.15	0.16	0.18	0.20	0.22
u/L/400	0.02	0.02	0.03	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.23	0.28	0.34	0.40	0.47	0.56	0.64
b=3 m																	
$f_{n_1}$ [Hz]	62.13	48.51	36.78	28.83	22.87	18.59	15.32	12.85	10.90	9.37	8.13	7.12	6.28	5.58	4.99	4.49	4.06
$f_{n_2}$ [Hz]	78.92	61.02	45.78	35.53	28.18	22.93	18.95	15.97	13.60	11.77	10.25	9.06	8.02	7.20	6.46	5.88	5.34
$f_{n_3}$ [Hz]	80.02	81.22	75.32	67.96	60.24	53.86	47.57	42.22	36.83	32.65	28.77	25.69	22.91	20.63	18.58	16.88	15.34
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	2.45	3.76
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.12	0.14	0.15	0.17	0.19	0.21	0.24	0.26	0.28
u/L/400	0.02	0.03	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.24	0.29	0.36	0.43	0.51	0.60	0.71	0.82
b=4 m																	
$f_{n_1}$ [Hz]	50.43	41.18	32.08	25.62	20.54	16.83	13.93	11.73	9.98	8.59	7.47	6.55	5.78	5.14	4.60	4.14	3.75
$f_{n_2}$ [Hz]	61.90	56.47	42.65	33.23	26.38	21.47	17.75	14.95	12.72	10.99	9.56	8.42	7.45	6.67	5.98	5.42	4.91
$f_{n_3}$ [Hz]	63.00	62.90	58.89	56.64	50.36	46.24	40.67	36.59	32.24	28.88	25.63	23.04	20.63	18.66	16.86	15.35	13.98
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.84	3.62	3.53
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.13	0.15	0.17	0.19	0.21	0.24	0.26	0.29	0.32	0.35
u/L/400	0.04	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.24	0.30	0.36	0.44	0.53	0.63	0.74	0.86	1.00
b=7 m																	
$f_{n_1}$ [Hz]	25.05	23.30	20.33	17.59	14.83	12.61	10.69	9.17	7.90	6.87	6.01	5.31	4.71	4.20	3.77	3.40	3.09
$f_{n_2}$ [Hz]	29.57	28.54	27.12	26.50	21.99	17.99	14.91	12.57	10.72	9.25	8.05	7.09	6.27	5.60	5.02	4.53	4.11
$f_{n_3}$ [Hz]	35.62	32.65	30.23	27.61	25.30	24.54	23.06	21.98	20.38	19.11	17.57	16.30	14.95	13.81	12.68	11.71	10.78
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.25	3.28	3.18	3.09	1.82
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.58	2.65
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.05	0.07	0.09	0.11	0.13	0.16	0.18	0.21	0.24	0.27	0.30	0.34	0.38	0.41	0.46	0.50	0.54
u/L/400	0.16	0.16	0.16	0.18	0.20	0.24	0.29	0.35	0.42	0.50	0.60	0.72	0.85	1.00	1.17	1.36	1.57

**Table A.4:** Natural frequencies and accelerations for class I bridges with HEA200 and 3 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	12.54	8.93	6.49	4.88	3.78	3.01	2.45	2.03	1.71	1.46	1.26	1.10	0.97	0.86	0.76	0.69	0.62
$f_{n_2}$ [Hz]	20.35	13.49	9.59	7.21	5.62	4.54	3.75	3.18	2.72	2.38	2.09	1.88	1.68	1.53	1.40	1.29	1.19
$f_{n_3}$ [Hz]	21.96	19.78	17.36	14.97	12.64	10.64	8.97	7.61	6.51	5.62	4.89	4.29	3.79	3.37	3.01	2.71	2.45
$a_{B1} [m/s^2]$	0.00	0.00	0.00	1.44	8.63	4.14	9.13	28.92	27.62	17.40	9.48	3.46	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	4.67	7.77	5.35	1.08	11.91	26.26	25.37	23.89	18.21	13.08	9.36	5.88
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92	5.66	6.11	5.66	2.95	0.77	6.36
UR	0.18	0.26	0.35	0.46	0.58	0.71	0.85	1.01	1.18	1.36	1.56	1.76	1.99	2.22	2.47	2.73	3.00
u/L/400	0.61	0.89	1.34	1.99	2.86	3.99	5.40	7.14	9.21	11.67	14.54	17.85	21.63	25.91	30.73	36.12	42.11
b=3 m																	
$f_{n_1}$ [Hz]	17.24	11.34	7.99	5.91	4.54	3.60	2.92	2.42	2.03	1.73	1.50	1.30	1.15	1.02	0.91	0.81	0.73
$f_{n_2}$ [Hz]	20.89	13.88	9.89	7.44	5.82	4.72	3.90	3.32	2.85	2.50	2.20	1.98	1.78	1.63	1.48	1.38	1.27
$f_{n_3}$ [Hz]	40.41	35.98	28.80	22.00	17.26	13.86	11.36	9.46	7.99	6.84	5.91	5.16	4.55	4.03	3.60	3.23	2.92
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	4.71	7.78	2.96	10.26	26.82	25.76	17.57	10.38	4.83	0.48	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	3.00	8.09	6.91	2.29	5.70	21.73	26.43	25.59	22.26	16.63	12.61	8.73
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.32	5.66	5.50	4.24	2.10
UR	0.14	0.19	0.25	0.32	0.40	0.49	0.59	0.70	0.82	0.95	1.09	1.23	1.39	1.55	1.73	1.91	2.10
u/L/400	0.30	0.52	0.86	1.32	1.94	2.74	3.74	4.96	6.42	8.15	10.17	12.50	15.17	18.19	21.58	25.37	29.59
b=4 m																	
$f_{n_1}$ [Hz]	14.96	10.00	7.11	5.29	4.08	3.24	2.63	2.18	1.83	1.56	1.35	1.18	1.03	0.92	0.82	0.73	0.66
$f_{n_2}$ [Hz]	18.63	12.53	8.94	6.71	5.23	4.22	3.47	2.93	2.50	2.18	1.91	1.71	1.52	1.39	1.26	1.16	1.06
$f_{n_3}$ [Hz]	29.77	24.39	21.78	18.90	14.98	12.14	10.02	8.39	7.12	6.11	5.30	4.63	4.08	3.63	3.24	2.91	2.63
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	7.68	5.73	0.27	21.68	24.58	18.99	11.33	5.51	1.04	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	8.07	7.80	3.05	5.64	23.04	26.38	25.48	18.47	13.17	8.52	5.12	1.97
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.61	5.44	5.24	4.07	1.93	0.20
UR	0.20	0.26	0.33	0.41	0.51	0.62	0.75	0.88	1.04	1.20	1.37	1.56	1.76	1.96	2.18	2.41	2.65
u/L/400	0.42	0.72	1.16	1.74	2.52	3.52	4.78	6.31	8.15	10.32	12.87	15.80	19.15	22.95	27.23	32.02	37.34
b=7 m																	
$f_{n_1}$ [Hz]	10.56	7.31	5.33	4.04	3.17	2.54	2.08	1.73	1.46	1.25	1.08	0.94	0.83	0.73	0.66	0.59	0.53
$f_{n_2}$ [Hz]	12.13	9.29	6.96	5.32	4.17	3.36	2.76	2.31	1.96	1.70	1.48	1.31	1.16	1.04	0.94	0.86	0.78
$f_{n_3}$ [Hz]	17.38	14.18	11.17	9.51	8.52	7.95	7.32	6.21	5.33	4.63	4.05	3.57	3.17	2.83	2.54	2.29	2.08
$a_{B1} [m/s^2]$	0.00	0.00	0.00	7.51	4.80	3.02	23.05	21.64	13.47	6.92	2.11	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	7.65	6.87	1.36	15.10	25.15	24.08	15.93	9.87	5.00	1.33	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.63	5.32	5.05	3.42	1.32	2.04	10.36	16.33
UR	0.39	0.52	0.66	0.80	0.95	1.11	1.30	1.50	1.73	1.97	2.24	2.52	2.83	3.16	3.54	3.96	4.46
u/L/400	0.88	1.47	2.30	3.35	4.67	6.31	8.33	10.77	13.71	17.18	21.24	25.93	31.30	37.38	44.24	51.90	60.43

**Table A.5:** Natural frequencies and accelerations for class I bridges with HEA400 and 3 main beams. UR is the Utilization Ratio, u/L/400 is the SLS deflection ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	51.17	34.16	24.21	18.02	13.91	11.05	8.98	7.45	6.27	5.35	4.62	4.03	3.54	3.14	2.80	2.52	2.27
$f_{n_2}$ [Hz]	58.32	39.78	28.84	21.99	17.37	14.18	11.82	10.08	8.71	7.66	6.79	6.11	5.51	5.05	4.62	4.29	3.96
$f_{n_3}$ [Hz]	117.28	96.85	80.00	64.83	51.20	41.52	34.13	28.58	24.22	20.79	18.02	15.77	13.91	12.36	11.05	9.94	8.98
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.55	5.19	5.02	3.30	1.21	3.03	11.74
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.04	3.72	4.05
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.03	0.04	0.05	0.07	0.09	0.10	0.12	0.15	0.17	0.20	0.22	0.25	0.28	0.31	0.35	0.38	0.42
u/L/400	0.03	0.05	0.08	0.12	0.17	0.25	0.33	0.44	0.57	0.72	0.90	1.11	1.34	1.60	1.90	2.24	2.61
b=3 m																	
$f_{n_1}$ [Hz]	44.78	30.48	21.76	16.28	12.59	10.02	8.16	6.77	5.70	4.87	4.20	3.67	3.23	2.86	2.55	2.29	2.07
$f_{n_2}$ [Hz]	52.17	35.78	25.82	19.55	15.30	12.36	10.18	8.58	7.32	6.36	5.56	4.95	4.41	3.99	3.61	3.32	3.03
$f_{n_3}$ [Hz]	94.38	85.78	69.48	56.13	44.85	36.84	30.44	25.64	21.78	18.76	16.28	14.27	12.60	11.21	10.02	9.02	8.16
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.87	5.02	4.86	3.68	1.48	1.70	10.66	16.85
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	3.54	4.64	4.51	3.92	2.29
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.04	0.06	0.08	0.09	0.12	0.14	0.17	0.19	0.23	0.26	0.29	0.33	0.37	0.41	0.46	0.50	0.55
u/L/400	0.04	0.07	0.10	0.16	0.23	0.32	0.43	0.57	0.73	0.93	1.16	1.42	1.72	2.06	2.45	2.88	3.35
b=4 m																	
$f_{n_1}$ [Hz]	39.48	27.37	19.72	14.85	11.52	9.19	7.49	6.22	5.25	4.48	3.87	3.38	2.97	2.64	2.35	2.11	1.91
$f_{n_2}$ [Hz]	47.07	32.75	23.68	17.93	14.00	11.27	9.25	7.76	6.59	5.70	4.96	4.38	3.88	3.50	3.14	2.87	2.61
$f_{n_3}$ [Hz]	72.11	61.03	54.21	48.96	39.56	32.84	27.32	23.16	19.75	17.07	14.85	13.05	11.53	10.27	9.20	8.29	7.50
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.18	4.71	4.42	2.05	0.19	8.21	15.75	15.76
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	3.79	4.76	4.62	3.04	1.47	0.04
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.06	0.08	0.10	0.12	0.15	0.18	0.21	0.24	0.28	0.32	0.36	0.41	0.46	0.51	0.56	0.62	0.68
u/L/400	0.06	0.09	0.13	0.20	0.29	0.40	0.53	0.70	0.90	1.14	1.42	1.74	2.11	2.52	2.99	3.51	4.09
b=7 m																	
$f_{n_1}$ [Hz]	28.30	20.35	15.07	11.60	9.14	7.38	6.06	5.07	4.29	3.68	3.18	2.78	2.45	2.18	1.94	1.75	1.58
$f_{n_2}$ [Hz]	32.69	25.23	18.95	14.61	11.50	9.28	7.62	6.38	5.41	4.65	4.04	3.55	3.13	2.80	2.51	2.27	2.05
$f_{n_3}$ [Hz]	42.60	36.41	29.00	24.96	22.36	20.96	19.87	17.49	15.10	13.22	11.61	10.29	9.16	8.22	7.39	6.69	6.08
$a_{B1} [\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.93	4.21	2.93	0.88	4.47	12.31	14.06	13.67	10.99
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.12	4.71	4.55	2.93	1.06	3.12	10.68	15.75
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.19	0.16	0.20	0.22	0.27	0.31	0.36	0.41	0.46	0.52	0.59	0.65	0.73	0.80	0.89	0.97	1.06
1 .	0.12	0.10	0.20	0.23	0.27	0.01	0.00	0.11	0.40	0.02	0.05	0.00	0.10	0.00	0.00	0.01	1.00

**Table A.6:** Natural frequencies and accelerations for class I bridges with HEA800 and 3 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	89.67	65.00	47.70	36.40	28.52	22.94	18.80	15.69	13.27	11.38	9.85	8.61	7.59	6.74	6.03	5.42	4.90
$f_{n_2}$ [Hz]	91.73	66.25	49.99	39.09	31.35	25.82	21.64	18.51	16.02	14.08	12.47	11.20	10.10	9.21	8.42	7.78	7.19
$f_{n_3}$ [Hz]	127.21	103.63	85.70	72.20	61.24	52.47	45.22	39.30	34.36	30.27	26.82	23.92	21.44	19.34	17.52	15.95	14.58
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.13	0.14	0.15	0.17
u/L/400	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.18	0.22	0.26	0.31	0.37	0.44	0.51
b=3 m																	
$f_{n_1}$ [Hz]	79.59	58.76	43.45	33.45	26.28	21.20	17.40	14.54	12.31	10.56	9.14	8.00	7.05	6.27	5.60	5.04	4.55
$f_{n_2}$ [Hz]	85.35	64.01	48.30	37.69	30.02	24.54	20.38	17.26	14.77	12.85	11.26	10.00	8.91	8.05	7.27	6.65	6.08
$f_{n_3}$ [Hz]	106.89	91.74	78.98	69.30	61.03	54.13	48.05	42.82	38.16	34.14	30.56	27.49	24.76	22.42	20.34	18.57	16.96
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.91
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.14	0.16	0.18	0.19	0.21
u/L/400	0.02	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.18	0.22	0.27	0.33	0.39	0.46	0.54	0.63
b=4 m																	
$f_{n_1}$ [Hz]	71.08	53.56	39.90	30.93	24.38	19.73	16.21	13.57	11.50	9.87	8.55	7.48	6.60	5.86	5.24	4.72	4.26
$f_{n_2}$ [Hz]	78.72	60.22	45.55	35.62	28.34	23.14	19.16	16.18	13.80	11.96	10.43	9.22	8.18	7.34	6.60	6.01	5.46
$f_{n_3}$ [Hz]	93.30	81.39	71.42	63.97	57.60	52.26	47.45	43.20	39.21	35.12	30.83	27.42	24.39	21.91	19.71	17.87	16.22
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18	3.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.11	0.12	0.14	0.16	0.18	0.19	0.21	0.23	0.26
u/L/400	0.02	0.02	0.03	0.04	0.06	0.08	0.10	0.14	0.17	0.22	0.27	0.32	0.39	0.47	0.55	0.64	0.75
b=7 m																	
$f_{n_1}$ [Hz]	53.17	41.13	31.41	24.87	19.88	16.28	13.48	11.35	9.65	8.31	7.22	6.33	5.59	4.98	4.45	4.01	3.63
$f_{n_2}$ [Hz]	60.60	48.83	37.77	29.99	24.01	19.68	16.32	13.78	11.75	10.16	8.84	7.79	6.89	6.16	5.53	5.00	4.53
$f_{n_3}$ [Hz]	66.81	59.77	52.89	48.11	44.13	41.71	39.21	35.97	31.47	28.04	24.82	22.27	19.93	18.02	16.28	14.82	13.50
$a_{B1} [\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	2.04	2.91	2.83
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.85
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.05	0.06	0.07	0.09	0.10	0.12	0.14	0.15	0.17	0.20	0.22	0.24	0.27	0.30	0.33	0.36	0.39
u/L/400	0.03	0.04	0.06	0.08	0.10	0.13	0.17	0.21	0.27	0.33	0.40	0.49	0.58	0.69	0.81	0.95	1.10
**Table A.7:** Natural frequencies and accelerations for class I bridges with HEA200 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	27.08	17.46	12.18	8.97	6.88	5.44	4.41	3.65	3.07	2.61	2.26	1.96	1.73	1.53	1.37	1.23	1.11
$f_{n_2}$ [Hz]	28.31	18.93	13.69	10.48	8.36	6.89	5.82	5.02	4.40	3.91	3.51	3.19	2.92	2.69	2.49	2.33	2.17
$f_{n_3}$ [Hz]	71.63	65.54	46.88	35.11	27.08	21.49	17.47	14.47	12.18	10.39	8.97	7.82	6.88	6.10	5.44	4.89	4.41
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	5.25	6.81	3.81	0.12	16.64	23.32	22.57	16.59	11.10	6.67	3.06
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.25	6.69	6.44	4.61	2.38	0.67	4.97	12.09	18.38
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73	3.71
UR	0.05	0.07	0.09	0.12	0.15	0.19	0.23	0.27	0.32	0.37	0.42	0.48	0.54	0.60	0.67	0.74	0.82
u/L/400	0.10	0.18	0.32	0.50	0.75	1.07	1.46	1.94	2.52	3.20	3.99	4.91	5.95	7.14	8.47	9.96	11.61
b=3 m	•																
$f_{n_1}$ [Hz]	24.64	15.87	11.07	8.15	6.25	4.94	4.01	3.31	2.79	2.37	2.05	1.78	1.57	1.39	1.24	1.11	1.00
$f_{n_2}$ [Hz]	25.38	16.71	11.90	8.97	7.04	5.71	4.75	4.04	3.48	3.06	2.71	2.43	2.20	2.01	1.84	1.70	1.58
$f_{n_3}$ [Hz]	43.11	37.44	34.57	31.97	24.66	19.55	15.89	13.15	11.07	9.44	8.15	7.11	6.25	5.54	4.94	4.44	4.01
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.50	6.68	5.67	1.41	10.58	22.53	21.76	17.09	11.36	6.78	3.10	0.11
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	2.43	7.32	7.01	3.85	0.87	8.37	19.65	23.57	22.90	22.29	17.90
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	3.44	4.72
UR	0.06	0.09	0.12	0.15	0.19	0.24	0.29	0.34	0.41	0.47	0.54	0.61	0.69	0.77	0.86	0.95	1.05
u/L/400	0.13	0.24	0.41	0.65	0.97	1.38	1.89	2.51	3.26	4.14	5.17	6.35	7.71	9.24	10.97	12.89	15.04
b=4 m																	
$f_{n_1}$ [Hz]	22.65	14.60	10.19	7.50	5.75	4.55	3.69	3.05	2.56	2.18	1.88	1.64	1.44	1.28	1.14	1.02	0.92
$f_{n_2}$ [Hz]	23.41	15.30	10.83	8.10	6.31	5.08	4.18	3.53	3.02	2.63	2.31	2.06	1.84	1.67	1.52	1.40	1.29
$f_{n_3}$ [Hz]	32.28	25.93	22.83	21.36	20.41	17.97	14.62	12.10	10.19	8.69	7.50	6.54	5.75	5.10	4.55	4.08	3.69
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	4.11	6.54	3.33	1.67	18.13	21.01	18.60	12.43	7.58	3.71	0.61	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	7.50	7.16	3.57	0.23	14.82	24.50	23.72	22.11	16.63	12.42	8.70
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.01	5.07	4.84
UR	0.07	0.10	0.14	0.19	0.24	0.29	0.35	0.42	0.50	0.57	0.66	0.75	0.84	0.94	1.05	1.16	1.28
u/L/400	0.16	0.31	0.52	0.81	1.20	1.69	2.31	3.07	3.99	5.07	6.33	7.79	9.45	11.33	13.45	15.81	18.44
b=7 m																	
$f_{n_1}$ [Hz]	18.21	11.85	8.32	6.15	4.73	3.74	3.04	2.51	2.11	1.80	1.56	1.36	1.19	1.06	0.94	0.85	0.76
$f_{n_2}$ [Hz]	19.25	12.66	8.94	6.65	5.14	4.11	3.36	2.80	2.38	2.05	1.78	1.57	1.39	1.25	1.13	1.03	0.94
$f_{n_3}$ [Hz]	21.90	15.44	12.03	10.15	9.02	8.34	7.89	7.61	7.40	7.11	6.15	5.37	4.73	4.19	3.75	3.37	3.04
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	2.45	6.57	3.32	3.82	20.13	19.14	14.15	8.62	4.50	1.27	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	7.12	6.39	1.63	11.07	23.73	22.87	18.05	12.05	7.48	3.68	0.76	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.73	4.88	4.68	4.29	2.38
UR	0.13	0.19	0.25	0.32	0.40	0.48	0.57	0.68	0.79	0.91	1.04	1.18	1.33	1.49	1.66	1.84	2.02
u/L/400	0.27	0.52	0.88	1.35	1.96	2.74	3.70	4.87	6.27	7.93	9.86	12.09	14.65	17.54	20.81	24.45	28.53

**Table A.8:** Natural frequencies and accelerations for class I bridges with HEA400 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	63.34	41.67	29.41	21.82	16.81	13.34	10.84	8.98	7.56	6.45	5.57	4.86	4.27	3.79	3.38	3.03	2.74
$f_{n_2}$ [Hz]	66.31	45.79	33.72	26.08	20.94	17.32	14.66	12.65	11.09	9.86	8.85	8.03	7.34	6.76	6.26	5.84	5.46
$f_{n_3}$ [Hz]	109.52	90.11	75.26	63.48	53.69	45.59	38.87	33.41	28.97	25.20	21.82	19.08	16.81	14.93	13.34	12.00	10.84
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	3.44	3.66	3.46	1.88	0.59
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR [%]	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.09	0.10	0.12	0.13	0.15	0.17	0.19	0.21	0.23	0.25
u/L/400	0.01	0.03	0.05	0.07	0.11	0.15	0.20	0.27	0.35	0.44	0.55	0.68	0.82	0.98	1.17	1.37	1.60
b=3 m																	
$f_{n_1}$ [Hz]	59.83	39.38	27.79	20.61	15.88	12.60	10.24	8.48	7.14	6.09	5.26	4.58	4.03	3.57	3.19	2.86	2.59
$f_{n_2}$ [Hz]	61.62	41.93	30.47	23.25	18.40	15.00	12.51	10.65	9.20	8.07	7.16	6.41	5.80	5.28	4.84	4.47	4.15
$f_{n_3}$ [Hz]	96.90	83.15	72.37	63.56	55.66	47.48	39.39	32.85	27.79	23.81	20.61	18.02	15.88	14.10	12.60	11.33	10.24
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.02	3.76	3.65	2.61	1.14	0.38
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	2.54	3.76
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR [%]	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.12	0.14	0.16	0.18	0.21	0.23	0.25	0.28	0.31
u/L/400	0.02	0.03	0.06	0.09	0.13	0.18	0.25	0.33	0.42	0.54	0.67	0.82	1.00	1.20	1.42	1.67	1.94
b=4 m																	
$f_{n_1}$ [Hz]	56.48	37.26	26.32	19.53	15.05	11.94	9.71	8.04	6.77	5.77	4.98	4.35	3.82	3.39	3.02	2.71	2.45
$f_{n_2}$ [Hz]	57.87	39.04	28.15	21.32	16.75	13.55	11.22	9.47	8.12	7.06	6.20	5.52	4.94	4.47	4.06	3.72	3.43
$f_{n_3}$ [Hz]	80.00	65.94	57.85	54.54	51.52	45.46	37.28	31.11	26.32	22.56	19.53	17.08	15.05	13.37	11.95	10.74	9.71
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	3.10	3.66	3.50	1.83	0.48	3.91
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	2.78	4.08	3.97	3.87
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR [%]	0.02	0.03	0.05	0.06	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36
u/L/400	0.02	0.04	0.07	0.10	0.15	0.21	0.29	0.38	0.50	0.63	0.79	0.96	1.17	1.40	1.66	1.95	2.28
b=7 m																	
$f_{n_1}$ [Hz]	47.20	31.75	22.59	16.86	13.03	10.37	8.43	6.99	5.89	5.03	4.34	3.79	3.33	2.95	2.64	2.37	2.14
$f_{n_2}$ [Hz]	49.11	33.34	23.89	17.97	14.00	11.23	9.21	7.71	6.54	5.64	4.91	4.32	3.83	3.43	3.08	2.80	2.54
$f_{n_3}$ [Hz]	55.67	40.84	32.24	27.45	24.35	22.60	21.27	20.58	19.90	19.44	16.86	14.78	13.04	11.59	10.37	9.33	8.44
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.08	3.50	3.01	1.41	0.14	5.65	10.92
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.52	3.68	4.20	4.07	2.38	0.94	1.72
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR [%]	0.04	0.06	0.08	0.09	0.12	0.14	0.16	0.19	0.22	0.25	0.28	0.32	0.36	0.40	0.44	0.48	0.53
u/L/400	0.04	0.07	0.10	0.16	0.23	0.32	0.42	0.56	0.72	0.90	1.12	1.38	1.67	2.00	2.37	2.78	3.25

**Table A.9:** Natural frequencies and accelerations for class I bridges with HEA800 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	85.09	65.55	51.29	41.15	32.97	26.45	21.66	18.05	15.27	13.08	11.32	9.89	8.72	7.74	6.92	6.22	5.62
$f_{n_2}$ [Hz]	110.80	76.49	55.65	42.17	33.76	28.30	24.15	20.95	18.41	16.38	14.71	13.33	12.16	11.17	10.31	9.57	8.92
$f_{n_3}$ [Hz]	130.68	101.55	82.63	69.23	58.91	50.78	44.13	38.68	34.11	30.31	27.07	24.33	21.98	19.96	18.21	16.69	15.35
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.06	0.06	0.07	0.08	0.09	0.10	0.11
u/L/400	0.00	0.01	0.01	0.02	0.02	0.03	0.04	0.06	0.08	0.09	0.12	0.14	0.17	0.21	0.25	0.29	0.34
b=3 m																	
$f_{n_1}$ [Hz]	84.45	66.68	52.22	40.49	31.66	25.40	20.80	17.33	14.66	12.55	10.87	9.50	8.37	7.43	6.64	5.97	5.40
$f_{n_2}$ [Hz]	106.27	73.45	53.47	41.55	33.70	27.91	23.53	20.16	17.51	15.40	13.69	12.28	11.10	10.12	9.28	8.56	7.93
$f_{n_3}$ [Hz]	116.79	90.67	75.45	65.15	57.15	50.71	45.23	40.58	36.50	32.97	29.84	27.10	24.67	22.54	20.65	18.98	17.49
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.05	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13
u/L/400	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.17	0.20	0.24	0.29	0.34	0.40
b=4 m																	
$f_{n_1}$ [Hz]	80.09	64.96	50.87	38.91	30.43	24.42	20.00	16.67	14.10	12.07	10.45	9.14	8.05	7.15	6.39	5.74	5.19
$f_{n_2}$ [Hz]	101.44	70.43	51.40	40.25	32.39	26.63	22.28	18.95	16.33	14.26	12.57	11.19	10.04	9.08	8.26	7.57	6.96
$f_{n_3}$ [Hz]	107.14	81.73	68.68	60.46	54.08	48.92	44.46	40.62	37.17	34.10	31.30	28.76	26.42	24.30	22.35	20.59	18.99
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.15
u/L/400	0.01	0.01	0.02	0.02	0.03	0.04	0.06	0.08	0.10	0.13	0.16	0.19	0.23	0.28	0.33	0.39	0.45
b=7 m																	
$f_{n_1}$ [Hz]	64.86	56.09	44.88	34.50	27.09	21.83	17.91	14.95	12.66	10.85	9.40	8.22	7.24	6.43	5.75	5.17	4.67
$f_{n_2}$ [Hz]	84.43	61.24	45.75	35.85	28.54	23.28	19.30	16.30	13.93	12.07	10.55	9.31	8.28	7.42	6.68	6.06	5.52
$f_{n_3}$ [Hz]	88.04	65.48	53.79	48.37	44.36	41.22	38.46	36.08	33.90	31.95	30.13	28.43	26.67	24.27	21.81	19.74	17.91
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.09	0.10	0.11	0.12	0.14	0.15	0.17	0.19	0.21
u/L/400	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.11	0.14	0.17	0.21	0.26	0.32	0.38	0.45	0.53	0.61

## A.2 Class III tables

**Table A.10:** Natural frequencies and accelerations for class III bridges with HEA200 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	16.30	10.87	7.69	5.70	4.39	3.48	2.83	2.34	1.97	1.68	1.45	1.26	1.11	0.98	0.88	0.79	0.71
$f_{n_2}$ [Hz]	23.01	15.24	10.89	8.25	6.49	5.29	4.41	3.77	3.26	2.88	2.55	2.31	2.09	1.92	1.76	1.64	1.52
$f_{n_3}$ [Hz]	39.36	32.78	25.97	20.55	16.37	13.25	10.90	9.10	7.70	6.60	5.71	4.99	4.39	3.90	3.48	3.13	2.83
$a_{B1} [\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.56	6.36	6.19	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.13	0.18	0.25	0.33	0.42	0.51	0.61	0.72	0.85	0.98	1.12	1.27	1.43	1.60	1.78	1.97	2.17
$u_{max}/u_{L/400}$	0.31	0.54	0.88	1.36	2.00	2.83	3.86	5.12	6.63	8.42	10.50	12.90	15.65	18.76	22.26	26.17	30.52
b=3 m																	
$f_{n_1}$ [Hz]	12.54	8.93	6.49	4.88	3.78	3.01	2.45	2.03	1.71	1.46	1.26	1.10	0.97	0.86	0.76	0.69	0.62
$f_{n_2}$ [Hz]	20.35	13.49	9.59	7.21	5.62	4.54	3.75	3.18	2.72	2.38	2.09	1.88	1.68	1.53	1.40	1.29	1.19
$f_{n_3}$ [Hz]	21.96	19.78	17.36	14.97	12.64	10.64	8.97	7.61	6.51	5.62	4.89	4.29	3.79	3.37	3.01	2.71	2.45
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.55	7.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.86	6.62	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.18	0.26	0.35	0.46	0.58	0.71	0.85	1.01	1.18	1.36	1.56	1.76	1.99	2.22	2.47	2.73	3.00
$u_{max}/u_{L/400}$	0.61	0.89	1.34	1.99	2.86	3.99	5.40	7.14	9.21	11.67	14.54	17.85	21.63	25.91	30.73	36.12	42.11
b=4 m																	
$f_{n_1}$ [Hz]	9.31	7.24	5.51	4.24	3.33	2.67	2.18	1.81	1.53	1.31	1.13	0.98	0.87	0.77	0.69	0.62	0.56
$f_{n_2}$ [Hz]	13.86	12.20	8.69	6.52	5.07	4.08	3.35	2.82	2.41	2.09	1.83	1.63	1.46	1.32	1.20	1.10	1.01
$f_{n_3}$ [Hz]	18.17	12.68	11.61	10.58	9.45	8.35	7.29	6.35	5.54	4.84	4.25	3.76	3.34	2.98	2.67	2.41	2.18
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.83	6.58	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.25	0.34	0.45	0.58	0.71	0.83	0.97	1.13	1.32	1.51	1.73	1.96	2.22	2.48	2.77	3.07	3.43
$u_{max}/u_{L/400}$	1.21	1.50	2.02	2.80	3.88	5.29	7.07	9.25	11.88	14.99	18.63	22.82	27.62	33.06	39.19	46.02	53.63
b=7 m																	
$f_{n_1}$ [Hz]	3.84	3.56	3.19	2.75	2.32	1.94	1.63	1.38	1.18	1.02	0.89	0.78	0.68	0.61	0.54	0.49	0.44
$f_{n_2}$ [Hz]	5.28	4.86	4.54	4.32	4.06	3.26	2.68	2.24	1.90	1.64	1.43	1.26	1.12	1.01	0.91	0.83	0.75
$f_{n_3}$ [Hz]	9.60	7.41	6.21	5.18	4.11	3.92	3.72	3.50	3.27	3.03	2.79	2.56	2.34	2.14	1.96	1.79	1.64
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	7.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.01	4.77	0.00
UR	0.25	0.34	0.45	0.58	0.71	0.83	0.97	1.13	1.32	1.51	1.73	1.96	2.22	2.48	2.77	3.07	3.43
$u_{max}/u_{L/400}$	6.79	6.51	6.82	7.68	9.12	11.14	13.81	17.16	21.26	26.16	31.93	38.62	46.32	55.04	64.90	75.92	88.20

**Table A.11:** Natural frequencies and accelerations for class III bridges with HEA400 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	42.35	29.26	21.00	15.76	12.21	9.73	7.92	6.58	5.54	4.73	4.09	3.57	3.14	2.78	2.49	2.23	2.02
$f_{n_2}$ [Hz]	55.40	37.56	27.02	20.43	15.98	12.93	10.65	9.00	7.68	6.70	5.86	5.24	4.66	4.25	3.83	3.54	3.23
$f_{n_3}$ [Hz]	81.31	76.46	62.92	52.49	42.50	35.24	29.24	24.73	21.04	18.15	15.77	13.84	12.22	10.88	9.73	8.77	7.93
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.62
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.04	0.06	0.07	0.09	0.12	0.14	0.17	0.20	0.23	0.27	0.30	0.34	0.38	0.42	0.47	0.52	0.57
u/L/400	0.05	0.07	0.11	0.16	0.23	0.33	0.44	0.59	0.76	0.96	1.19	1.47	1.77	2.12	2.52	2.96	3.45
b=3 m																	
$f_{n_1}$ [Hz]	33.45	24.51	18.06	13.76	10.74	8.60	7.03	5.85	4.94	4.22	3.65	3.19	2.80	2.49	2.22	2.00	1.80
$f_{n_2}$ [Hz]	49.47	33.87	24.37	18.39	14.34	11.53	9.46	7.94	6.73	5.83	5.07	4.49	3.97	3.58	3.21	2.94	2.67
$f_{n_3}$ [Hz]	51.91	49.96	44.35	39.59	33.65	28.94	24.54	21.10	18.14	15.79	13.79	12.16	10.77	9.61	8.62	7.77	7.04
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.41	3.32
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.06	0.08	0.10	0.13	0.16	0.19	0.23	0.27	0.31	0.36	0.41	0.46	0.51	0.57	0.63	0.69	0.76
u/L/400	0.08	0.11	0.16	0.23	0.32	0.45	0.60	0.79	1.01	1.28	1.59	1.94	2.35	2.81	3.33	3.91	4.56
b=4 m																	
$f_{n_1}$ [Hz]	9.31	7.24	5.51	4.24	3.33	2.67	2.18	1.81	1.53	1.31	1.13	0.98	0.87	0.77	0.69	0.62	0.56
$f_{n_2}$ [Hz]	13.86	12.20	8.69	6.52	5.07	4.08	3.35	2.82	2.41	2.09	1.83	1.63	1.46	1.32	1.20	1.10	1.01
$f_{n_3}$ [Hz]	18.17	12.68	11.61	10.58	9.45	8.35	7.29	6.35	5.54	4.84	4.25	3.76	3.34	2.98	2.67	2.41	2.18
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.83	4.65	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.07	0.10	0.13	0.17	0.20	0.25	0.29	0.34	0.39	0.45	0.51	0.57	0.64	0.71	0.79	0.87	0.95
u/L/400	0.15	0.19	0.24	0.33	0.44	0.59	0.78	1.01	1.28	1.61	1.99	2.44	2.94	3.51	4.16	4.88	5.67
b=7 m																	
$f_{n_1}$ [Hz]	11.09	10.27	9.09	7.86	6.65	5.62	4.75	4.04	3.47	3.00	2.62	2.30	2.04	1.81	1.62	1.46	1.32
$f_{n_2}$ [Hz]	13.94	13.01	12.26	11.82	10.77	8.68	7.13	5.96	5.05	4.34	3.76	3.30	2.91	2.60	2.32	2.10	1.90
$f_{n_3}$ [Hz]	19.33	16.52	14.77	13.73	11.29	10.87	10.29	9.78	9.13	8.54	7.88	7.29	6.69	6.16	5.65	5.20	4.78
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.10	3.95	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.63
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.12	0.17	0.22	0.28	0.34	0.41	0.48	0.56	0.64	0.73	0.83	0.93	1.04	1.15	1.27	1.39	1.52
u/L/400	0.85	0.81	0.82	0.92	1.06	1.28	1.55	1.90	2.32	2.82	3.41	4.10	4.89	5.78	6.79	7.92	9.18

**Table A.12:** Natural frequencies and accelerations for class III bridges with HEA800 and 2 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	75.36	56.53	42.01	32.48	25.56	20.66	16.97	14.19	12.02	10.31	8.94	7.82	6.90	6.13	5.48	4.93	4.45
$f_{n_2}$ [Hz]	88.77	64.59	48.26	37.37	29.67	24.19	20.05	16.98	14.51	12.64	11.05	9.83	8.75	7.92	7.14	6.55	5.98
$f_{n_3}$ [Hz]	104.85	102.14	85.65	72.73	61.98	53.26	45.95	39.95	34.90	30.72	27.16	24.19	21.63	19.48	17.60	16.00	14.58
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.15	0.16	0.18	0.20	0.22
u/L/400	0.02	0.02	0.03	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.23	0.28	0.34	0.40	0.47	0.56	0.64
b=3 m																	
$f_{n_1}$ [Hz]	62.13	48.51	36.78	28.83	22.87	18.59	15.32	12.85	10.90	9.37	8.13	7.12	6.28	5.58	4.99	4.49	4.06
$f_{n_2}$ [Hz]	78.92	61.02	45.78	35.53	28.18	22.93	18.95	15.97	13.60	11.77	10.25	9.06	8.02	7.20	6.46	5.88	5.34
$f_{n_3}$ [Hz]	80.02	81.22	75.32	67.96	60.24	53.86	47.57	42.22	36.83	32.65	28.77	25.69	22.91	20.63	18.58	16.88	15.34
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.12	0.14	0.15	0.17	0.19	0.21	0.24	0.26	0.28
u/L/400	0.02	0.03	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.24	0.29	0.36	0.43	0.51	0.60	0.71	0.82
b=4 m																	
$f_{n_1}$ [Hz]	50.43	41.18	32.08	25.62	20.54	16.83	13.93	11.73	9.98	8.59	7.47	6.55	5.78	5.14	4.60	4.14	3.75
$f_{n_2}$ [Hz]	61.90	56.47	42.65	33.23	26.38	21.47	17.75	14.95	12.72	10.99	9.56	8.42	7.45	6.67	5.98	5.42	4.91
$f_{n_3}$ [Hz]	63.00	62.90	58.89	56.64	50.36	46.24	40.67	36.59	32.24	28.88	25.63	23.04	20.63	18.66	16.86	15.35	13.98
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.13	0.15	0.17	0.19	0.21	0.24	0.26	0.29	0.32	0.35
u/L/400	0.04	0.04	0.05	0.07	0.09	0.12	0.15	0.19	0.24	0.30	0.36	0.44	0.53	0.63	0.74	0.86	1.00
b=7 m																	
$f_{n_1}$ [Hz]	25.05	23.30	20.33	17.59	14.83	12.61	10.69	9.17	7.90	6.87	6.01	5.31	4.71	4.20	3.77	3.40	3.09
$f_{n_2}$ [Hz]	29.57	28.54	27.12	26.50	21.99	17.99	14.91	12.57	10.72	9.25	8.05	7.09	6.27	5.60	5.02	4.53	4.11
$f_{n_3}$ [Hz]	35.62	32.65	30.23	27.61	25.30	24.54	23.06	21.98	20.38	19.11	17.57	16.30	14.95	13.81	12.68	11.71	10.78
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.05	0.07	0.09	0.11	0.13	0.16	0.18	0.21	0.24	0.27	0.30	0.34	0.38	0.41	0.46	0.50	0.54
u/L/400	0.16	0.16	0.16	0.18	0.20	0.24	0.29	0.35	0.42	0.50	0.60	0.72	0.85	1.00	1.17	1.36	1.57

**Table A.13:** Natural frequencies and accelerations for class III bridges with HEA200 and 3 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	12.54	8.93	6.49	4.88	3.78	3.01	2.45	2.03	1.71	1.46	1.26	1.10	0.97	0.86	0.76	0.69	0.62
$f_{n_2}$ [Hz]	20.35	13.49	9.59	7.21	5.62	4.54	3.75	3.18	2.72	2.38	2.09	1.88	1.68	1.53	1.40	1.29	1.19
$f_{n_3}$ [Hz]	21.96	19.78	17.36	14.97	12.64	10.64	8.97	7.61	6.51	5.62	4.89	4.29	3.79	3.37	3.01	2.71	2.45
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.55	7.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.86	6.62	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.18	0.26	0.35	0.46	0.58	0.71	0.85	1.01	1.18	1.36	1.56	1.76	1.99	2.22	2.47	2.73	3.00
u/L/400	0.61	0.89	1.34	1.99	2.86	3.99	5.40	7.14	9.21	11.67	14.54	17.85	21.63	25.91	30.73	36.12	42.11
b=3 m																	
$f_{n_1}$ [Hz]	17.24	11.34	7.99	5.91	4.54	3.60	2.92	2.42	2.03	1.73	1.50	1.30	1.15	1.02	0.91	0.81	0.73
$f_{n_2}$ [Hz]	20.89	13.88	9.89	7.44	5.82	4.72	3.90	3.32	2.85	2.50	2.20	1.98	1.78	1.63	1.48	1.38	1.27
$f_{n_3}$ [Hz]	40.41	35.98	28.80	22.00	17.26	13.86	11.36	9.46	7.99	6.84	5.91	5.16	4.55	4.03	3.60	3.23	2.92
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.00	6.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.90	6.68	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.14	0.19	0.25	0.32	0.40	0.49	0.59	0.70	0.82	0.95	1.09	1.23	1.39	1.55	1.73	1.91	2.10
u/L/400	0.30	0.52	0.86	1.32	1.94	2.74	3.74	4.96	6.42	8.15	10.17	12.50	15.17	18.19	21.58	25.37	29.59
b=4 m																	
$f_{n_1}$ [Hz]	14.96	10.00	7.11	5.29	4.08	3.24	2.63	2.18	1.83	1.56	1.35	1.18	1.03	0.92	0.82	0.73	0.66
$f_{n_2}$ [Hz]	18.63	12.53	8.94	6.71	5.23	4.22	3.47	2.93	2.50	2.18	1.91	1.71	1.52	1.39	1.26	1.16	1.06
$f_{n_3}$ [Hz]	29.77	24.39	21.78	18.90	14.98	12.14	10.02	8.39	7.12	6.11	5.30	4.63	4.08	3.63	3.24	2.91	2.63
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.89	6.65	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.20	0.26	0.33	0.41	0.51	0.62	0.75	0.88	1.04	1.20	1.37	1.56	1.76	1.96	2.18	2.41	2.65
u/L/400	0.42	0.72	1.16	1.74	2.52	3.52	4.78	6.31	8.15	10.32	12.87	15.80	19.15	22.95	27.23	32.02	37.34
b=7 m																	
$f_{n_1}$ [Hz]	10.56	7.31	5.33	4.04	3.17	2.54	2.08	1.73	1.46	1.25	1.08	0.94	0.83	0.73	0.66	0.59	0.53
$f_{n_2}$ [Hz]	12.13	9.29	6.96	5.32	4.17	3.36	2.76	2.31	1.96	1.70	1.48	1.31	1.16	1.04	0.94	0.86	0.78
$f_{n_3}$ [Hz]	17.38	14.18	11.17	9.51	8.52	7.95	7.32	6.21	5.33	4.63	4.05	3.57	3.17	2.83	2.54	2.29	2.08
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	6.02	5.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.26
UR																	
	0.39	0.52	0.66	0.80	0.95	1.11	1.30	1.50	1.73	1.97	2.24	2.52	2.83	3.16	3.54	3.96	4.46

**Table A.14:** Natural frequencies and accelerations for class III bridges with HEA400 and 3 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	51.17	34.16	24.21	18.02	13.91	11.05	8.98	7.45	6.27	5.35	4.62	4.03	3.54	3.14	2.80	2.52	2.27
$f_{n_2}$ [Hz]	58.32	39.78	28.84	21.99	17.37	14.18	11.82	10.08	8.71	7.66	6.79	6.11	5.51	5.05	4.62	4.29	3.96
$f_{n_3}$ [Hz]	117.28	96.85	80.00	64.83	51.20	41.52	34.13	28.58	24.22	20.79	18.02	15.77	13.91	12.36	11.05	9.94	8.98
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.03	0.04	0.05	0.07	0.09	0.10	0.12	0.15	0.17	0.20	0.22	0.25	0.28	0.31	0.35	0.38	0.42
u/L/400	0.03	0.05	0.08	0.12	0.17	0.25	0.33	0.44	0.57	0.72	0.90	1.11	1.34	1.60	1.90	2.24	2.61
b=3 m	•																
$f_{n_1}$ [Hz]	44.78	30.48	21.76	16.28	12.59	10.02	8.16	6.77	5.70	4.87	4.20	3.67	3.23	2.86	2.55	2.29	2.07
$f_{n_2}$ [Hz]	52.17	35.78	25.82	19.55	15.30	12.36	10.18	8.58	7.32	6.36	5.56	4.95	4.41	3.99	3.61	3.32	3.03
$f_{n_3}$ [Hz]	94.38	85.78	69.48	56.13	44.85	36.84	30.44	25.64	21.78	18.76	16.28	14.27	12.60	11.21	10.02	9.02	8.16
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.40
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.04	0.06	0.08	0.09	0.12	0.14	0.17	0.19	0.23	0.26	0.29	0.33	0.37	0.41	0.46	0.50	0.55
u/L/400	0.04	0.07	0.10	0.16	0.23	0.32	0.43	0.57	0.73	0.93	1.16	1.42	1.72	2.06	2.45	2.88	3.35
b=4 m																	
$f_{n_1}$ [Hz]	39.48	27.37	19.72	14.85	11.52	9.19	7.49	6.22	5.25	4.48	3.87	3.38	2.97	2.64	2.35	2.11	1.91
$f_{n_2}$ [Hz]	47.07	32.75	23.68	17.93	14.00	11.27	9.25	7.76	6.59	5.70	4.96	4.38	3.88	3.50	3.14	2.87	2.61
$f_{n_3}$ [Hz]	72.11	61.03	54.21	48.96	39.56	32.84	27.32	23.16	19.75	17.07	14.85	13.05	11.53	10.27	9.20	8.29	7.50
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.12
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.06	0.08	0.10	0.12	0.15	0.18	0.21	0.24	0.28	0.32	0.36	0.41	0.46	0.51	0.56	0.62	0.68
u/L/400	0.06	0.09	0.13	0.20	0.29	0.40	0.53	0.70	0.90	1.14	1.42	1.74	2.11	2.52	2.99	3.51	4.09
b=7 m																	
$f_{n_1}$ [Hz]	28.30	20.35	15.07	11.60	9.14	7.38	6.06	5.07	4.29	3.68	3.18	2.78	2.45	2.18	1.94	1.75	1.58
$f_{n_2}$ [Hz]	32.69	25.23	18.95	14.61	11.50	9.28	7.62	6.38	5.41	4.65	4.04	3.55	3.13	2.80	2.51	2.27	2.05
$f_{n_3}$ [Hz]	42.60	36.41	29.00	24.96	22.36	20.96	19.87	17.49	15.10	13.22	11.61	10.29	9.16	8.22	7.39	6.69	6.08
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.67	3.57	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.11
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.12	0.16	0.20	0.23	0.27	0.31	0.36	0.41	0.46	0.52	0.59	0.65	0.73	0.80	0.89	0.97	1.06
u/L/400	0.11	0.18	0.26	0.38	0.52	0.69	0.90	1.16	1.47	1.83	2.25	2.74	3.30	3.93	4.65	5.44	6.33

**Table A.15:** Natural frequencies and accelerations for class III bridges with HEA800 and 3 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	89.67	65.00	47.70	36.40	28.52	22.94	18.80	15.69	13.27	11.38	9.85	8.61	7.59	6.74	6.03	5.42	4.90
$f_{n_2}$ [Hz]	91.73	66.25	49.99	39.09	31.35	25.82	21.64	18.51	16.02	14.08	12.47	11.20	10.10	9.21	8.42	7.78	7.19
$f_{n_3}$ [Hz]	127.21	103.63	85.70	72.20	61.24	52.47	45.22	39.30	34.36	30.27	26.82	23.92	21.44	19.34	17.52	15.95	14.58
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.02	0.02	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.13	0.14	0.15	0.17
u/L/400	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.18	0.22	0.26	0.31	0.37	0.44	0.51
b=3 m																	
$f_{n_1}$ [Hz]	79.59	58.76	43.45	33.45	26.28	21.20	17.40	14.54	12.31	10.56	9.14	8.00	7.05	6.27	5.60	5.04	4.55
$f_{n_2}$ [Hz]	85.35	64.01	48.30	37.69	30.02	24.54	20.38	17.26	14.77	12.85	11.26	10.00	8.91	8.05	7.27	6.65	6.08
$f_{n_3}$ [Hz]	106.89	91.74	78.98	69.30	61.03	54.13	48.05	42.82	38.16	34.14	30.56	27.49	24.76	22.42	20.34	18.57	16.96
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.14	0.16	0.18	0.19	0.21
u/L/400	0.02	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.18	0.22	0.27	0.33	0.39	0.46	0.54	0.63
b=4 m																	
$f_{n_1}$ [Hz]	71.08	53.56	39.90	30.93	24.38	19.73	16.21	13.57	11.50	9.87	8.55	7.48	6.60	5.86	5.24	4.72	4.26
$f_{n_2}$ [Hz]	78.72	60.22	45.55	35.62	28.34	23.14	19.16	16.18	13.80	11.96	10.43	9.22	8.18	7.34	6.60	6.01	5.46
$f_{n_3}$ [Hz]	93.30	81.39	71.42	63.97	57.60	52.26	47.45	43.20	39.21	35.12	30.83	27.42	24.39	21.91	19.71	17.87	16.22
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.11	0.12	0.14	0.16	0.18	0.19	0.21	0.23	0.26
u/L/400	0.02	0.02	0.03	0.04	0.06	0.08	0.10	0.14	0.17	0.22	0.27	0.32	0.39	0.47	0.55	0.64	0.75
b=7 m																	
$f_{n_1}$ [Hz]	53.17	41.13	31.41	24.87	19.88	16.28	13.48	11.35	9.65	8.31	7.22	6.33	5.59	4.98	4.45	4.01	3.63
$f_{n_2}$ [Hz]	60.60	48.83	37.77	29.99	24.01	19.68	16.32	13.78	11.75	10.16	8.84	7.79	6.89	6.16	5.53	5.00	4.53
$f_{n_3}$ [Hz]	66.81	59.77	52.89	48.11	44.13	41.71	39.21	35.97	31.47	28.04	24.82	22.27	19.93	18.02	16.28	14.82	13.50
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.05	0.06	0.07	0.09	0.10	0.12	0.14	0.15	0.17	0.20	0.22	0.24	0.27	0.30	0.33	0.36	0.39
u/L/400	0.03	0.04	0.06	0.08	0.10	0.13	0.17	0.21	0.27	0.33	0.40	0.49	0.58	0.69	0.81	0.95	1.10

**Table A.16:** Natural frequencies and accelerations for class III bridges with HEA200 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	1.42	17.46	12.18	8.97	6.88	5.44	4.41	3.65	3.07	2.61	2.26	1.96	1.73	1.53	1.37	1.23	1.11
$f_{n_2}$ [Hz]	28.31	18.93	13.69	10.48	8.36	6.89	5.82	5.02	4.40	3.91	3.51	3.19	2.92	2.69	2.49	2.33	2.17
$f_{n_3}$ [Hz]	71.63	65.54	46.88	35.11	27.08	21.49	17.47	14.47	12.18	10.39	8.97	7.82	6.88	6.10	5.44	4.89	4.41
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.09	5.89	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.05	0.07	0.09	0.12	0.15	0.19	0.23	0.27	0.32	0.37	0.42	0.48	0.54	0.60	0.67	0.74	0.82
u/L/400	0.10	0.18	0.32	0.50	0.75	1.07	1.46	1.94	2.52	3.20	3.99	4.91	5.95	7.14	8.47	9.96	11.61
b=3 m	•																
$f_{n_1}$ [Hz]	24.64	15.87	11.07	8.15	6.25	4.94	4.01	3.31	2.79	2.37	2.05	1.78	1.57	1.39	1.24	1.11	1.00
$f_{n_2}$ [Hz]	25.38	16.71	11.90	8.97	7.04	5.71	4.75	4.04	3.48	3.06	2.71	2.43	2.20	2.01	1.84	1.70	1.58
$f_{n_3}$ [Hz]	43.11	37.44	34.57	31.97	24.66	19.55	15.89	13.15	11.07	9.44	8.15	7.11	6.25	5.54	4.94	4.44	4.01
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.88	5.68	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.15	5.98	5.82	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.06	0.09	0.12	0.15	0.19	0.24	0.29	0.34	0.41	0.47	0.54	0.61	0.69	0.77	0.86	0.95	1.05
u/L/400	0.13	0.24	0.41	0.65	0.97	1.38	1.89	2.51	3.26	4.14	5.17	6.35	7.71	9.24	10.97	12.89	15.04
b=4 m																	
$f_{n_1}$ [Hz]	22.65	14.60	10.19	7.50	5.75	4.55	3.69	3.05	2.56	2.18	1.88	1.64	1.44	1.28	1.14	1.02	0.92
$f_{n_2}$ [Hz]	23.41	15.30	10.83	8.10	6.31	5.08	4.18	3.53	3.02	2.63	2.31	2.06	1.84	1.67	1.52	1.40	1.29
$f_{n_3}$ [Hz]	32.28	25.93	22.83	21.36	20.41	17.97	14.62	12.10	10.19	8.69	7.50	6.54	5.75	5.10	4.55	4.08	3.69
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.49	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.40	6.19	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.07	0.10	0.14	0.19	0.24	0.29	0.35	0.42	0.50	0.57	0.66	0.75	0.84	0.94	1.05	1.16	1.28
u/L/400	0.16	0.31	0.52	0.81	1.20	1.69	2.31	3.07	3.99	5.07	6.33	7.79	9.45	11.33	13.45	15.81	18.44
b=7 m																	
$f_{n_1}$ [Hz]	18.21	11.85	8.32	6.15	4.73	3.74	3.04	2.51	2.11	1.80	1.56	1.36	1.19	1.06	0.94	0.85	0.76
$f_{n_2}$ [Hz]	19.25	12.66	8.94	6.65	5.14	4.11	3.36	2.80	2.38	2.05	1.78	1.57	1.39	1.25	1.13	1.03	0.94
$f_{n_3}$ [Hz]	21.90	15.44	12.03	10.15	9.02	8.34	7.89	7.61	7.40	7.11	6.15	5.37	4.73	4.19	3.75	3.37	3.04
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6.20	5.97	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.13	0.19	0.25	0.32	0.40	0.48	0.57	0.68	0.79	0.91	1.04	1.18	1.33	1.49	1.66	1.84	2.02
u/L/400	0.27	0.52	0.88	1.35	1.96	2.74	3.70	4.87	6.27	7.93	9.86	12.09	14.65	17.54	20.81	24.45	28.53

**Table A.17:** Natural frequencies and accelerations for class III bridges with HEA400 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	63.34	41.67	29.41	21.82	16.81	13.34	10.84	8.98	7.56	6.45	5.57	4.86	4.27	3.79	3.38	3.03	2.74
$f_{n_2}$ [Hz]	66.31	45.79	33.72	26.08	20.94	17.32	14.66	12.65	11.09	9.86	8.85	8.03	7.34	6.76	6.26	5.84	5.46
$f_{n_3}$ [Hz]	109.52	90.11	75.26	63.48	53.69	45.59	38.87	33.41	28.97	25.20	21.82	19.08	16.81	14.93	13.34	12.00	10.84
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.07	0.09	0.10	0.12	0.13	0.15	0.17	0.19	0.21	0.23	0.25
u/L/400	0.01	0.03	0.05	0.07	0.11	0.15	0.20	0.27	0.35	0.44	0.55	0.68	0.82	0.98	1.17	1.37	1.60
b=3 m																	
$f_{n_1}$ [Hz]	59.83	39.38	27.79	20.61	15.88	12.60	10.24	8.48	7.14	6.09	5.26	4.58	4.03	3.57	3.19	2.86	2.59
$f_{n_2}$ [Hz]	61.62	41.93	30.47	23.25	18.40	15.00	12.51	10.65	9.20	8.07	7.16	6.41	5.80	5.28	4.84	4.47	4.15
$f_{n_3}$ [Hz]	96.90	83.15	72.37	63.56	55.66	47.48	39.39	32.85	27.79	23.81	20.61	18.02	15.88	14.10	12.60	11.33	10.24
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.11	0.12	0.14	0.16	0.18	0.21	0.23	0.25	0.28	0.31
u/L/400	0.02	0.03	0.06	0.09	0.13	0.18	0.25	0.33	0.42	0.54	0.67	0.82	1.00	1.20	1.42	1.67	1.94
b=4 m																	
$f_{n_1}$ [Hz]	56.48	37.26	26.32	19.53	15.05	11.94	9.71	8.04	6.77	5.77	4.98	4.35	3.82	3.39	3.02	2.71	2.45
$f_{n_2}$ [Hz]	57.87	39.04	28.15	21.32	16.75	13.55	11.22	9.47	8.12	7.06	6.20	5.52	4.94	4.47	4.06	3.72	3.43
$f_{n_3}$ [Hz]	80.00	65.94	57.85	54.54	51.52	45.46	37.28	31.11	26.32	22.56	19.53	17.08	15.05	13.37	11.95	10.74	9.71
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.03	0.05	0.06	0.07	0.09	0.11	0.13	0.15	0.17	0.19	0.22	0.24	0.27	0.30	0.33	0.36
u/L/400	0.02	0.04	0.07	0.10	0.15	0.21	0.29	0.38	0.50	0.63	0.79	0.96	1.17	1.40	1.66	1.95	2.28
b=7 m																	
$f_{n_1}$ [Hz]	47.20	31.75	22.59	16.86	13.03	10.37	8.43	6.99	5.89	5.03	4.34	3.79	3.33	2.95	2.64	2.37	2.14
$f_{n_2}$ [Hz]	49.11	33.34	23.89	17.97	14.00	11.23	9.21	7.71	6.54	5.64	4.91	4.32	3.83	3.43	3.08	2.80	2.54
$f_{n_3}$ [Hz]	55.67	40.84	32.24	27.45	24.35	22.60	21.27	20.58	19.90	19.44	16.86	14.78	13.04	11.59	10.37	9.33	8.44
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.04	0.06	0.08	0.09	0.12	0.14	0.16	0.19	0.22	0.25	0.28	0.32	0.36	0.40	0.44	0.48	0.53
u/L/400	0.04	0.07	0.10	0.16	0.23	0.32	0.42	0.56	0.72	0.90	1.12	1.38	1.67	2.00	2.37	2.78	3.25

**Table A.18:** Natural frequencies and accelerations for class III bridges with HEA800 and 7 main beams. u/L/400 is the SLS deflection ratio, UR is the Utilization Ratio,  $a_{B1}$  is the maximum acceleration for the first bending mode,  $a_{T1}$  for the first torsion mode, and  $a_{B2}$  for the second bending mode.

L [m]	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b=2 m																	
$f_{n_1}$ [Hz]	85.09	65.55	51.29	41.15	32.97	26.45	21.66	18.05	15.27	13.08	11.32	9.89	8.72	7.74	6.92	6.22	5.62
$f_{n_2}$ [Hz]	110.80	76.49	55.65	42.17	33.76	28.30	24.15	20.95	18.41	16.38	14.71	13.33	12.16	11.17	10.31	9.57	8.92
$f_{n_3}$ [Hz]	130.68	101.55	82.63	69.23	58.91	50.78	44.13	38.68	34.11	30.31	27.07	24.33	21.98	19.96	18.21	16.69	15.35
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.06	0.06	0.07	0.08	0.09	0.10	0.11
u/L/400	0.00	0.01	0.01	0.02	0.02	0.03	0.04	0.06	0.08	0.09	0.12	0.14	0.17	0.21	0.25	0.29	0.34
b=3 m																	
$f_{n_1}$ [Hz]	84.45	66.68	52.22	40.49	31.66	25.40	20.80	17.33	14.66	12.55	10.87	9.50	8.37	7.43	6.64	5.97	5.40
$f_{n_2}$ [Hz]	106.27	73.45	53.47	41.55	33.70	27.91	23.53	20.16	17.51	15.40	13.69	12.28	11.10	10.12	9.28	8.56	7.93
$f_{n_3}$ [Hz]	116.79	90.67	75.45	65.15	57.15	50.71	45.23	40.58	36.50	32.97	29.84	27.10	24.67	22.54	20.65	18.98	17.49
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.01	0.02	0.02	0.03	0.03	0.04	0.05	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13
u/L/400	0.00	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.09	0.11	0.14	0.17	0.20	0.24	0.29	0.34	0.40
b=4 m																	
$f_{n_1}$ [Hz]	80.09	64.96	50.87	38.91	30.43	24.42	20.00	16.67	14.10	12.07	10.45	9.14	8.05	7.15	6.39	5.74	5.19
$f_{n_2}$ [Hz]	101.44	70.43	51.40	40.25	32.39	26.63	22.28	18.95	16.33	14.26	12.57	11.19	10.04	9.08	8.26	7.57	6.96
$f_{n_3}$ [Hz]	107.14	81.73	68.68	60.46	54.08	48.92	44.46	40.62	37.17	34.10	31.30	28.76	26.42	24.30	22.35	20.59	18.99
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.01	0.02	0.02	0.03	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.15
u/L/400	0.01	0.01	0.02	0.02	0.03	0.04	0.06	0.08	0.10	0.13	0.16	0.19	0.23	0.28	0.33	0.39	0.45
b=7 m																	
$f_{n_1}$ [Hz]	64.86	56.09	44.88	34.50	27.09	21.83	17.91	14.95	12.66	10.85	9.40	8.22	7.24	6.43	5.75	5.17	4.67
$f_{n_2}$ [Hz]	84.43	61.24	45.75	35.85	28.54	23.28	19.30	16.30	13.93	12.07	10.55	9.31	8.28	7.42	6.68	6.06	5.52
$f_{n_3}$ [Hz]	88.04	65.48	53.79	48.37	44.36	41.22	38.46	36.08	33.90	31.95	30.13	28.43	26.67	24.27	21.81	19.74	17.91
$a_{B1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{T1} [m/s^2]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$a_{B2}[\mathrm{m/s^2}]$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UR	0.02	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.09	0.10	0.11	0.12	0.14	0.15	0.17	0.19	0.21
u/L/400	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.11	0.14	0.17	0.21	0.26	0.32	0.38	0.45	0.53	0.61