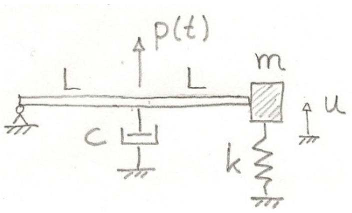
Theory exam in Structural Dynamics 2013-03-05 kl. 10-12

The test consists of 6 questions giving the maximum of 15 points. Each question should be answered on a separate paper. No helping aids are permitted on this test, except calculator. Do not forget to write your name on each submitted paper.

1) (2 p)

A weightless rigid bar is connected to a discrete mass in one end and a frictionless joint in the other end according to the figure.



Show by using free body diagrams that the equation of motion is

$$m\ddot{u}+\frac{c}{4}\dot{u}+ku=\frac{1}{2}p(t)$$

2) (2 p)

Determine the steady state response $u(t)$ for $p(t)=p_0\sin(\omega t)$ at resonance for the system in 1) if $c/4m=2\zeta\omega_n$ and $\zeta=0.1$. Use complex analysis or the deformation response factor as shown in Appendix to determine the vibration amplitude u_0.

3) (2 p)

A system with N dofs is given in general terms as $\boldsymbol{m}\ddot{\boldsymbol{u}}+\boldsymbol{k}\boldsymbol{u}=\boldsymbol{p}(t)$ and the full modal expansion is given by $\boldsymbol{u}=\boldsymbol{\Phi}\boldsymbol{q}=\sum\boldsymbol{\phi}_n q_n$.

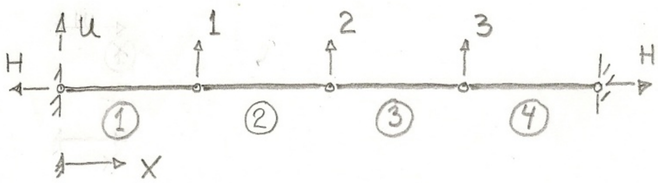
a) Establish the uncoupled system of equations given by $\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{K}\boldsymbol{q}=\boldsymbol{P}(t)$ i.e. determine the matrices ***M, K,*** and the vector ***P***.

b) Determine initial values for the modal coordinates i.e. determine $\boldsymbol{q}(0)$ and $\dot{\boldsymbol{q}}(0)$ in terms of $\boldsymbol{u}(0)$ and $\dot{\boldsymbol{u}}(0)$.

c) A solution $\boldsymbol{q}(t)$ has been found. How are the physical displacements determined?

4) (3p)

A model of a string is established by using four linear elements with mass and stiffness matrices given below. The mass matrix can be either lumped or consistent.



The element matrices are given by: $\frac{m}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, or $\frac{m}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$, and $k\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ $(k=H/L_e)$

a) Show that the assembled system matrices becomes

$$\boldsymbol{m}^l = m\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}, \quad \text{or} \quad \boldsymbol{m}^c = \frac{m}{6}\begin{bmatrix}4 & 1 & 0\\1 & 4 & 1\\0 & 1 & 4\end{bmatrix}, \quad \text{and} \quad \boldsymbol{k} = k\begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}$$

b) A damping matrix is introduced in the case of the consistent mass matrix as

$$\boldsymbol{c} = c\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}.$$ Is this classical damping? Motivate your answer.

5) (3p)

Consider the system (without damping) given in question 4), in the case of the lumped mass matrix. The eigen-frequencies and vectors are

$$\omega_1^2 = (2 - \sqrt{2}\,)\frac{k}{m}, \quad \omega_2^2 = 2\frac{k}{m}, \quad \text{and} \quad \omega_3^2 = (2 + \sqrt{2})\frac{k}{m}$$

$$\boldsymbol{\phi}_1 = \begin{bmatrix}1\\\sqrt{2}\\1\end{bmatrix}, \quad \boldsymbol{\phi}_2 = \begin{bmatrix}1\\0\\-1\end{bmatrix}, \text{ and } \quad \boldsymbol{\phi}_3 = \begin{bmatrix}-1\\\sqrt{2}\\-1\end{bmatrix}$$

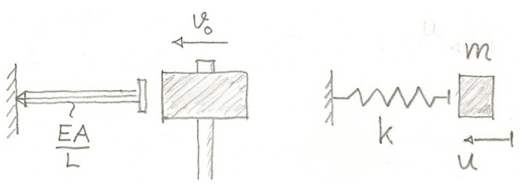
a) Use a linear deflection $\boldsymbol{\Psi}_1 = \begin{bmatrix}1\\2\\1\end{bmatrix}$ to determine an estimate of the first eigen-frequency for the system above.

b) Use also a second vector $\boldsymbol{\Psi}_2 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$ to set up a reduced system. Answer with the reduced systems matrices $\tilde{\boldsymbol{m}}$ and $\tilde{\boldsymbol{k}}$.

c) The eigen-frequencies of the reduced system are $\tilde{\omega}_1^2 = 0.586\,\frac{k}{m}$, and $\tilde{\omega}_2^2 = 3.41\,\frac{k}{m}$. Compare with the exact frequencies and vectors. Give comments on the comparison.

6) (3 p)

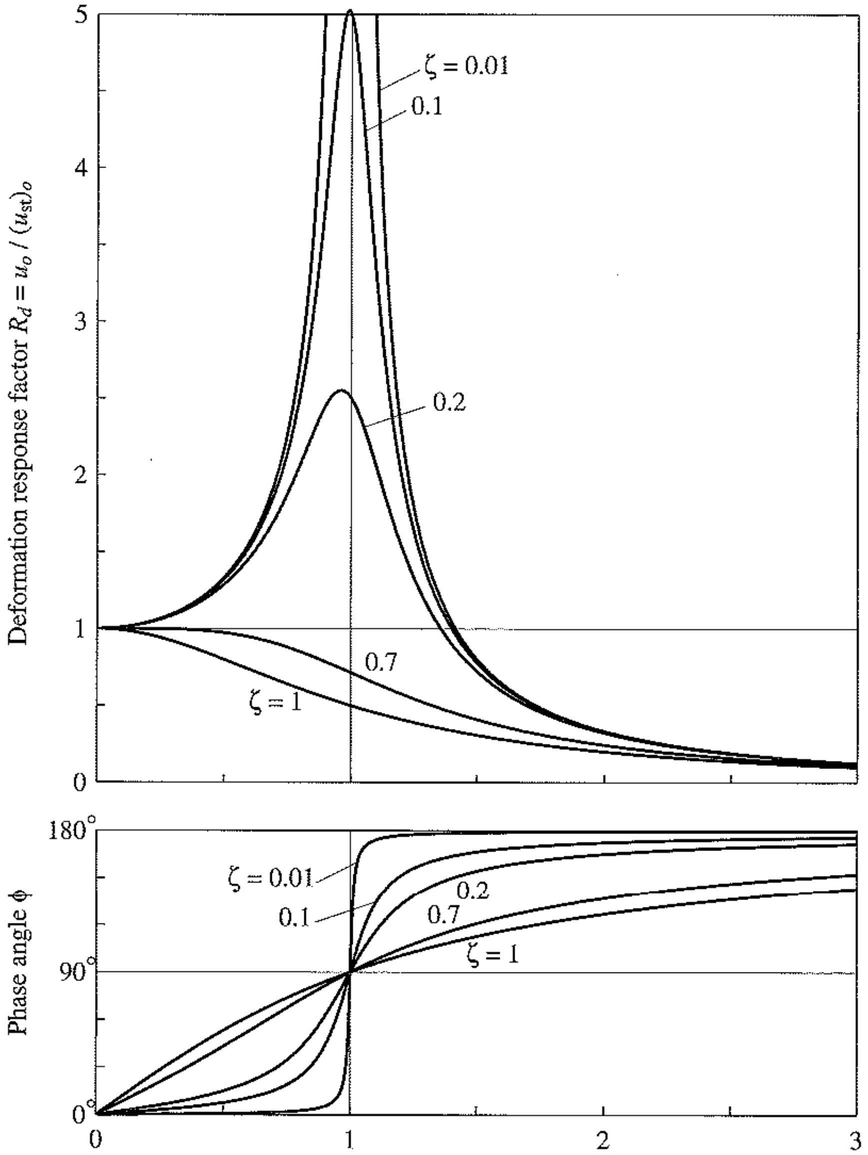
A nail and a hammer are modeled by a linear spring and an incident mass according to the figure. (The nail is supposed to be driven into a piece of very hard wood and cannot move into the wood for the velocity given to the hammer.)



The hammer has an initial velocity v_0=5m/s and a mass m=0.7kg. The nail is made of steel E=210GPa and the cross section is square A= 2x2mm, with a length L=0.1m.

Determine the maximum force F_{max} reached in the contact between hammer and nail and the duration Δt of the force pulse.

Appendix:



Deformation response factor $R_d = u_o / (u_{st})_o$
5
4
3
2
1
0
$\zeta = 0.01$
0.1
0.2
0.7
$\zeta = 1$
Phase angle ϕ
180°
90°
0°
$\zeta = 0.01$
0.1
0.2
0.7
$\zeta = 1$
0
1
2
3