Design Principles For Glass Used Structurally

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Preface

Mine is the first step and therefore a small one, though worked out with much thought and hard labour. You my readers or hearers of my lectures, if you think I have done as much as can fairly be expected of an initial start…will acknowledge what I have achieved and will pardon what I have left for others to accomplish.
Aristotle (384-322 BC)

At the start of this research project I thought that finding out the strength of glass and describing its behaviour during loading would be a very interesting and quiet easy challenge, regarding my background as a Civil Engineer. It would also give me the possibility of developing my skills as a design engineer and increase my self confidence. I must confess as the project has evolved I have had to question my knowledge and skills and what used to seem easy and understandable was now getting very complex and difficult to describe in one simple and easy way. There were many times when I could not see where all these new influences could be headed. The end of this project seemed VERY far away! So now when I am writing the last sentences of this report, I feel there are some people that I need to express my gratitude towards. These people have been backing me up and guided me through this journey. Their help and devotion have given me motivation to keep on going and enjoying my work meanwhile.

Professor Lars Sentler, my supervisor, who has been exposed to my up and down moods and believed in me when I did not. Lars this journey would certainly have not been as interesting and challenging without your motivations and high demands! I would also want to thank Professor Bertil Fredlund for listening to me and guiding me through this sometimes complex academic world.

In addition, the following people have contributed through discussions of the application of glass: Mr Anders Jacobsson, Pilkington, Mr Thomas Grange, MTK, who has also read through and given me comments on this thesis, Mr Tim MacFarlane, Dewhurst & Macfarlane, Mr Andrea Compagno, who has held very interesting courses and workshops about glass, Mr Anthony Smith Ove Arup and finally Mrs Saga Hellberg and Mr Mikael Ödesjö, Glasbranschföreningen, for being my face towards the industry and promoting my project.

As a researcher it is very difficult to set work limits and leave work at work. I am glad that my family and my friends have been around and at times forced me to take time off and focus on other matters of life. The joy which this thesis means to me I want to share with all of you!

Abstract

Glass is a material which is used in increasing demanding applications. This is not only as structural glazing in facades but also as structural elements like beams and columns. In all these applications there is a need for a good understanding of the material glass. This concerns the strength properties which need to be characterized appropriately to reflect the actual behaviour. Design principles should provide the necessary reliability.

The response of glass is considered to be linearly elastic. This means that the theory of elasticity is directly applicable to analyse the response of glass. But typical for structural units of glass plates is that the thickness is small compared to the in plane dimensions. This may result in structural complications. When a glass plate is loaded perpendicular to its plane there will be both bending and membrane responses.

The rupture strength of glass is complex. This reflects the influence of flaws, mainly in the surface regions, which will reduce the rupture strength to a fraction of the theoretical strength. In addition the rupture stress will exhibit dependencies where in particular a size and a time dependence need to be addressed. This will further reduce the long term rupture stress of glass.

The analysis of the rupture behaviour of glass can be done based on fracture mechanics or the stochastic mechanics. This report summarizes and presents available knowledge based on these add on theories to the theory of elasticity.

TABLE OF SYMBOLS

A Area

B Risk function

D_0 reference duration

E_{tot} Total energy

F Failure probability function

G Energy release rate

K_t Stress concentration factor

K_I Stress intensity factor

K_{IC} Critical stress intensity factor

L Length of the beam

M Moment

P Load acting on the body

P_f Probability of failure

$T_{\min}$ Time to failure

U Internal elastic strain energy

V Volume

V_0 reference volume

W Work done

W_s Energy required to create a new crack surface

Y Dimensionless parameter dependent on crack geometry

a Crack length

b Half crack width

b Thickness of the beam

c_0 Normalization constant

e Distance between loads acting on the beam

h Weibull parameter, time dependence

h Height of the beam

m Weibull parameter, size dependence

n Number of elements

$E[\sigma]$ Mean value of the stress

$Var[\sigma]$ Variance of the stress

Δ Displacement

ρ Radius of the curvature of the flaw

u Elastic strain energy density

σ_0 Stress far away from the flaw tip

σ_m Stress at the flaw tip

σ_0 minimum strength of a defective element

ν Poisson's ratio

μ Shear modulus of the material

Π Potential of the body

1 ARCHITECTURAL GLASS

1.1 Introduction

Glass is a material which has been favoured by architects from the time it was introduced in buildings. There are several reasons to this but the transparency of glass is often presented as an important characteristic. Glass could not be produced in larger sizes of reasonable quality until the beginning of the 20th century. A new production technique was introduced at the turn of the century which was soon improved to a continous production of drawn glass. This opened up new potentials which was soon realized by many architects. One of the first examples is the Fagus factory by Walter Gropius of the Bauhaus group from 1911. In this building large portions of the facade was completely covered by glass, something which was later termed glass skin facades. This new architectural style opened up a building and gave it a new character.

Modern glass of high quality known as float glass from its production technique was introduced in the 1960th. This glass can be produced in very large sizes with an extremely high flatness. Within the production technique the surface finish is improved and glass sections with visible internal defects are removed. The production technique requires that residual stresses are introduced. This creates compressive stresses in surface regions which is an advantage in practical use. Float glass has opened the possibility to use glass in new and demanding applications. A new series of glass products known as structural glass has been introduced. This type of glass dominates completely today. Based on float glass various products have been introduced in the market like:

- Laminated glass
- Heat strengthened glass
- Toughened glass
- Insulating glass
- Coated glass

and there is a continuous development regarding glass products to meet new needs on the market. Some of the mentioned products have been on the market for longer period of time but not being used in structurally demanding applications.

Glass has traditionally not been seen as a structural material even if it has a structural function when used as a window pane. Glass producers traditionally have given recommendations for appropriate glass thicknesses of a pane given the overall dimensions. But the possibility to use very large window panes and glass in increasing demanding applications raises the question of how glass should be designed. The thickness of glass is normally small in relation to length and width. Deformations can be large in relation to the thickness which requires more advanced calculation methods than normally is used in structural designs. Glass is a true brittle material which fails suddenly when the failure stress is exceeded. There are many questions which need to be addressed and given an appropriate answer, some of which will be addressed in this report.

1.2 Glass as a load bearing component

Architects soon found out the potential with the qualities of float glass. This together with further developments within the glass industry, for instance heat strengthened glass, have made it possible to use glass in increasing demanding applications.

For further dematerialisation of the support structure, it is possible to use the transparent material itself as a load bearing component. This started with suspended glass walls in the 1960^{th}. The glass panes were fixed at the upper edge by means of clamps attached to a horizontal beam. This technique was rapidly adopted throughout the world. Some examples have window panes which are 13 metres high like in the new terminals at the airport Charles de Gaulle. The difficulties in manufacturing, transporting and installing such oversized panes have led to more manageable sizes.

Suspended glazing with bolted corner plate fixing points, called “patch fittings” was developed by Foster and Partners in the 1970^{th} in connection with the design of a headquarter in Ipswich in England, see Figure 1.1. The panes with the size 2 x 2,5 m with 12 mm toughened glass are suspended from the corners of the panes above by means of brass patch fitting plates. The uppermost panes are connected to the roof structure along the top edge. The result is a facade with a height of 15 m which is suspended from the roof. Lateral support against wind loads is achieved by means of glass fins inside the building suspended from the intermediate floor. The glass fins will act as cantilevers affected by a point load at the free end when the building is subjected to wind loadings.

Figure 1.1 Headquarter in Ipswich in England.

The patch fitting method was further developed during the 1980th with bolted fixing systems. Several different types of such systems were developed for different projects. For the most common system a hole was drilled at each corner of the window pane. To minimize the influence of bending and torsional stresses around the holes, spherical bearings in plane with the glass had to be developed. For horizontal stabilization cable systems were introduced as trusses. In these trusses only members in compression had to be designed for buckling. All other members were high strength cables to maximize the visibility.

During the 1980th glass started to be used in more demanding applications. This can be seen as an extension of the fin principle visualised in Figure 1.1. A one storey house was built in Almere in the Netherlands where walls are made up of 12 mm heat treated glass and stabilized by 15 mm thick fins of heat treated glass. The fins are fixed at the floor and the roof with aluminium shoes. The glass fins also serve as columns for a roof of low weight.

A more spectacular use of glass started to develop during the 1990th. This can be seen as a natural development but a new philosophy was also adopted about how glass should be used. When a solid glass pane failed it did so in a brittle manner and normally no warning could be expected. Laminated glass where two or more glass panes were glued together with a plastic membrane behaves differently. If one pane brakes in a laminated pane with two or more glass panes it can be expected that this will not lead to a complete failure for the whole system. It will lead to increased deflections and if no action is taken this might eventually lead to a creep failure. One of the pioneers in this development was Tim Macfarlane at Dewhurst, Macfarlane and Partners, a London based company which has specialised in glass applications.

An impressive use of glass is the Local Authority Office in St-Germain-en-Laye near Paris where a glass roof is supported by glass columns. The architects were J. Brunet and E. Saunier. The 24 x 24 m glass roof is supported by 8 cross shaped columns 220 x 220 mm made up of three layers of laminated heat treated glass. In one direction the panes are continuous while in the opposite direction they are spilt up and glued to the other panes. Only the inner panes of glass are intended to contribute to the load bearing capacity and the outer panes act as a shield. The cross shaped columns are approved for a loading of 60 kN. Calculations indicate that the actual failure load is around 500 kN for the column as a whole. This gives a level of safety of around 3 if only the inner panes are contributing. In reality the level of safety is higher. These columns represent the first structural units subject to substantial continuous high sustained loads.

Approximately at the same time the same architects as above created a glass roof with glass beams for the Workshops at the Musée de Louvre in Paris. The roof covers a three storey light-well which admits daylight into the underground extension of the museum. Laminated panes of four 15 mm heat treated glass panes were used for the 4 x 16 m glass roof. Beams are 600 mm high and are made of four 15 mm heat treated glass panes.

Tests of the beams revealed that the failure load was 122 to 140 kN which corresponds to a maximum stress between 33,9 to 38,9 MPa. The required load carrying capacity was 50 kN which corresponds to a maximum stress of 13,9 MPa. A calculation shows that the dead load will give rise to a maximum stress of 0,42 MPa. With a live load of 4 kN/m^2 the maximum stress will be 1,96 MPa. These stress levels indicate that each of the four panes can carry both the dead load and the live load. The actual safety level is extremely high.

Figure 1.2 Local Authority Office in St-Germain-en-Laye.

In Rotterdam a foot bridge between two buildings built completely in glass was erected in 1993 which is shown in Figure 1.3. The 3,2 m long bridge consists of panes of laminated glass joined together with point fixings of stainless steel. The floor slab consists of 2 x 15 mm laminated together and the beams are made up of 3 x 10 mm glass strips. The side walls and roofs are made of 10 mm toughened glass and 6 mm heat strengthened glass laminated together.

Figure 1.3 Footbridge in Rotterdam.

A calculation of the maximum stresses which will occur for the dead load alone is just below 1,0 MPa and for a design load of 4,0 kN/m^2 the maximum stress will be 3,1 MPa. This indicates that each of the three glass panes can carry the design load and there will still be a high degree of reliability for each glass pane. Because of the height of the beam the deflection will be very small and hardly noticeable.

In 1996 a glass canopy at the entrance to the Yurakucho underground railway station was finished, see Figure 1.4. The design is by Rafael Viñoly architects together with Dewhurst Macfarlane and Partners. This cantilevered glass structure is 10,6 m long, 4,8 m wide and 4,8 m high at the apex. The support structure consists of three parallel, cantilevered beams, composed of several triangular shaped laminated 2x19 mm glass panes. The variously shaped blades are bolted to interlock with each other with one blade at the apex and four blades at the bearing point. The roof is made of 1,9 to 2,5 m long and 4,8 m wide laminated sheets. The thickness is 2 x 15 mm toughened glass which are fixed at junctions with the cantilevered beam.

Figure 1.4 Canopy in Tokyo.

This structure was a challenge for the engineering profession and the society. The architectural design was made during a short period, roughly around a week. The technical analysis and testing took half a year. The location of the canopy in Tokyo made it necessary to design it to withstand heavy earthquakes and high wind speeds from typhoons. Besides, the building authorities wanted a fail safe structure.

The canopy had to be put together in a number of pieces because it was not possible to toughen larger lengths than around 5m . These pieces are put together with bolts which will create stress concentrations. In this case the stress concentrations are perpendicular to the main direction of tensile stresses. A number of questions arose: How close to the surface could a hole be placed without creating too much additional stresses? How should the details in the transfer zone between an axle and the hole be manufactured? To answer these questions Asahi Glass

Company started a project with tests to find the answers, (S. Wakui,1999). Three different hole diameters were tested (36,5 mm, 55 mm and 68 mm) and the result showed that if the large hole was used there was no reduction if the distance from the centre to the edge was larger than 180 mm. It was known that from previous experience that an aluminium ring was appropriate as a bezel. The actual function including a plastic ring or an epoxy adhesive was tested. This information was used in the design of the holes in the blades.

The most stressed glass blade was tested in full scale up to the design load to determine the stress field around the hole subject to the highest force. The stresses around the hole seem to have been only marginally higher than that caused by bending.

Before erection each glass blade was proof loaded up to the design load. The actual failure load was estimated to be at least 2,6 times the design load, in reality much higher. To make the structure fail proof additional blades of plexiglass was added to the structure.

1.3 Design aspects

Glass is a material which has not been used in demanding structural applications until the last decades. The reason for this is that a failure almost always is initiated by a tensile stress and this failure is brittle. No prior warning can be expected. This introduces questions concerning how the design and the reliability should be considered.

Glass has been used in windows for a very long time. It was not until the beginning of the 20th century before it was possible to produce glass panes in larger sizes of good quality. Such windows had to be designed for wind loads. From producers of glass, recommendations are given of the thicknesses needed for a specific window size. Such recommendations can vary between producers. The reason to this reflects the lack of codes for the design procedure of glass. Standards available are mainly concerned with surface finish and flatness. It can be expected that most glass producers have carried out tests on glass to determine the rupture strength. It is also very likely that such tests have revealed that the rupture strength is both size and time dependent. The rupture strength will decrease for increasing sizes of a window pane and thickness. In the same way the rupture strength will decrease for decreasing loading rates. This is not unique for glass but these dependancies are more pronounced. It is not likely that two manufacturer of glass have carried out their tests in the same way. The interpretation of the results may also differ. This can be one reason to explain differences in recommendations. But the manufacturers also have to consider the reliability or the consequences of a failure. It is reasonable to assume that glass manufacturers tend to be conservative in their evaluation of test results.

When new materials are introduced, like fibre reinforced plastics as reinforcement for concrete structures, or old materials are beginning to be used in new ways, like glass, there is a need to review the design process. This concerns

- material properties
- design methodology
- failure mode analysis

where it is often necessary to provide information for end users. Often the intention with the new or improved material is to meet certain demands or requirements on the market. In the case of glass as a construction material the need reflects architectural design ideas of creating transparent structures. There is no obvious technical advantage. On the contrary, ordinary glass is a material which can be fairly strong during short durations of a load application. But the long term properties for a sustained load are poor.

Since there was a need on the market for structural use of glass it was necessary for the glass industry to make further developments. Heat strengthened glass is such a development. Heat strengthened glass was developed to avoid the risk of spontaneous granulation because of uneven temperatures. Such glass has been available for several decades but only for special applications at high price levels. An improved production technique was necessary and new control methods to reveal impurities which could generate delayed failures. Laminated glass is also a way to improve structural characteristics which alters the over all failure behaviour.

Glass is not a material which is included in codes as a structural material. Hence, there is no information about how the design should be made. For other structural materials like steel, concrete, wood and masonry information is readily available about design principles and strength values which can be used in applications.

It is not obvious that a design of a glass structure should be made in the same way as for ordinary structural materials. On the contrary, designs made in the failure state based on the partial safety factor methodology is not directly applicable unless certain modifications are added. For loads exceeding a certain threshold stress creep will be initiated which eventually will lead to a failure. This can be considered in a similar manner as for structural wood where loads are graded in relation to their duration.

Given the knowledge that a glass failure will be initiated by a flaw in relation to a stress field there is a possibility to use other design techniques. Fracture mechanics was developed to analyse the influence of flaws in glass fibres. During the 1950th fracture mechanics evolved as a potential analysis technique for glass but in particular for steel in mechanical applications. For steel subject to fatigue stresses fracture mechanics is an established technique to estimate the time to failure. During the 1990th a number of persons have advocated the use of fracture mechanics for the analysis of glass (Jacob et al, 1997).

Another analysis technique is based on stochastic mechanics. The original theory (Weibull, 1937) makes the rupture strength size dependent. This theory can easily be extended to include a time dependence which will be conditional on the size dependence. The complexity in analysing thin plates where both the bending theory and the membrane theory have to be considered has made this technique too complex until the introduction of computers and finite element programs. Today this technique has been used in USA to produce design charts for any window pane geometry (Beason, 2002).

The basic failure mode of glass is brittle failure. This is hardly a desirable situation in a structural design but this situation is not unique. There are several ways to solve this situation which was demonstrated in the previous section. The most direct way is to use a high factor of safety. But this is not enough. Given the pronounced time dependence of the rupture load it is necessary to limit the maximum stress. The solution depends on the strategy chosen.

Laminated glass gives the potential to alter the over all failure mode from being brittle. If two or three glass panes are laminated together, where each pane can carry the design load there will not be a brittle failure if one of the panes fail. Deformations would increase and the failed unit had to be replaced.

Tempered glass where the residual stresses have been substantially increased is an alternative where no failure should be expected as long as stresses from design loads are kept below the compressive residual stress in the surface region. But this requires that no internal inclusion of foreign material is present in the interior where high tensile stresses are present.

Laminated tempered glass seems to be the ultimate solution. This makes it possible to utilize relatively high tensile stresses under long term loading conditions. Should one pane fail it is not likely that this will lead to a total failure.

The strategy which is chosen to handle the failure mode analysis cannot be seen independent from information about the properties of glass and how the design is made. It is the total strategy which will determine the reliability conditions. Each of these aspects: material information, design methodology and failure mode analysis will be presented in the following chapters in relation to available information.

1.4 Reliability implications

The statistically based design procedure was introduced mainly because of two reasons: (1) there was no realistic way to determine the actual reliability with the safety factor principle and (2) there was a general belief that most structures were over designed. To introduce a complete statistical design procedure was not possible because of the complexity this would have lead to. Instead the partial safety factor method was introduced as a compromise. In large this design procedure has lead to several improvements and most structures are more economical than if the old design procedure had been used.

Glass used as windows will be subject to wind loads and when used in roofs it has to be designed for the dead load and eventual snow loads. In these applications the duration of high wind loads is short, snow loads will have a longer duration and the dead load will be permanent. The size and time dependence of the rupture load therefore has to be dealt with separately. Besides, a window glass failure will not affect the reliability of the building itself. It may, of course, be a hazard to the surroundings where people and property may be affected in a negative way.

The examples which were presented in the section 1.2 show that glass has started to be used in structural demanding applications. Since glass is an appreciated material among architects it can be expected that this trend will continue. Glass is not a material which is included in codes as a structural material. Hence, there is no information about how the reliability should be considered. For other structural materials like steel, concrete, wood and masonry information is readily available about strength values which can be used in applications. The reliability is obtained with statistically based principles known as the partial safety factor system.

There are discrepancies with respect to size and time aspects between standardized tests and real structural members. Standardized tests are performed on well prepared small specimens in a ramp loading test. Real members are larger and in some cases extremely much larger and this will cause the rupture strength to be lower than assumed. Long term loading will cause creep to take place which in some cases is of secondary type. This will further reduce the rupture strength. This is not included properly in the present version of the partial safety factor method. So, even if a structural unit is designed in accordance with the partial safety principle, a failure may occur because of creep to failure. This latter problem can be resolved by using an appropriate statistical distribution to characterize the strength where the size and time dependencies are included. This results in an analysis based on the theory of stochastic mechanics.

There are in principle three different ways to analyse a structural member. In addition to an ordinary design based on the partial coefficient method an analysis can be made based on fracture mechanics or a stochastic mechanics. In the first case the reliability analysis is based on a fail-safe concept. By limiting the maximum stress to a level where cracks will not grow and cause a failure the structural member will be safe. In the second case the reliability is based on an allowable stress which is determined from a predetermined failure probability where size and time aspects are included. These two different principles will be applied on a beam of glass subject to four point bending to show how a design is made in each case and the result.

The question concerning brittle failure versus ductile failure will be discussed and analysed in Chapter 5 where the results from the two design principles are compared.

GLASS PROPERTIES

2.1 Introduction

Glass is a uniform material, a liquid that has solidified by cooling to a rigid state without crystallizing, which means that the molecules are in a completely random order and hence do not form a crystal lattice. It is a solid with an amorphous, non-crystalline structure and it is far too rigid to flow at normal temperatures as would a supercooled liquid do. This explains why glass is transparent. The material consists of a combination of various bonds, and hence there is no specific chemical formula. There is no melting point but instead, upon applying heat, the material gradually changes from a solid state to a plastic-viscous and finally to a liquid state. The glass used today for building purposes is a soda-lime-silica glass.

The theoretical strength of a glass is determined by the strength of the bonds between the individual components. The strength of a piece of flat glass should on atomic bond strength calculations be around 21 GPa, (Pilkington,1993). Window glass usually fails at stress levels less than 100 MPa, (Pilkington,1993). In practice, therefore, the amount of stress needed to start a crack growth in glass is very much less than expected considering the forces needed to break the interatomic bonds.

2.2 Glass production methods

There are two main flat glass manufacturing methods for producing the basic glass from which all processed glass products are made: the drawn glass process and the float glass process. Since the introduction of the float process in 1959 by Pilkington it has gradually replaced other processing techniques.

More than 90% of the world's flat glass is now made by the float process, where molten glass, at approximately 1000ºC, is poured continuously from a furnace on to a large shallow bath of molten tin. The liquid glass floats on the tin, spreads out and forms a level surface. Since the melting point of the tin is much less than that for glass, the glass solidifies as it slowly cools on top of the molten tin. Thickness is controlled by the speed at which the solidifying glass ribbon is drawn off the bath. Once the glass solidifies, it is fed into an annealing lehr where it is slowly cooled in a process where the residual stresses are controlled. This process results in the production of an annealed float glass with residual compressive stresses around 8 MPa in the surface. After annealing the glass emerges as a "fire" polished product with virtually parallel surfaces. This method, in which the glass pane is formed by floating the melt on a bath of liquid tin, revolutionized the manufacture of high-quality glass and large sizes. Float glass is available in thicknesses ranging from 2 mm up to 25 mm.

Primary processing is a treatment of the basic glass after its manufacture. Since surface flaws only lead to fracture when a tensile stress opens them, any method of putting the glass surface into permanent compression is advantageous. An applied tensile stress would have to overcome this built-in compression before it begins to open up a flaw and hence the glass would be able to resist higher loads. Toughened glass and heat strengthened glass use this principle. The stress distribution in toughened glass enables it to withstand tensile stresses of much higher levels than ordinary annealed glass. Annealed glass has a residual surface compression stress of around 8-10 MPa, (Sobek and Kutterer, 1999), because of production reasons. Any external stress level has to exceed this threshold stress to cause a failure which will be time and size dependent. The thickness of the glass may influence the actual residual compressive stress. Toughened glass, or tempered glass as it is also known as, is first cut to its final size and it is edge treated and drilled if required. Afterwards the glass pane is heated to approximately 650ºC, at which point it begins to soften. Its outer surfaces are then cooled rapidly, creating in them a high compression stress, where the rate of the cooling will determine the amount of built-in compression stress and hence the final strength of that glass. Its bending strength is usually increased by a factor of 4 or 5 to that of annealed glass and hence a new and raised threshold stress has been achieved. The maximum tensile stress in the middle is half of the surface compressive stress. When broken, it fractures into small harmless dice and it is known as safety glazing material. Heat strengthened glass is similarly produced, but with strengths approximately half that of toughened glass and without the safety glazing characteristic. Toughened glass cannot be subsequently surface or edge worked or cut because this would initiate a failure. Figure 2.1 shows the stress distribution across the thickness of toughened glass.

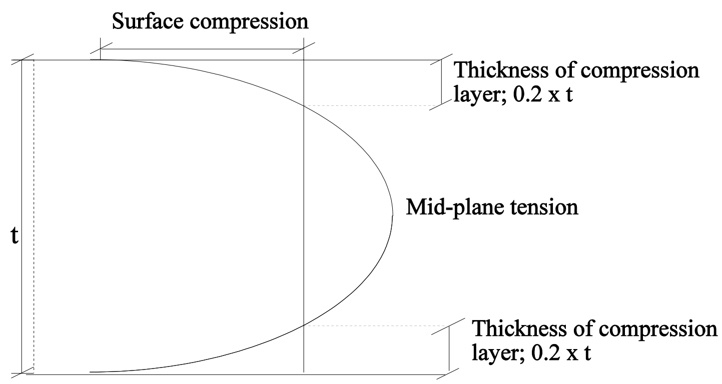

Figure 2.1 Stress distribution across the thickness of toughened glass.

Toughened glass offers an advantage because an external stress can be much higher than for annealed glass. As long as the sum of the compressive residual stress and the tensile stress from an external load is less than zero no failure is possible. At least in theory, see Figure 2.2. In reality the situation is more complex because of the presence of impurities in the interior. Most well known is NiS which exists in two forms, α or β phase. When a transition takes place from an α phase to a β phase there will be an expansion. If this expansion takes place where tensile stresses dominate this will lead to a failure, often delayed in time. Such failures can be forced by increasing the temperature to a certain level for a certain amount of time in what is called as a Heat Soak Test, HST, where a failure is forced at an elevated temperature. But NiS is not the only impurity which can create problems in terms of delayed failures. There could be other forms of impurities or processes involved which could cause a failure. The knowledge about this is far from complete. It can be anticipated that the "static fatigue" phenomenon which is characterized by the time dependence might be slower within a glass plate because it takes place without any influence of water.

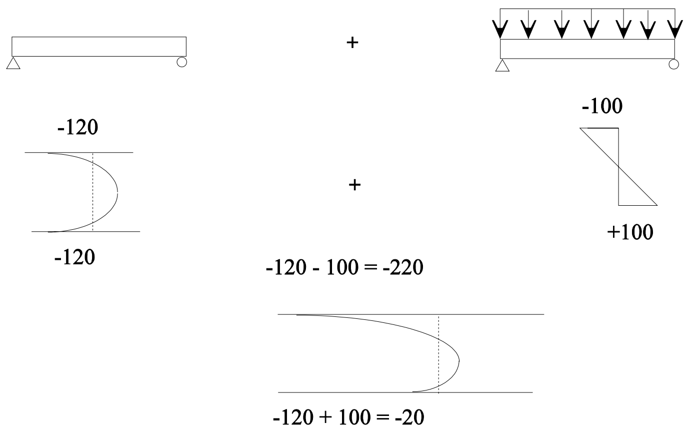

Figure 2.2 Summation of stresses in bending in toughened glass.

Chemically toughened glass is an alternative to heat treated glass. This technique to achieve higher strength in glass is based on the chemical exchange of larger radius K^+ ions, (potassium), for Na^+ ions, (sodium), in the surface of a sodium containing silicate glass. The compressive stress of the silicate network at the thin top layer produces a product known as chemically strengthened glass, where compressive stresses of up to 300 MPa, can be reached at the surface. However, this is only in a very thin boundary layer which is easily penetrated by scratches. The immersion procedure also strengthens the edges of the pane. Chemically strengthened glass exhibits a high resistance to mechanical and thermal loads. Its fracture behaviour corresponds to that of float glass and it may be cut, however, a cut edge only has the strength of normal glass.

The elastic modulus, E, of glass will not be affected by any surface treatment. As a consequence, deformation characteristics will not be influenced.

The edges of glass may be finished in a variety of ways. The normal cut edge represents the simplest form. Such edges are used whenever the edge of the glass is placed in a frame and there is no danger of being injured by the sharp edge. The edges of glass members are usually ground to remove major flaws and reduce the variation in crack size along the cut edges.

Laminated glass is produced by bonding two or more panes of glass together with a plastic material, polyvinyl butyral (PVB) or in some cases the use of liquid resins. When a PVB interlayer is used, the foil is placed between the panes and the whole unit pressed together in an autoclave under the action of heat and pressure. Laminates can incorporate most thicknesses of glass and plastics to give a selection of products with a range of mechanical, fire resistant and optical properties. When a laminated glass is broken the interlayer tends to hold the fragments of broken glass in place, and it may be named a safety glazing material. Examples of laminated glass are anti-vandal, anti-intruder and bullet-resistant glazing.

2.3 Strength characteristics

The strength of any structural material will exhibit size, time, temperature and sometimes humidity dependencies. This is explained by the presence of various types of defects which are present in all materials. Glass is no exception, on the contrary, flaws in the surface of glass will have a pronounced influence on the strength of glass. The size and time dependancies can be illustrated as is shown in Figure 2.3. For glass the size dependence is better explained with an area dependence instead of a volume dependence. The reason is the existence of defects in the surface region.

For an increasing size or area or increasing duration of a stress the failure stress will decrease. The dotted line reflectss a threshold below which no failure will take place. For annealed glass this will correspond to approximately 8 MPa and for heat strengthened glass the compression stress in the surface. The shaded area is the strength behaviour in time for a given volume under stress. The result given in figure 2.3 is in relative values.

The reason to why the strength decrease with increasing area is normally explained as that there will be an increased possibility of the existence of a major defect with increasing area. The time dependence reflect the time for a defect, or crack, to grow to a critical level which will cause a failure. This crack growth will be discussed in subsequent sections.

In (Weissman, 1997) it has been proposed that the mechanical strength is determined by micro flaws in the surface. These flaws act as stress concentrators and the critical breaking stress depends on the depth of the flaws. Flaws are created during the forming, cooling and final handling processes. The main source is the contact of the freshly produced glass with other materials. During the forming process float glass has no contact with solid materials. By that reason, freshly produced float glass has a higher strength compared with ground and polished plate glass. The grinding process simply reduces the depth of flaws but do not completely remove them.

The flaw distribution function, n(s), is the mean number of flaws per unit area, that are acting as fracture origins at stresses < s, where s is the stress at the crack tip. Fig 2.4 shows the variation of n(s) for the two float glass surfaces in comparison with polished plate glass. The results verify the higher density of surface flaws of the polished plate. Moreover the tin and atmosphere sides show a slight difference in their strength that comes from the contact of the tin side with transport rollers, (Weissmann, 1997).

The strength of glass can be based on the well-known Griffith's flaw theory (Jacob, 1999). This gives an interpretation of experimental data on the strength of glass. According to these ideas, the observed strength is always reduced below a theoretical strength by the presence of flaws and the measured strength is determined by the stress concentration acting on the "worst" suitably oriented flaw at its apex. This accounts directly for the wide variation in experimental values of strength obtained under any given condition (since all flaws will not be equally dangerous) and must obviously be considered in the interpretation of all the strength phenomena.

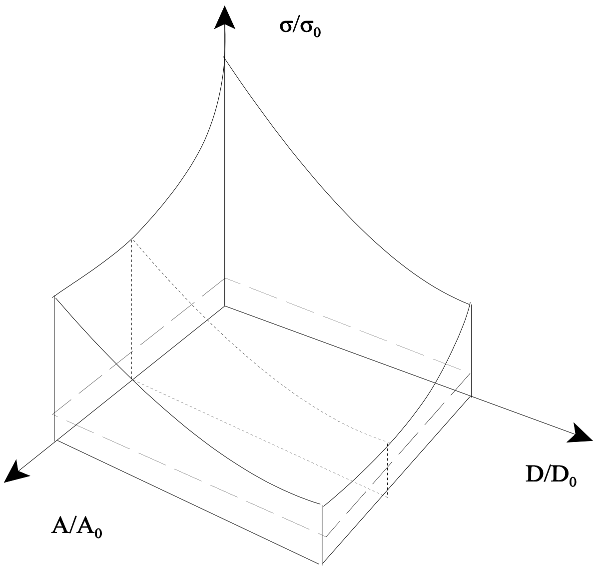

Figure 2.3 The rupture stress as a function of volume and duration of a stress.

It is simply stated here that the factor by which the "theoretical" strength exceeds the ordinary experimental values is usually considered to be in the order of 500.

The Griffith flaws, which have a strong influence on the strength of glass, are assumed random in size, distribution and orientation. The only way to establish a value for the strength of glass is to mechanically load the glass to breakage. This establishes the strength of that piece, tested at that time and in that particular way. Statistical analysis of a set of scattered values of glass strength measurements allows the derivation of a value below which there will be relatively few failures. Statistical treatment of test results can be used to obtain a design value at which there is sufficiently low risk of the glass being weaker than that design value. This value can be used to select a glass thickness for a given situation.

Tests are made to gather information regarding a variable, which in this case would be the strength of glass. This information would then indicate the strength of that "test package". Different methodologies can be utilised for further analysis, see chapter 3 and 4.

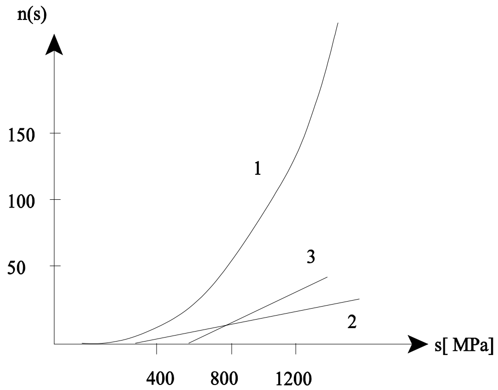

Figure 2.4 Number of surface flaws/m^2. 1-polished plate glass, 2-float glass tin side, 3-atmosphere side.

The strength of glass is also modified by the presence of larger surface flaws. Under stress these can be the origin of cracks since the glass may be unable to accomodate the local stress concentrations they cause. The presence of major flaws, for example scratches, on the surface or the edges of the glass can generate local stress concentrations when the glass is put under strain, which may lead to crack formation. It is however not easy to predict whether any individual flaw is likely to do this. Small defects in the glass are likely starting points for crack propagation as they produce unacceptable stress concentrations. The presence of large flaws is one reason why the edge region of a piece of glass is usually weaker than the surface, where it is much more prone to damage from accidental contact with the surroundings. Methods of cutting glass and edge finishing may also lead to the presence of flaws at the edge.

Since time duration of loading is important in determining the failure load for a given glass panel, it is essential that loadings be defined in a time-dependent, rather than static form. Experimental data, (Minor, 1974), which highlights this phenomen are presented in Fig 2.5.

The temperature dependence was analysed by (Charles, 1958) And the result is presented in Figure 2.6. It can be seen that the strength will decrease with increasing temperatures. Below a certain temperature where no moisture is present there seem to be no or very little time dependence. The influence of the humidity or water on the rupture behaviour is often termed static fatigue for glass. This will be analysed separately in chapter 2.4. Hence the stress will depend on the following external factors:

$$\sigma_{glass}(\text{Duration}, \text{Size}, \text{Temperature}, \text{Relative Humidity}) \propto f(D, V, T, RF)$$

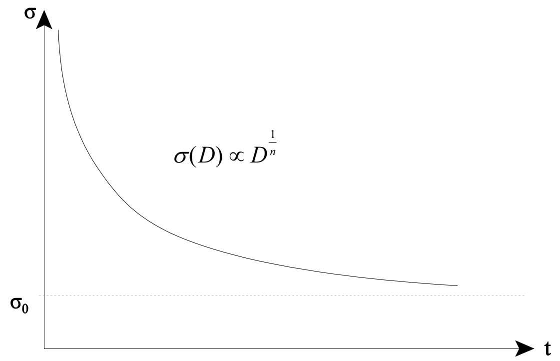

Figure 2.5 The time dependence of the strength.

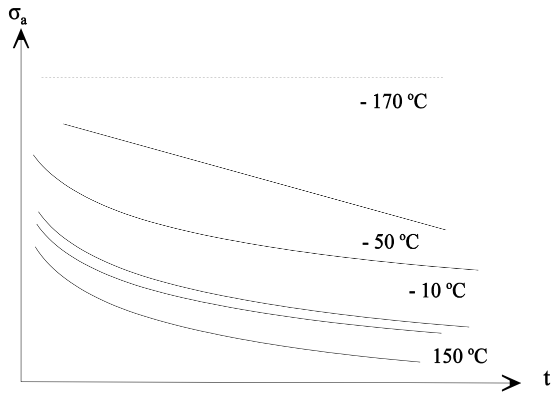

Figure 2.6 Delayed failure curves for soda-lime glass at different temperatures.

2.4 Static fatigue

Ceramics and glasses can demonstrate a loss of strength over time. This degradation of the strength can take place under constant stress conditions. In the glass community this phenomena is often termed static fatigue and it is a sensitive function of the environment. Chemical attack by water vapour permits a pre-existing flaw to grow to critical dimensions and bring about spontaneous crack propagation. Depending on environmental conditions, glass exhibits time-delayed failure where fracture may occur some time after the initial application of a load. Static fatigue is basically caused by the subcritical crack growth that can take place in glass, even in a weak tension field.

Small amounts of water vapour, normally found in the atmosphere will react with glass under stress to cause a time-dependent reduction in strength. By chemically reacting with the silicate network, an H_2O molecule generates two Si-OH units. The hydroxyl units are not bonded to each other, leaving a break in the silicate network. When this reaction occurs at the tip of a surface crack, the crack is lengthened by one atomic-scale step, see Figure 2.7 and 2.8.

Static fatigue results from a stress-dependent chemical reaction between water vapour and the surface of glass. The rate of reaction depends on the state of stress at the surface and the temperature. The rate increases mainly with increasing stress. The stress is greatest at the root of small cracks and consequently the reaction proceeds at its greatest rates from these roots. Since the reaction products do not have the strength of the unreacted glass, the small cracks gradually lengthen and failure occurs when the cracks are long enough.

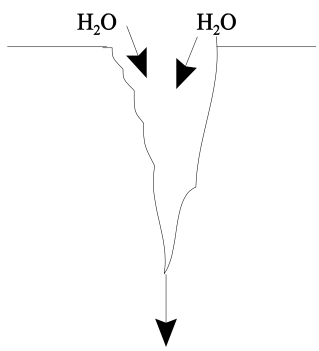

Figure 2.7 Crack growth by chemical breaking of oxide network.

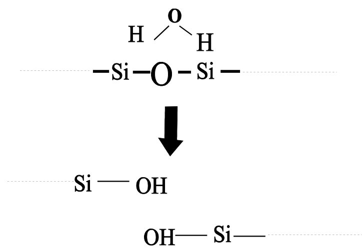

Figure 2.8 A break in the network.

It seems well established that the general failure of glass is determined by surface flaws. Stress fields around a flaw in conjunction with corrosion mechanisms dependent on stress provide a ready answer to the question of how a flaw may grow in an isotropic material, such as glass, and bring about delayed failure. The amount of growth is determined by the stress concentration that is necessary to bring the applied stress up to a critical value whereas the differential rate of growth is dependent on the stress distribution around the flaw at any instant and the composition, pressure, and temperature of the surrounding atmosphere.

Figure 2.9 shows the effect of water vapour on crack motion in glass at room temperature (Wiederhorn, 1970). When the loading is at low levels the combination of load and moisture affect the strength, whereas at high levels of loading, around 1,0 kg, the moisture has almost no effect on the strength. The crack velocity is divided into three different regions. According to Wiederhorn these conclusions were made:

- In region I crack propagation is due to corrosive attack of water vapour on the glass at the crack tip.

- In region II the crack velocity is nearly independent of the applied force, and the position of each curve shifts to lower velocities as the water in the environment decreases.

- In region III the crack velocity is again exponentially dependent on the applied force , however, the slope of the curve is considerably greater than in region I, 3 times. This change in slope leads to the conclusion that a third crack propagation mechanism occurred.

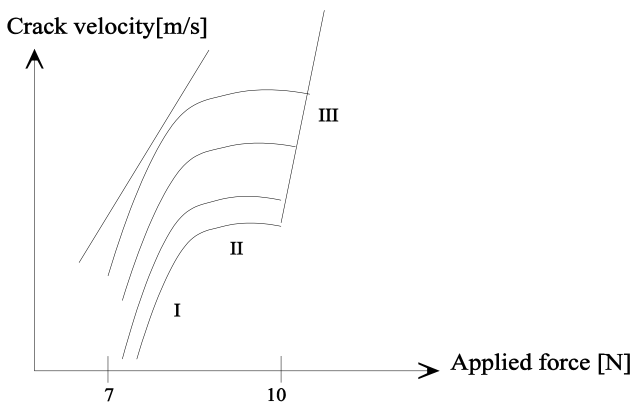

Figure 2.9 Crack velocity versus load.

The fact that the curves of region I and II blend to form a single curve in region III indicates that this new mechanism is independent of water concentration in the environment.

The load below which glass would be free of fatigue is not well known and moreover, static fatigue which is very sensitive to the environment is accelerated by temperature and atmospheric humidity, (Charles, 1958).

It was suggested (Preston, 1942, Orrowan,1941) that cracks might be assisted to spread slowly in a prolonged test by atmospheric action of the types already discussed. Baker and Preston have shown that glass specimens baked and tested in vacuum show practically no reduction in strength under prolonged loading.

2.5 Experiments on glass beams

There are two interesting reports on tests of glass used structurally. In both cases it concerns beams and the time behaviour. In Carré, (1996) a large number of beams have been tested to failure in ramp loading tests with different rates of loading. The surface treatment of the edge in tension has been what is considered the normal preparation but there is also a new improved preparation. Each beam had a free span of 230 mm, a height of 37,5 mm and a thickness of 19 mm. The loading arrangement was a symmetric four point loading scheme. The sample size is 28 beams for a loading rate of 0,5 MPa/s and two sets of 14 beams with a loading rate of 0,05 MPa/s respectively 5 MPa/s. In both cases the predicted failure behaviour based on a Weibull distribution is in good agreement with the obtained actual failure behaviour.

The main result is summarized in Table 2.1. It can be seen that both for the normal surface treatment and the improved surface treatment the rupture stress will increase with increasing rates of loading. With increasing loading rates the duration of a test will decrease.

In the report by Carré (1996), there is also an analysis of the effect of heat treatment made in a finite element analysis. The conclusion is that no failure could take place for bending tensile stresses from external load which were less than the compressive residual stresses in the surface.

Table 2.1 The main result from tests of beams

Failure stress MPa	Normal surface treatment			Improved surface treatment	
Rate of loading MPa/s	0,05	0,5	5,0	0,5	5
Mean value MPa	41,2	45,4	50,1	54,7	56,1

Based on the information from (Carré, 1996) a sustained load test was carried on a heat treated beam out by Saint Gobain (Gy, 1999). In this case the beam was substantially larger, 4 m long, the height 621 mm the width 19 mm. Also in this case a four point bending arrangement was used. The residual compressive stresses was determined to be around 120 MPa. The applied load resulted in around 80 MPa in tensile stresses which is less than the residual stress at the edges. (According to this calculation, the glass weight plus an external applied load of 4000 N, lead to a maximum tensile stress of 72 MPa. This is below the residual compressive level and hence it should prevent the beam from suffering from static fatigue during a long experiment). In addition to direct deformation at loading the creep deformation in the middle of the beam was measured during 1 year. This deformation is shown in Figure 2.10. It seems as if the creep deformation reflects primary creep which can be seen as a delayed elastic response. But primary creep is associated with what is termed a subcritical crack growth. There is an indication that there could be a start of secondary creep which reflects a crack growth. Such a crack growth can take place internally where large tensile stresses exist. This could be because of the existence of some impurity. It could also be an edge crack growth. After 1 year the beam was unloaded and the deformation recovery was continued to be measured over another year. Most of the delayed deformation was recovered during this period but a small deformation still remained. No explicit information was given to this. Personal communication indicated that this most likely was a measuring error or deformation of the supports.

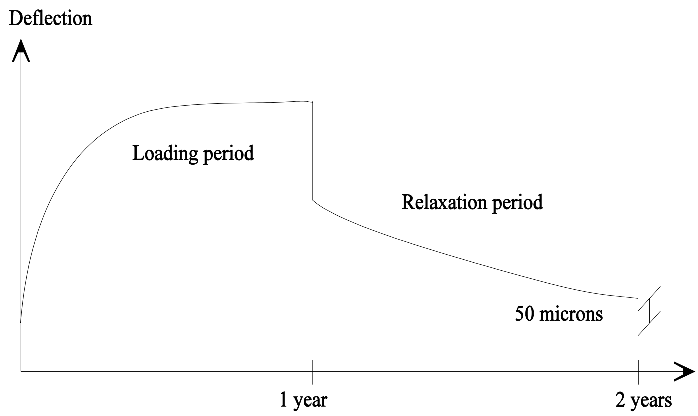

Figure 2.10 Deformation versus time.

The remaining deformation after the test was completed indicates that some damage accumulation may have taken place. Damage accumulation may reflect a crack growth in the bottom tensile face of the glass beam. This will be analysed and explained in more detail in chapter 3, fracture mechanics. If this damage accumulation really takes place there is a potential problem with heat strengthened glass.

FRACTURE MECHANICS APPROACH

3.1 Introduction

Fracture mechanics is the theory about how solids rupture, primarily structural components of different types. Such ruptures can take place in several different ways. Typical for these ruptures are that one or a few cracks propagate through the material and finally results in a rupture. Fracture mechanics deals with the mechanisms involved and methods to characterize them. Fractures are found to originate at flaws or cracks of finite size, most of which are at the surface. The mechanism, crack propagation, begins when the local stress at the crack exceeds a minimum value. Dependent on material behaviour and the type of loading fracture mechanics can be represented in three main categories.

Linear elastic fracture mechanics deals with existing cracks which, at a critical stress, grow momentarily which results in a brittle rupture. Typical is that the rupture planes do not deform and the broken parts can be put together again where the rupture line may not be directly visible. This is a common failure behaviour for glass and ceramics. But also steel may rupture in this manner, especially at low temperatures. The additional energy needed to cause the crack to grow is normally very limited.

Non-linear fracture mechanics also deals with an existing crack but the crack growth takes place in a plastic zone. Also in this case the crack growth can be momentarily if the applied stress is sufficiently high or the initial crack is large. But it is more common that the crack growth is slow and accompanied with plastic deformation. This failure behaviour is associated with metallic materials subject to relatively steady state loading conditions.

Fatigue initiated fracture mechanics cause existing cracks to grow in relation to the load spectrum which affects the component with a crack. This crack growth can initially be extremely slow and start at cracks which are not easily detectable. All structural elements contain defects in terms of cracks already after production. In a surface of a material there is likely to be irregularities between groups of atoms or molecules. This might be sufficient to start a crack growth. This type of crack growth is mainly associated with non-linear fracture mechanics. But there are examples where a crack growth takes place for a brittle material response. Cracks can grow in a slow manner in glass under certain loading conditions.

Fracture mechanics is a powerful tool in many applications. It can answer the following questions:

- How large a crack can be tolerated for a specified stress condition before a failure can be anticipated?
- How fast will a crack grow when subject to fatigue for a specified load spectrum?

Dependent on the answer the structural part can be modified, the stress field altered or some type of regular inspection procedure can be introduced to detect cracks before they will cause a failure. In many advanced applications where materials are used to their limit, for instance turbines, regular inspections is part of the maintenance scheme.

3.2 Some historical notes

Problems with unexpected failures due to crack growths, especially in connection with stress concentrations, was recognized already in the 19th century. Sharp corners and defects in form of different types of cracks were often associated with the initiation of a failure. The first published theoretical article on this subject is by Inglis (1913). For a flaw which is represented by half an ellipse with opening 2*b* and depth *a*, see Figure 3.1, the stress at a flaw tip is given as

$$\sigma_m = \sigma_0 \left(1 + 2 \frac{a}{b} \right) \tag{3.1}$$

where σ_m is the concentrated stress at the flaw tip, σ_0 is applied stress. For a circular flaw, a = b, the stress increase is 3. When *a* increases or *b* decreases σ_m will approach infinity. This is not possible in reality so instead there will be a crack growth. The same stress intensity increase is obtained also for a whole elliptically flaw in the surface.

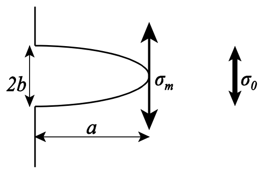

Figure 3.1 Half elliptical flaw.

The expression within the parenthesis in (3.1) can be seen as a stress concentration factor K_t. This can be generalized as

$$K_t = 1 + \frac{a}{b} = 1 + 2\sqrt{\frac{a}{\rho}} \tag{3.2}$$

where $\rho = b^2/a$ is the radius of curvature of the ellipse at the vortex. This expression can be applied to other shapes than circular or elliptical flaw tips if the curvature is known. When ρ goes towards zero K_t will approach infinity.

Equation 3.1 is applicable to an analysis of the influence of flaws on the strength of glass. This was demonstrated by Charles (1958,1) in an analysis of the conditions for a crack growth. He assumed that for a theoretical strength of glass of $14 \cdot 10^3$ MPa a ratio a/b = 40 is necessary for a failure at an applied stress of 172 MPa (Note that this high stress level refers to small bars of glass subject to bending). This will alter the initial crack relations and make the ratio a/b to continuously increase. Based on his experiments there is a critical value of a/b ≈ 200, Charles(1958, 1).

The theory of brittle fractures was extended by Griffith (1920) who made use of Inglis stress concentration expression around a crack. The novelty in Griffith´s work is that he made an attempt to formulate a linear elastic theory for crack propagation based on considerations of the global balance of energy in a body.

For an linear elastic body the internal elastic strain energy U is

$$U = \int_V u \, dV = \int_V \frac{\sigma_{ij}\,\varepsilon_{ij}}{2}\, dV \tag{3.3}$$

where u is the elastic strain energy density, and σ and ε are stress and strain respectively and the indices refers to x and y in a Cartesian system. With Hooke´s law

$$\varepsilon_{ij} = \frac{1}{2\,\mu}\left(\sigma_{ij} = \frac{\nu}{1+\nu}\,\sigma_{ij}\,\delta_{ij}\right) \tag{3.4}$$

where μ is the shear modulus of the material, $\mu = E/2(1 + \nu)$, E is the elastic modulus and ν is Poisson´s ratio. Combining (3.3) and (3.4) the elastic strain energy is given by

$$U = \int_V \frac{1}{4\,\mu}\left(\sigma_{ij}\sigma_{ij} = \frac{\nu}{1+\nu}\,\sigma_{ij}\sigma_{ij}\right) dV \tag{3.5}$$

This is the total elastic strain energy stored in a loaded body made of linear elastic material.

For a body subject to a prescribed load P, where P can be any type of loading such as force, moment or distributed force, the potential Π of the body is defined as

$$\Pi = U - P\Delta = U - W \tag{3.6}$$

where Δ is the displacement if P is a force, and Δ is rotation if P is a moment. For a fracture to occur, the energy stored in a body including the work done by the forces acting on the body, must be sufficient to overcome the surface energy needed to create a new crack surface. The total energy is given by

$$E_{tot} = \Pi + W_s \tag{3.7}$$

where $W_s = 2Aw_s$ is the energy required to create a new crack surface. Here A is the crack area and w_s is the surface energy per unit area. When a crack propagates no net change in the total energy takes place, thus

$$\frac{dE_{tot}}{dA} = \frac{d\Pi}{dA} + \frac{dW_s}{dA} = 0 \tag{3.8}$$

This results in the condition

$$- \frac{d\Pi}{dA} = \frac{dW_s}{dA} \tag{3.9}$$

which must be met to make the crack propagation possible. Crack growth is possible when the energy release rate G reaches a critical value

$$G = G_{critical} = G_c = \frac{dW_s}{dA} \tag{3.10}$$

The parameter G_c is a measure of the fracture toughness of the material. This is also known as the Griffith criterion and can be expressed as

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi c_0}} \tag{3.11}$$

where γ is the and c_0 is the
It is often considered a material property. In reality it will be temperature dependent and in the case of glass also humidity dependent.

3.3 Linear fracture mechanics
Stresses and deformations in a body in front of a crack tip depends on how the body is loaded. The crack may be affected in three different ways or modes which are illustrated in Figures 3.2 - 4.

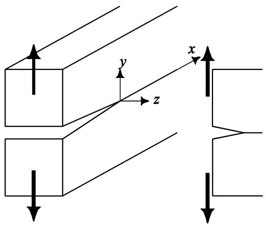

Figure 3.2 Crack loaded in Mode 1.

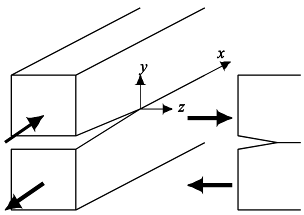

Figure 3.3 Crack loaded in Mode 2.

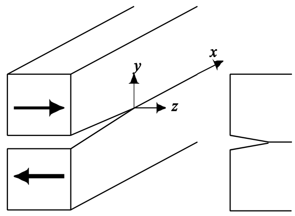

Figure 3.4 Crack loaded in Mode 3.

The analysis of stresses and deformations around the crack tip is normally made in a polar coordinate system which is defined in Figure 3.5. Mode 1 and Mode 2 solutions can be found with the help of the Airy stress function (Timoshenko, 1959). Mode 3 solutions are based on equilibrium, deformation and constitutive equations. Solutions result in series expansions which can be found in books dealing with in depth analysis of fracture mechanics. Most available fracture mechanics solutions are based on Mode 1 failures. This is natural when stresses from bending moments dominate. Mode 2 and 3 failures are hardly dealt with in practice. But it cannot be ruled out that the crack growth process which takes place in static fatigue is influenced by Mode 2 or Mode 3 stress conditions. The main results which are used in research can be summarized in the following expressions (Dahlberg, et al, 2002):

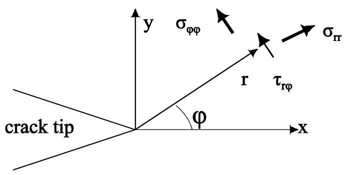

Figure 3.5 Polar coordinate system and stress components.

Mode 1:

$$\sigma_{xx} = \sigma_{rr}(\varphi = 0) = \frac{K_I}{\sqrt{2\pi x}} \tag{3.12}$$

$$\sigma_{yy} = \sigma_{\varphi\varphi}(\varphi = 90) = \frac{K_I}{\sqrt{2\pi x}} \tag{3.13}$$

$$\tau_{xy} = \tau_{r\varphi}(\varphi = 0) = 0 \tag{3.14}$$

$$u_y = u_\varphi(\varphi = \pm\pi) = \pm\frac{\kappa+1}{2\mu}K_I\sqrt{\frac{-x}{2\pi}} \tag{3.15}$$

Mode 2:

$$\sigma_{xx} = \sigma_{yy} = 0 \tag{3.16}$$

$$\tau_{xy} = \tau_{r\varphi}(\varphi = 0) \quad \frac{K_{II}}{\sqrt{2\pi x}} \tag{3.17}$$

$$u_x = u_r(\varphi = 0) = \pm\frac{\kappa+1}{2\mu}K_{II}\sqrt{\frac{-x}{2\pi}} \tag{3.18}$$

Mode 3:

Mode 3:

$$\tau_{xz} = \tau_{rz}(\varphi=0) = 0 \tag{3.19}$$

$$\tau_{yz} = \tau_{\varphi z}(\varphi=0) = \frac{K_{III}}{\sqrt{2\pi x}} \tag{3.20}$$

$$u_z = u_r(\varphi=\pm\pi) = \pm\frac{\kappa+1}{2\mu}K_{III}\sqrt{\frac{-x}{2\pi}} \tag{3.21}$$

where

$$\kappa = \frac{3-\nu}{1+\nu} \qquad \textit{for plane stress} \tag{3.22}$$

$$\kappa = 3-4\nu \qquad \textit{for plane strain} \tag{3.23}$$

Plane stress is expected for a thin plate where the stress component perpendicular to the plane is zero, i.e. $\sigma_{zz} = 0$ and $\tau_{rz} = \tau_{\varphi z} = 0$. For a plate which is not very thin the stress σ_{zz} will not necessarily be zero. If the lateral strain ε_{zz} is prevented at the crack tip then $\varepsilon_{zz} = 0$ and $\tau_{rz} = \tau_{\varphi z} = 0$. This condition is termed plane strain.

Even if a glass plate can be considered thin there will be significant stresses perpendicular to the plane. There will always be residual stresses which indicate that a plane stress assumption might not be the most optimal solution for glass. Still, this is the general assumption made in all publications available. This is a simplification which need to be analysed in a more objective way.

The factors K_I, K_{II} and K_{III} give a measure of the strength of the stress singularities at the crack tips. These factors are called stress intensity factors and play an important role in fracture mechanics. These factors can be expressed as

$$K_I = \sigma_{y,0}\sqrt{\pi a}\,f \qquad K_{II} = \tau_{xy,0}\sqrt{\pi a}\,g \qquad K_{III} = \tau_{yz,0}\sqrt{\pi a}\,h \tag{3.24a, b, c}$$

where a is the crack length and the index 0 refers to stresses far away (compare Figure 3.1). The functions *f*, *g* and *h* will depend on the geometry and the type of loading. Values of *f*, *g* and *h* can be found in handbooks dealing with fracture mechanics (Lawn,1995).

The fracture criterion is normally defined for plane strain conditions. Since Mode 1 normally is most dangerous a failure is expected when

$$K_I = K_{Ic} \tag{3.25}$$

where K_{Ic} is the fracture toughness of the material. This fracture toughness is determined experimentally. There are three conditions which have to be met:

$$(t, a, W\text{-}a) \quad \geq 2{,}5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 \tag{3.26}$$

where t is the thickness of the plate, a is the crack depth and W is the height of the member in the same direction as the flaw.

In the case of plane stress the failure condition is written

$$K_I = K_c \tag{3.27}$$

where K_c is determined experimentally.

Failure occurs when the stress intensity factor reaches a critical value, K_{IC}. This critical stress intensity factor is a constant for glass and provides a fixed basis on which to conduct design, rather than using an allowable stress which has to vary with time. The loss in strength over time is a result of slow crack growth which occurs when the stress intensity factor is less than the critical value. There is a threshold stress intensity factor limit below which no slow crack growth occurs.

Glass fails when the stress intensity factor reaches the critical stress intensity factor. This failure is sudden and results in very rapid crack growth leading usually to loss of structural integrity. If the stress intensity factor is less than the critical value then the crack grows. The speed of this growth is given by

$$v = v_0\left(\frac{K_I}{K_{IC}}\right)^n \tag{3.28}$$

where n is the static fatigue constant and has a value between 12 and 20, but a often used value is 16 and it depends on environmental conditions.

3.4 Fracture mechanics of glass windows

The fracture of glass is associated with the existence of flaws. Griffith advanced a theory that brittle bodies fail under a relatively low nominal stress because of the presence of flaws or cracks which have the characteristics of extreme narrowness with respect to their length or depth. Consequently they produce high local stress concentrations when oriented at right angles to a tensile component of stress.

There are two ways of looking at crack growth for glass. First, from the point of view of the stress intensity factor, K_I, which acts as a resistance to cracking and it is called fracture toughness. The crack can grow when its stress intensity factor reaches the fracture toughness of the material. A typical value for soda-lime glass is 0,78 MPa $m^{1/2}$.

A series of 3 point bending tests were conducted on 70mm x 40mm x 6mm thick glass samples consisting of annealed, tempered and laminated glass (Jacob, 1999). Control samples without any surface damage was used to compare the magnitude of strength reduction caused by the application of surface scratches on the glass samples. Figures 3.6 to 3.8 demonstrate the impact a small scratch has on the reduction in the bending strength.

Surface scratches to glass up to 0,008 mm in depth were found to be the limiting scratch. Using this as a limiting scratch, the fracture stress can be determined from eq 3.11. In this analysis the fracture toughness is assumed to be K_{IC} = 0,851 $MPam^{1/2}$.

The following conclusions were made from the analysis:

- the limiting flaw based on a permissible design stress of 15,7 MPa is 0,745 mm
- the limiting stress based on a maximum flaw depth of 0,008 mm is 47,9 MPa
- using a safety factor of 2,5 for permissible glass design, the design stress will be 19,16 MPa.

These conclusions seem somewhat arbitrary. From reality it is known that the failure stress is both size and time dependent. The safety factor principle is hardly used within building component designs any more. It has been replaced by the statistically based partial safety factor method. It had been possible with a more advanced analysis where the permissible stress had been determined in a statistical analysis.

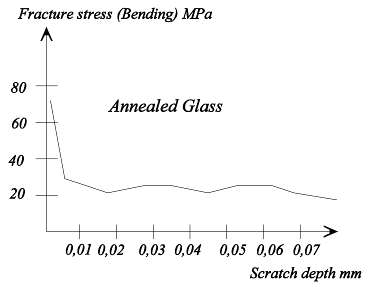

Figure 3.6 Fracture stress versus scratch depth for annealed glass.

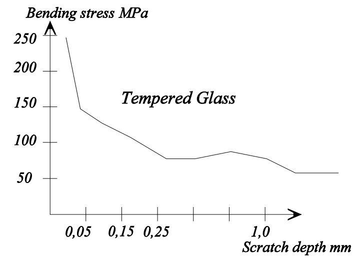

Figure 3.7 Fracture stress versus scratch depth for tempered glass.

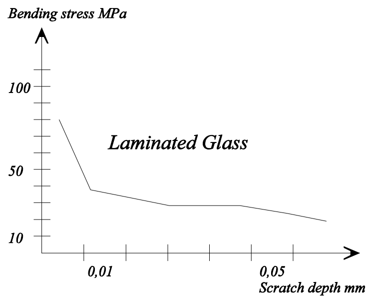

Figure 3.8 Failure stress versus scratch depth for laminated glass.

Failures in glass panels generally start from the surface or the edge depending on the loading and support conditions. The environmental contribution to crack growth results in a time dependence of strength known as static fatigue. Static fatigue only occurs in the presence of water, which reacts chemically with the strained bonds at the crack tip causing bond rupture. The rate of crack growth is determined by the rate of chemical reaction, and the time to failure is determined by the time required for the crack to grow from a sub-critical to a critical size, at which point fracture will be immediate. Once the crack growth has been characterised then the time to failure can be calculated provided the size of the critical flaw can be determined.

It was suggested purely on grounds of geometry (Littleton and Preston, 1922) that surface flaws can be roughly twice as dangerous as internal flaws of the same shape and orientation. Upon exceeding a "critical (tensile) stress" the crack begins to grow at the tip of a notch or crater. In some circumstances this growth only takes place in small steps, coming to a halt in between. In fracture mechanics this slow or "stable" crack growth is regarded as sub-critical, and is primarily determined by the duration of the load. Short-term loads lead to higher allowable stresses than long-term loads. The sub-critical crack growth is influenced by chemical reactions at the tip of the crack. Once the critical crack growth velocity has been exceeded, a crack becomes "unstable", i.e. the widening process accelerates rapidly. This then leads to sudden failure of the glass element.

For an applied stress which leads to a stress intensity factor $K_I < K_{IC}$, a crack growth may still be possible due to the effect of the environment. Crack growth under these conditions is called "sub-critical crack growth" or "static fatigue", see Chapter 2.4, and may ultimately lead to fracture some time after the initial application of the load. The growth of the crack over the lifetime of the structure is modelled based on the expected stresses, and the time to failure can be estimated as:

$$T_{\min} = 2\sigma_a^{-2}(K_{IC}\sigma_a / \sigma_p)^{2-n} / AY^2 n - 2) \tag{3.29}$$

where Y is a dimensionless parameter, see Appendix A, that depends on both the specimen and crack geometry and sigma is an applied stress and a is crack length. The exponent n determines the time dependence.

According to (Fischer-Cripps and Collins, 1995) the minimum strength of glass is related to a threshold stress intensity factor, rather than a unique minimum stress. If the initial crack size is known then a minimum long-term stress strength can be determined. If, however, during the loading history of the member this stress is exceeded, then cracks will grow resulting in a lower subsequent minimum strength, even if the stress then reverts to its initial value.

When the deformation of the pane exceeds around 75% of its thickness the action of membrane stresses come to affect the total stress distribution on the surface. The membrane stresses grow towards the middle section of the pane. The extra contribution of shear stresses leads to a maximum stress that is some distance away from the middle. The effect of membrane stresses depend upon the supporting solution.

3.5 Analysis of beams

Beams and other structural elements subject to tensile stresses can be analysed with a fracture mechanics approach. In this analysis only beams subject to four point symmetric bending is dealt with. Such a beam is shown in Figure 3.9. Between the two point loads the maximum bending stress will occur and this is most likely to result in a Mode I failure. Between the supports and each point load the maximum shear stress will occur and this may result in a Mode II failure. In all normal analysis a Mode I failure will be the dominant failure mode. This failure mode is the only one dealt with in the literature dealing with fracture mechanics of glass.

The maximum load effects are given by

$$V_{\max} = \frac{P}{2}$$
$$M_{\max} = \frac{P}{2} \cdot \frac{L-e}{2}$$

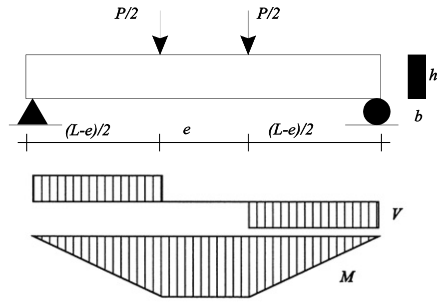

Figure 3.9 Beam loaded with two point loads and the resulting shear and moment diagrams.

and with the bending resistance W

$$W = \frac{b\,h^2}{6}$$

the maximum stress can be derived as

$$\sigma = \frac{M}{W} = \frac{3P(L-e)}{2bh^2}$$

As an example, assume the following values for the beam L= 2 m, h = 0,400 m and b = 0,006 m. It is subject to two point loads with e = 0,500 m. The total load is P = 6 kN. This will result in a maximum stress of

$$\sigma = \frac{M}{W} = \frac{3 \cdot 6(2{,}000 - 0{,}500)}{2 \cdot 0{,}006 \cdot 0{,}400^2} = 14{,}06 \cdot 10^3 \quad kPa$$

which is a stress that should not cause a direct failure. Because of a crack, real or assumed, in the section of maximum stress there will be stress increase which can be analysed with eq. 3.11. Values of K_I as well as additional information concerning calculation of K_I can be found in Appendix A. The value of K_I can be expressed as:

$$K_I = \sigma_0 \sqrt{\pi a} f\left(\frac{a}{w}\right) \quad \Rightarrow$$

$$\sigma_0 = \frac{K_I}{\sqrt{\pi a}\, f\left(\frac{a}{w}\right)}$$

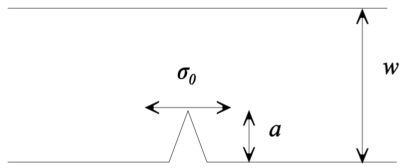

Figure 3.10 A crack.

where σ_o is the stress at the tip of the crack. Assume that $K_{IC} = 0{,}851 \cdot 10^6 \mathrm{Pam}^{1/2}$.

For a crack depth of a = 0,008 mm, which seems to be the smallest crack depth which can be measured directly in a sheet of glass, the following calculation can be made

$$K_I = \sigma_0 \sqrt{(\pi a)} f\left(\frac{a}{w}\right) \leq K_{IC}$$

$$\sigma_0 = 14 \cdot 10^3 \,\mathrm{kPa} = 14 \mathrm{MPa}$$

$$f\left(\frac{a}{w}\right) = f\left(\frac{0{,}008}{400}\right) = 1{,}12 \quad \Rightarrow$$

$$K_I = 14 \cdot 10^6 \sqrt{\left(\pi \cdot 0{,}00810^{-3}\right)} 1{,}12 \Rightarrow$$

$$K_I = 0{,}0786 \cdot 10^6 < K_{IC} = 0{,}851 \times 10^6$$

$$\Rightarrow \ OK$$

For a crack depth of a = 0,1 mm

$$K_I = \sigma_0 \sqrt{(\pi a)} f\left(\frac{a}{w}\right) \leq K_{IC}$$

$$\sigma_0 = 14 \times 10^3 \,\mathrm{kPa} = 14 \mathrm{MPa}$$

$$f\left(\frac{a}{w}\right) = f\left(\frac{0{,}01}{400}\right) = 1{,}12 \quad \Rightarrow$$

$$K_I = 14 \times 10^6 \sqrt{\left(\pi \cdot 0{,}110^{-3}\right)} 1{,}12 \Rightarrow$$

$$K_I = 0{,}278 \times 10^6 < K_{IC} = 0{,}851 \times 10^6$$

$$\Rightarrow \ OK$$

$$K_I = \sigma_0 \sqrt{(\pi a)} f\left(\frac{a}{w}\right) = K_{IC} \quad \Rightarrow$$
$$\sigma_0 = 14 \times 10^3 \, \text{kPa} = 14 \text{MPa}$$
$$f\left(\frac{a}{w}\right) = ? \qquad \Rightarrow$$
$$K_I = 14 \times 10^6 \sqrt{(\pi \cdot a)} f\left(\frac{a}{w}\right) = K_{IC} = 0{,}851 \times 10^6$$
$$\Rightarrow \textit{critical cracksize}?$$

Obviously the initial size of a crack is a very important factor for the failure stress. Another way to address the problem is to find the critical crack size. This method will be an iterative process, since when the crack length, a, is explicit it will still be a function of itself and hence a non-explicit solution will be obtained. Because the crack depth, a, does not affect the height of the structure, w, to any larger extent until it reaches about 30% of w, this iterative process can be done quickly.

$$a_c = \left[\frac{K_I}{\sigma_0 f\left(\frac{a}{w}\right) \pi} \right]^2 \tag{3.30}$$

$$K_I = \sigma_0 \sqrt{\pi a} f_6\left(\frac{a}{H}\right)$$
$$f_6\left(\frac{a}{H}\right) \approx 1{,}122$$
$$K_{IC} = 0{,}851$$
$$\sigma_0 = 20 \, MPa \Rightarrow$$
$$a = \left(\frac{K_{IC}}{\sigma_0 f \frac{a}{w}} \right)^2 \cdot \frac{1}{\pi} = \left(\frac{0{,}851 \cdot 10^3}{20 \cdot 10^6 \cdot 1{,}122} \right)^2 \cdot \frac{1}{\pi} = 0{,}4578 \cdot 10^{-3}$$

For the following stresses in the crack tip the critical crack depth will be

σ_0 = 50 MPa $\quad \Rightarrow \quad a = 0{,}073246 \; mm$

σ_0 = 100 MPa $\quad \Rightarrow \quad a = 0{,}018311 \; mm$

σ_0 = 150 MPa $\quad \Rightarrow \quad a = 0{,}000812 \; mm$

Example: A glass plate contains an atomic-scale surface crack. (Take the crack tip radius ≈ diameter of an O^{2-} ion). Given that the crack is 1 μm long and the theoretical strength of the defect-free glass is 70,0 GPa, calculate the breaking strength of the plate.
Solution:

$$\sigma_m \cong 2\sigma\sqrt{\frac{a}{\rho}} \quad \Rightarrow \quad \sigma = \frac{1}{2}\sigma_m\sqrt{\frac{\rho}{a}}$$

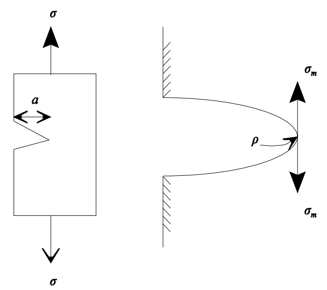

Figure 3.11

Table, (see Appendix):

$$\rho(O^{2-}) = 2 \cdot r(O^{2-}) = 2 \cdot 0{,}132\ nm = 0{,}264\ nm$$

$$\sigma = \frac{1}{2} 70 \cdot 10^{9} \sqrt{\frac{0{,}264 \cdot 10^{-9}}{1 \cdot 10^{-6}}} = 57{,}0\, MPa$$

STOCHASTIC MECHANICS APPROACH

4.1 Introduction

Stochastic mechanics is the theory about how to consider the uncertainty in the rupture strength of materials. Such uncertainty concerns both the magnitude of a rupture and the variability. The reason to this uncertainty reflects the presence of flaws in a material. An analysis of the rupture strength can be done in two principally different ways. It can be based on the influence of flaws or the consequence of flaws. In the latter case it is more adequate to term it a statistical analysis.

All materials contain flaws. They can be a natural part of a material or a consequence of the production, handling and use. In theory the tensile strength of glass is around 20 000 MPa. In reality the rupture strength of window panes is in the range of 30 - 80 MPa. This dramatic reduction of the rupture strength is not unique for glass. Similar reductions can be found for any type of structural material.

If a material had been perfect, i.e. all internal components were in perfect order, then a tensile failure had been expected for a tensile stress. With internal disruptions between the components there will be a new failure mode, a shear failure mode which can take place for an external tensile stress. This has been studied extensively for metallic materials where this shear behaviour is associated with viscous creep. For other materials where this type of creep is not possible a crack growth takes place instead. For glass and other ceramics this crack growth seems to be associated with primary creep which can be seen as a delayed elastic response.

Flaws are often assumed to be random in size and orientation. A consequence of this assumption is that there is an increasing probability of finding a more dangerous flaw for increasing volumes under stress. This introduces a size dependence of the rupture strength. A size dependence of the rupture load is a well known phenomenon for structural materials as well as for glass.

Existing flaws can grow in time or new flaws can be created. This alters the original flaw characteristics and thus the failure stress, which in turn introduces a time dependence of the rupture stress. Such a time dependence of the rupture load is of concern if a material is subject to a high sustained stress, for instance pre-stressing tendons in concrete beams. For glass there is a noticeable time dependence which needs to be considered.

The environment in terms of the humidity also has a pronounced effect on the strength of glass. Cracks will grow in size faster when water molecules are present, something which is termed static fatigue for glass.

Stochastic mechanics is also a powerful tool in many applications. It can answer the following questions:

- What is the failure probability for a structural unit subject to specified stress condition?

- How will the rupture strength decrease for an increasing volume under stress?
- How will the rupture strength decrease for an increasing duration of a stress?

Dependent on the answer the structural part can be modified or the stress field altered.

4.2 Some historical notes

The presence of a size dependence of the rupture stress was first noticed by Leonardo da Vinci for wrought iron in the 15th century. This does not seem to have given rise to any practical use. In the beginning of the 20th century it was recognized that it was necessary to standardize test methods in terms of specimen size and rate of loading to make fair comparisons possible.

For brittle materials Weibull(1939) introduced a theory that could characterize the size dependence of the rupture load. He assumed that a bar subject to a tensile stress can be split up in a large number of units or volumes. For each such volume, V_i, there is a probability density function, pdf, giving the probability of failure. The necessary condition for the occurrence of fracture in the body at a certain definite load is that the ultimate strength shall be exceeded at a single point/element, i.e. the weakest element. This probability is equal to the product of the probabilities in every elementary volume of the body that fracture will occur. If two independent elements are put in series, the new failure probability, P, will be

$$P(\text{system fails}) = 1-(1-F_1)\cdot(1-F_2) = \{F_1 = F_2\} = 1-(1-F_1)^2 \qquad (4.1)$$

This can be done for n elements which results in the following failure probability distribution

$$P(\text{system with n elements fail}) = F_{nv} = 1-(1-F_1)^n \qquad (4.2)$$

After taking the logarithm and differentiating, the result will be

$$F_{nv} = 1-(1-F_1)^n \qquad (4.3)$$

$$1-F_{nv} = (1-F_1)^n$$

$$\ln(1-F_{nv}) = n\ln(1-F_1) \qquad (4.4)$$

$$\frac{d}{dV}\ln(1-F_{nv}) = \frac{n}{1-F_{v1}} \qquad (4.5)$$

$$d\ln(1-F_{nv}) = \frac{n}{1-F_{v1}}dV \tag{4.6}$$

$$\ln(1-F_{nv}) = \int\frac{n}{1-F_{v1}}dV \tag{4.7}$$

$$1-F_{nv} = e^{\int\frac{n}{1-F_{v1}}dV} \tag{4.8}$$

$$F_{nv} = 1-e^{\int\frac{n}{1-F_{v1}}dV} \tag{4.9}$$

The failure probability function will be:

$$F_{nv} = 1-e^{\int g(\sigma)dV} \tag{4.10}$$

For the stress function g(σ) Weibull suggested

$$g(\sigma) = -\left(\frac{\sigma}{c_o}\right)^m \tag{4.11}$$

where c_o is a normalization constant. The exponent m is often referred to as a size parameter since it determines the size dependence. It reflects the influence of various types of defects where cracks is one type. It can also be irregularities in the atomic structure which are not directly measurable. The statistical distribution (4.10) with eq. (4.11) is the well known two parameter Weibull distribution.

4.3 Stochastic mechanics for brittle materials

There are various approaches to the statistical theory of brittle fracture. These differ in the method of reasoning and the assumption of the postulated form of the distribution function of the strength of the primary elements. It is also possible to start with a hazard function, which is the probability of failure given that no failure has taken place before. A solution based on an asymptotic distribution for minimum values of sufficiently large sets will be given below. This approach is a generalization of the Weibull method, being at the same time justifiable from a theoretical point of view. Weibull apparently proposed the distribution function out of convenience to fit available experimental information, which has been a vulnerable spot in his theory.

The principle utilized in section 4.2 can be applied for any type of statistical distribution and when n goes towards infinity this will lead to an extreme value statistical distribution. Based on the original distribution there are three possible extreme value distribution alternatives, type 1, type 2 and type 3. For each of them there is a distribution for a minimum and a maximum value. This theory of statistical distributions of extreme value was not available at the time when Weibull developed his statistical distribution.

If the original distribution cannot take negative values, i.e. the breaking tensile stress is positive. The tail of the distribution will to some extent reflect stresses between the components in the material which often are represented with a hyperbolical expression. With these two conditions it can be expected that the extreme value distribution is of type 3 and minimum value. This statistical distribution can be expressed as

$$F(\sigma) \approx 1 - \exp[\ \ knV(\sigma - \sigma_0)^m] \tag{4.12}$$

where σ_0 is a minimum strength of a defective element. For nV sufficiently large the approximate sign can be substituted with an equal sign. When $\sigma_0 = 0$ the result is the same statistical distribution proposed by Weibull. The conditions for eq. (4.12) to hold are fairly general and a broad class of initial statistical distributions can be utilized. With a change of the constants k and m it is possible to obtain a statistical distribution with a wide variety of form and scale. The Weibull distribution has many interesting features which has made it attractive in many fields of application. If a reference volume V_0 and a constant c_0 are introduced as

$$kn = \frac{1}{V_0 c_0^{\,m}} \tag{4.13}$$

then eq. (4.12) can be expressed as

$$F(\sigma) = 1 - \exp\left(-\frac{V}{V_0}\left(\frac{\sigma - \sigma_0}{c_0}\right)^m\right) \tag{4.14}$$

which is common way to express the Weibull distribution in structural applications. But since it is known that the failure stress also is time dependent the constant kn can be chosen to take this into account. From the information given in Chapter 2 this time dependence is also hyperbolical in terms of the mean value. To obtain this result the constant kn is chosen as

$$kn = \left(\frac{D}{D_0}\right)^{\frac{m}{h}} \frac{1}{V_0 c_0^{\,m}} \tag{4.15}$$

where D is an effective duration and D_0 is a reference duration. This will result in

$$F(\sigma) = 1 - \exp\left(-\frac{V}{V_0}\left(\frac{D}{D_0}\right)^{\frac{m}{h}}\left(\frac{\sigma - \sigma_0}{c_0}\right)^m\right) \tag{4.16}$$

Equation (4.16) can be applied both for loading directly to failure or creep to failure. The effective volume under stress and the effective duration of stress need attention under non uniform stress or not constant stress conditions in time. Any volume can be split up in small units as well as a duration in time units and the evaluation of eq. (4.16) can be expressed as

$$F(\sigma) = 1 - \exp\left(-\frac{1}{V_0}\left(\frac{1}{D_0}\right)^{\frac{m}{h}} \sum_i^p \Delta V_i \sum_j^q \Delta D_j^{\frac{m}{h}} \left(\frac{\sigma(x,y,z,t) - \sigma_0}{c_0} \right)^m \right) \tag{4.17}$$

which is how the analysis will take place in a computer evaluation based for instance on the finite element method. When p and q goes towards infinity the summations can be replaced by integrals which results in

$$F(\sigma) = 1 - \exp\left(-\frac{1}{V_0}\left(\frac{1}{D_0}\right)^{\frac{m}{h}} \int_{\sigma(V)} xyz \int_{\sigma(t)} t^{\frac{m}{h}} \left(\frac{\sigma(x,y,z,t) - \sigma_0}{c_0} \right)^m dV dt \right) \tag{4.18}$$

where it can be expected that only very simple stress conditions can be evaluated analytically. If desirable the reference volume V_0 and reference duration D_0 can be given the value 1.

The mean value of (4.16) is obtained by means of variable transformations and integrations. For the simplest load case of uniform stress which is constant in time the mean value can be derived as

$$E[\sigma] = \sigma_0 + c_o \left(\frac{V_0}{V} \right)^{\frac{1}{m}} \left(\frac{D_0}{D} \right)^{\frac{1}{h}} \Gamma\left(1 + \frac{1}{m} \right) \tag{4.19}$$

and the variance as

$$Var[\sigma] = c_o^2 \left(\frac{V_0}{V} \right)^{\frac{2}{m}} \left(\frac{D_0}{D} \right)^{\frac{2}{h}} \left(\Gamma\left(1 + \frac{2}{m} \right) - \Gamma^2\left(1 + \frac{1}{m} \right) \right) \tag{4.20}$$

where Γ is the gamma function which can be evaluated from a series expansion, from tables or mathematical computer programs. The choice of constants was motivated by a desire to obtain eq. (4.19) as a mean value. A similar technique can be utilized to include the temperature dependence found in (Charles, 1958), see Chapter 2. The size and time dependencies of the rupture stress of brittle materials like glass found in experimental information are characterized well with eq. (4.20). No other statistical distribution has this ability to include dependencies in this way. The result of eq. (4.19) can be visualized as a stress failure surface in a size - time domain which is presented in Figure 4.1. The dotted line reflect the minimum strength σ_0. The marked area reflects the time dependence of the rupture strength for a given volume under stress. For increasing volumes under stress and increasing durations the rupture stress will go towards σ_0.

The coefficient of variation, c.o.v., which is the square root of eq. (4.20) divided by eq. (4.21) can in the case when $\sigma_0 = 0$ be expressed as

$$c.o.v. = \sqrt{\frac{\Gamma(1+2/m)}{\Gamma^2(1+1/m)} - 1} \approx \frac{\pi}{m\sqrt{6}} \tag{4.21}$$

which provides a means to estimate the size parameter *m* if the mean value and standard deviation are known from experiments. The accuracy of eq. (4.21) depends on the relation between σ_0 and c_0 and m. The approximation given in eq. (4.21) is acceptable for values of $m > 4$ - 5. The variability in experiments may be affected by external sources which means that eq. (4.21) can only give guidance in a possible value of the size parameter.

For ordinary plate glass it is normally not the defects within the volume which are of main concern. Instead it is surface defects or edge defects which will cause a rupture to occur. For glass it seems reasonable to substitute V with the surface area under stress, A, or the edge length, L, under stress. This leads to the following expressions for glass strength evaluations

$$F(\sigma) = 1 - \exp\left(- \frac{A}{A_0} \left(\frac{D}{D_0} \right)^{\frac{m_A}{h}} \left(\frac{\sigma - \sigma_0}{c_0} \right)^{m_A} \right) \tag{4.22}$$

$$F(\sigma) = 1 - \exp\left(- \frac{L}{L_0} \left(\frac{D}{D_0} \right)^{\frac{m_L}{h}} \left(\frac{\sigma - \sigma_0}{c_0} \right)^{m_L} \right) \tag{4.23}$$

where A_0 and L_0 are reference values. The exponent m_A is in the range 7 - 9 and the exponent m_L is in the range 3 - 4 (Ontario Research Foundation, 1981). The threshold stress σ_0 reflects residual compressive stresses in the surface region which are introduced during the production because the surface cools down faster then the interior. In the float glass production the cooling is controlled and σ_0 will be around 8 MPa. Along the edge it is likely that σ_0 is close to zero. For heat strengthened glass σ_0 will correspond to the compressive stress introduced in the surface which can be well over 100 MPa.

In an evaluation of the effective volume under stress and an effective duration of stress by hand it is an advantage to disregard the threshold stress. In the evaluation of the volume under stress the influence of existing defects are taken into account. Then the evaluation can be seen as the evaluation of a "reduced volume" and "reduced time". This can be expressed as

$$V^* = \int_V f(x,y,z)^m dx\,dy\,dz \tag{4.24}$$

The time dependence reflects the growth of existing defects with time and the stress level. This "reduced time" is evaluated from

$$D^* = \int_D f(t)^h \, dt \tag{4.25}$$

where *f(t)* describes the stress variation in space or in time for a body subject to stress. The resulting mean value and variance are given by eqs (4.19) and (4.20) with $\sigma_0 = 0$ which is an acceptable approximation in most applications. With $\sigma_0 = 0$ a linear stress variation over a body can be evaluated analytically. An example of such a calculation will be given in the last section of this Chapter. For a stress which is increasing continuously until a failure occurs eq. (4.25) gives the result that the reduced time is $D^* = t/(h + 1)$. For glass where $h \approx 16$ the reduced time is 1/17 of the actual time. This can be interpreted as that only the time spent on a high stress level will be of importance. This implies strongly that eq. (4.24) and eq. (4.25) can be used together with eq. (4.19) and eq. (4.20) without any loss of accuracy even if the threshold stress is important.

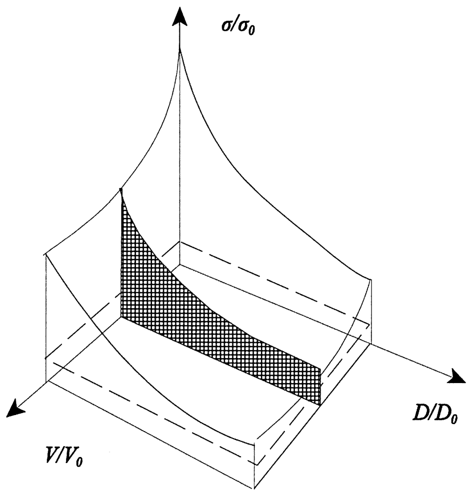

Figure 4.1 The rupture stress as a function of volume and duration of a stress.

4.4 Failure prediction for glass windows

A glass window will normally be subject to wind loads acting perpendicular to its plane. For a glass pane which is simply supported along its edges this will create bending stresses and membrane stresses. For small deformations, less than the thickness of the glass plate, bending stresses will dominate. For larger deformations membrane stresses will increase in importance. Modern analysis with finite element analysis (FEM) shows that at failure bending tensile stresses and membrane stresses will have approximately the same magnitude. The maximum tensile stress will not occur at the centre of a plate but at four points some distance away towards the corners. Many old stress analysis of glass plates seem to have been based on assumptions which do not account for the membrane stresses. This is understandable if calculations had to be made by hand since this leads to extensive evaluations.

A wind load which is acting on a window pane is complex. During shorter periods of time it can be assumed to have a stationary component and a fluctuating component. There will also be a random fluctuation of the wind over a glass area. For increasing durations of a wind loading the importance of the fluctuating component will decrease. The optimal design duration of a wind load is not easy to define in a consistent way. Besides, the building itself will affect the wind structure and create turbulence which will also be influenced by the wind direction and surrounding buildings. Therefore the design wind speed is assumed to be the average wind speed during a short duration, normally related to available wind statistics. In the United States the duration is 1 minute. In Sweden the duration is 10 minutes but this value is adjusted to account for a dynamic influence.

When glass plates are tested the load is increased slowly until a failure occurs. It was realized by (Brown, 1974) that the failure stress depended on the rate of loading or the duration of a test. To account for this discrepancy the following expression was proposed to normalize the rupture stress result

$$\sigma_{\mathrm{d}} = \left[\frac{\int \sigma(\mathrm{t})^{\mathrm{n}}}{\mathrm{t}_{\mathrm{d}}} dt \right]^{1/n} \tag{4.27}$$

where σ(t) is the failure stress in an experiment with duration t and n is a constant chosen as 16 from experiments.

A large research project started in Lubbock, Texas, in the 1970 to analyse window glass failures and to develop a rational tool for the design of windows. The first aspect was a consequence of glass failures in connection with tornados. It was not just high wind speeds but the debris following a tornado which could increase the flaw density of widows and make them rupture at a lower wind speed than otherwise would be the case. The second aspect reflected a need for a new design guide for window glass. The increasing use of large glass windows in tall buildings made this desirable. Existing design guides were not consistent and there had also been a transition from plate glass to float glass with different strength characteristics.

The desired final result was to obtain a failure probability for a given window size and thickness subject to certain wind load with 1 minute duration. This made it suitable with a statistical approach. To include a known size and time dependence of the rupture load a Weibull distribution was chosen. The failure probability was initially expressed as

$$P_f = 1 - e^{-B} \tag{4.28}$$

where B is a function which reflects the risk of failure. The risk of failure is assumed independent of flaw orientation for this case because a flaw of any orientation is exposed to the same stress, (Beason, 1980). The risk function is expressed as

$$B = k(\sigma_{\max})^m A_0 \tag{4.29}$$

where k is a normalizing constant and m is a surface flaw parameter, σ_{max} is the maximum equivalent principal tensile stress, and A_0 = glass plate surface area exposed to tensile stress.

In a more advanced model the failure probability is derived from

$$P_f = 1 - e^{-\kappa R(m,q,a/b)} \tag{4.30}$$

where κ accounts for a specific plate geometry and load duration, and R(m,q, a/b) is a nonlinear relationship, which cannot be stated explicitly, (Beason, 1980).

In reality the risk function can be evaluated from

$$B = k \int_{Area} (\sigma\ (x,y))^m\, dA \tag{4.31}$$

where the integral can be substituted with a summation and the analysis can be performed in a FEM program which accounts both for bending stresses and membrane stresses.

The result from a number of tests of old windows and one set of new float glass is presented in Table 4.1. Direct comparisons of these data suggest that there are significant differences in the surface flaw characteristics of different glass samples.

Values of material parameter *m* and normalizing constant *k* presented in Table 4.1 are unfortunately estimates based on the failure load and not the failure stress. This means that the normalizing constant *k* will be completely wrong if a stress analysis is desired. Only the value of the parameter m can be used.

It is important to note that it is only the last result, New float glass, which reflects glass produced today. All other tests reflect sheet glass which are present in old buildings. There is a significant difference between these two types of glass. The low load capacity and m value of sheet glass may not necessarily be a consequence of the glass being used in 20 - 25 years. It could simply be the characteristics of sheet glass which could not be produced with the same high quality as float glass.

For float glass where the failure is initiated from the surface it is generally accepted that the m value should be in the range 7-9. The most commonly used value is m=8. For failures which are initiated from an untreated edge the m value will be reduced to roughly half of that for a surface failure.

Table 4.1 Tests of old and new window glass.

Location of window	**Size in mm**	**Number of specimens**	**Mean kPa**	**Stand. dev kPa**	**m**	**k**
Great Plains Life Building (GPL) in Lubbock, Texas 1975;20 years of exposure of sheet glass	724x1540x5,56	20	3,78	0,88	6	6,33x 10^{-15} $mm^{10}N^{-6}$
	724x724x5,56	20	8,05	1,80		
Johnson Chevrolet Building, Dallas 1980; 20 years of exposure of sheet glass	413x502x3,18	22	10,99	2,97	6	3,01x 10^{-15} $mm^{10}N^{-6}$
Public School in Anton, Texas 1980; 25 years of exposure of sheet glass	356x921x3,18	132	6,43	1,61	5	9,60x 10^{-14} $mm^{8}N^{-5}$
New Float Glass from PPG in Lubbock, Texas	413x502x3,18		20,49	3,71	9	1,32x 10^{-21} $mm^{16}N^{-9}$

4.5 Analysis of beams

Structural elements like beams can be analysed with the stochastic mechanics approach. As a comparison the same type of beam as was used in the fracture mechanics approach will be utilized here. Also the loading conditions will be the same with two symmetric point loads.

The analyses can be made in several ways. Flaws exist both in the interior as inclusions of foreign material and as surface scratches. The importance of such flaws will depend on the tensile stress field around the flaw.

The moments and corresponding stresses which act on the beam are shown in Figure 4.2 and are given below.

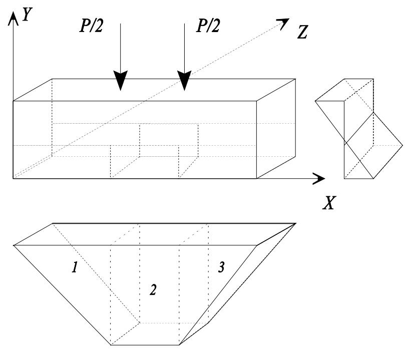

Figure 4.2 The loading conditions for the beam.

$$M_{x1} = \frac{Px}{2} = M_{x3}$$

$$\sigma_1 = \frac{M_{x1}}{I} y = \frac{Px}{2I} y = \frac{Px \cdot 12}{2bh^3} y = \frac{6P}{bh^3} xy = \sigma_3$$

$$M_{x2} = \frac{Px}{2} - \frac{P}{2}\left(x - \frac{l-e}{2} \right) = \frac{P}{4}(l-e)$$

$$\sigma_2 = \frac{M_{x2}}{I} y = \frac{P(l-e) \cdot 12}{4bh^3} y = \frac{3P(l-e)}{bh^3} y$$

where the indices 1, 2 and 3 refer to the different sections indicated in Figure 4.2. The maximum stress occurs between the two point loads where flaws will have the most pronounced influence. Between the supports and the point loads the moment will increase from zero to its maximum value. This will give flaws a lesser influence based on the moment curve. But still it will influence the failure probability.

The volume under stress is obtained from integrating the stress variation in space. First, the risk function will be

$$B_V = \int\int\int \sigma(x,y,z)^m dxdydz = b\int\int \sigma(x,y)^m dxdy = \int\int \left(\frac{M(x,y)y}{I} \right)^m dxdy$$

where a derivation is performed in Appendix B and the final result can be expressed as:

$$B_V = bhL\left(\frac{\sigma}{c_0}\right)^m\left(1+\frac{e}{L}m\right) = \ldots = bhL\left(\frac{3P(L-e)}{2c_0bh^2}\right)^m\left(1+\frac{e}{L}m\right)$$

$$P_f = F(\sigma) = 1 - e^{-B}$$

An estimate of the value of k or c_0 can be obtained from the results given in (Carré, 1996) which were presented in Chapter 2. For the lowest loading rate, 0,05 MPa/s the mean value rupture strength was 41,2 MPa for an ordinary surface treatment. The effective duration based on eq. (4.19) with h = 16 is 48 seconds. The normalization constant in eq. (4.10) can now be determined as c_0 = 17,74 MPa for σ_0 = 0 MPa. With this choice of the value of c_0 the tests carried out by (Carré, 1996) will be established as a reference volume under stress with a corresponding mean value rupture stress.

In this first example the following dimensions are assumed: L = 2 m, e = 0,5 m, h = 0,400 m and b = 0,006 m. The external load is P = 6 kN. This will result in a stress of 14 MPa. Just to show the influence of different parameters the width is altered, the distance between the loads and the load level. The results are presented later in Table 4.2.

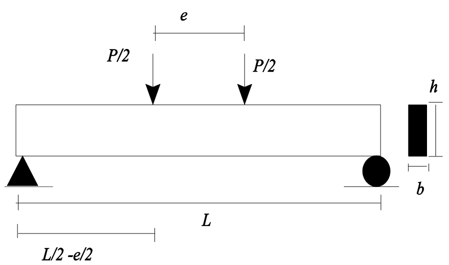

Figure 4.3 Beam under evaluation.

$$M = \frac{P}{2}\left(\frac{L}{2} - \frac{e}{2}\right) = \frac{P}{4}\left(L-e\right)$$

$$W = \frac{bh^2}{6}$$

$$\sigma = \frac{M}{W} = \frac{P}{4}\frac{\left(L-e\right)6}{bh^2} = \frac{3P}{2}\frac{\left(L-e\right)}{bh^2}$$

$$P = 6kN \Rightarrow$$

$$\sigma = \frac{3\times 10^3\cdot 6}{2}\frac{\left(2-0{,}5\right)}{0{,}006\cdot 0{,}4^2} = 14{,}1\,MPa$$

It can be seen from the tables the probability of failure will decrease with increasing value of e. As the distance between the forces increase the moment decreases and hence the stress is decreasing. When the width is increased the bending stiffness increases which results in reduced failure probability. As it would be expected when the load is increases so is the failure probability.

Table 4.2 Results for failure analysis of beams where the failure probability has a volume dependence.

P= 6 kN	B	P_f
e=0,5 m b=6 mm	0,0023	$23{,}0 \times 10^{-4}$
e=0,5 m b=8 mm	0,00030	$3{,}0 \times 10^{-4}$
e=1,0 m b=6 mm	0,00015	$1{,}5 \times 10^{-4}$
e=1,0 m b=8 mm	0,000019	$0{,}19 \times 10^{-4}$
e=1,5 m b=6 mm	0,00000080	$0{,}008 \times 10^{-4}$
e=1,5 m b=8 mm	0,00000011	$0{,}0011 \times 10^{-4}$

P= 12 kN	B	P_f
e=0,5 m b=6 mm	0,575	4370×10^{-4}
e=0,5 m b=8 mm	0,0767	740×10^{-4}
e=1,0 m b=6 mm	0,0374	370×10^{-4}
e=1,0 m b=8 mm	0,00499	$49{,}8 \times 10^{-4}$
e=1,5 m b=6 mm	0,000204	$2{,}04 \times 10^{-4}$
e=1,5 m b=8 mm	0,0000273	$0{,}273 \times 10^{-4}$

P= 18 kN	B	P_f
e=0,5 m b=6 m	14,730	0,9999
e=0,5 m b=8 mm	1,966	0,860
e=1,0 m b=6 mm	0,9579	0,616
e=1,0 m b=8 mm	0,1279	0,12
e=1,5 m b=6 mm	0,00524	0,00523
e=1,5 m b=8 mm	0,000699	0,0007

A fractile value can be obtained from:

$$P_f = \textit{the fractile required} = 1 - e^{-B}$$

Often a lower 5% fractile value is used as a characteristic value. This is obtained for b=0,006 m; e=0,5 m:

$$P_f = 0{,}05 \quad \Rightarrow \quad B = 0{,}0513$$

$$B = 2{,}0 \times 0{,}4 \times 0{,}006\left(\frac{\sigma}{17{,}74}\right)^8 \times \left(1 + \frac{0{,}5}{2{,}0}8\right)$$

$$\sigma = 20{,}79 \ \text{MPa}$$

An evaluation of the surface area under tensile stress can be evaluated as

$$B_A = \int\int \sigma(x,y)^m dxdy + \int\int \sigma(x,z)^m dxdz$$

where the first part reflects the sides and the last part the bottom side. This is shown and evaluated further in Appendix 2.
For the first term, which is dominating, the risk function is

$$B_A = 2hL\left(\frac{\sigma - \sigma_0}{c_0}\right)^m \left(1 + \frac{e}{L}m\right)$$

$$B_{ATOT} = \left(\frac{3P(l-e)}{2bh^2}\right)^m \frac{1}{(m+1)^2}\left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1}e(m+1)\right]$$

$$\left(\frac{1}{c_{0A}}\right)^m = \frac{1}{(m+1)^2}$$

$$B_{ATOT} = \left(\frac{3P(l-e)}{2bh^2 c_{0A}}\right)^m \left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1}e(m+1)\right]$$

$$P_f = 1 - e^{-B_{ATOT}} \qquad \Rightarrow \quad B_{ATOT} = -\ln\left(1 - P_f\right)$$

$$\left(\frac{3P(l-e)}{2bh^2c_{0A}}\right)^m \left[b(l-e)(m+1)+eb(m+1)^2+h(l-e)+h^{m+1}e(m+1)\right] = -\ln(1-P_f)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots c_{0A} = 35{,}3\ MPa$$

Example on the same beam again but this time with the area under stress:

Table 4.3 Results for failure analysis of beams where the failure probability has a area dependence.

P= 6 kN	$\mathbf{B_{ATOT}}$	$\mathbf{P_f}$
e=0,5m and b= 6 mm	679,0x10-6	$6300{,}0\text{x}10^{-6}$
e=0,5m and b=8 mm	65,6x10-6	$65{,}6\text{x}10^{-6}$
e=1,0 m and b=6 mm	23,3x10-6	$23{,}3\text{x}10^{-6}$
e=1,0 m and b=8 mm	2,78x10-6	$2{,}78\text{x}10^{-6}$

P=12 kN	$\mathbf{B_{ATOT}}$	$\mathbf{P_f}$
e=0,5m and b= 6 mm	0,15023	$1390{,}0\text{x}10^{-4}$
e=0,5m and b=8 mm	0,01679	$166{,}0\text{x}10^{-4}$
e=1,0 m and b=6 mm	5,97x10-3	$57{,}7\text{x}10^{-4}$
e=1,0 m and b=8 mm	7,12x10-4	$7{,}12\text{x}10^{-4}$

The result will be different when the volume under stress is compared with the area under stress. In the mean value sense the result will be similar for the beam tests carried out in (Carré, 1996) because this is the reference results used. When a larger beam is evaluated the scale factor will be different for the volume under stress and the area under stress. Therefore it cannot be expected that the results of the failure probability are different.

A comparison of the results in Table 4.2 or Table 4.3 show what can be expected. When the distance e increases the volume or area under stress will increase. At the same time the maximum moment will decrease faster and the resulting failure probability will decrease. Differences between Table 4.2 and Table 4.3 are a consequence of different scale factors for volume and area respectively.

With a knowledge of the parameters c_0 and m it is possible to establish the failure distribution for any type of beam. If the loading condition is a uniformly distributed load it is not possible to make the calculations by hand. Instead the evaluation is made with the help of some suitable computer software.

The result in Table 4.2 is supposed to be the failure load for a sustained stress with a duration of 48 seconds, let´s say approximately 1 minute. For longer durations the failure stress will be reduced. If it is assumed that the time parameter n = 16, which is a value often found in the literature, this reduction can be evaluated from eq. (4.26) for a constant stress. For a variable stress which can be expressed in mathematical terms eq. (4.27) can be used. In Table 4.4 are some reduction values given for different durations of a constant stress that would cause a failure.

Table 4.4 The reduction of the failure stress for different durations of a stress.

Duration	Relative reduction
1 minute	1
1 hour	0,996
1 day	0,635
1 year	0,439
10 years	0,380
100 years	0,329

As an example, the mean rupture stress for sustained stress over 100 years will be reduced to:

$$\left(\frac{D_0}{D^*}\right)^{\frac{1}{n}} = \left(\frac{1}{D^*}\right)^{\frac{1}{16}} \qquad \textit{Example } 100 \textit{ years}$$

$$\left(\frac{1}{100\times 365\times 24\times 60}\right)^{\frac{1}{16}} = 0{,}329$$

CONCLUSIONS

5.1 Introduction
Glass is a material which is used in increasing more complex and demanding applications. This in spite of a lack of official codes which can give guidance about proper design procedures and material strength characteristics. Glass as a material has many characteristics in common with ordinary structural materials. But there are also differences which need attention. Glass will fail in a brittle manner which is considered unacceptable in most applications. The use of glass in structural applications therefore need special attention concerning

- Design strategy/reliability
- Material strength characterization
- Design methodology

where each aspect can be analysed separately but there will be a connection between them. Design strategy and reliability has been discussed in connection with application in Chapter 1. Material strength characterization was presented in Chapter 2. Design methodology as extensions to normal design was given in Chapters 3 and 4. A synthesis of this information is presented below.

5.2 Design strategy
The oldest design principle is based on the safety factor principle. This means that the allowable stress is determined as the mean rupture stress divided by a safety factor. This design principle has been used since formal designs of structural members started during the 18th century. The safety factor was initially chosen arbitrarily to be large enough. The first person who made a scientific analysis of this safety factor was a French engineer, Vicat (1828), at the French Road Administration. The background was the continuous increasing deflections of steel hangers in an arch bridge. He performed creep tests where the applied stresses corresponded to 25%, 50% and 75% of the failure stress of a rod hanger. After a number of years he noticed that it was only for a stress of 25% of the failure stress that the increase of the deflection decreased with time. For a stress level of 75% of the failure stress the deflection increased linearly in time. For a stress level of 50% of the failure stress the deflection still increased with time but not as fast as linear with time. Results of this type and the consequences were not analysed and mathematically described until almost 100 years later. In view of this it was natural for Vicat to draw the conclusion that it was not appropriate to use more than something between 25% and 50% of the rupture stress in an application. He suggested a safety factor of 3. Even if this experiment reflects very special loading conditions with a sustained stress this proposal for a safety factor had a pronounced influence on the engineering profession. For this reason a safety factor of 3 has

been in use since then until the introduction of the statistically based design procedure.

One of the most well known designers in the field of glass is Tim McFarlane. In designs he uses an admissible design stress which is determined from an experiment where the mean failure stress is divided with a safety factor of 3. This seems natural in view of the result of Vicat. The factor 3 is not a universal figure even if it seems as if it has been applied in this way. It has to be material dependent and as such specific to a material composition and manufacturing. It may also be influenced in time by the environment.

The result obtained by Vicat shows that there is a threshold stress between safe and unsafe use. This threshold stress reflects the dividing line between two different material responses. Below the threshold stress the initial deformation is followed by primary creep which can be seen as a delayed elastic response. Above the threshold stress the deformation will end in secondary creep which can be seen as a damage accumulation in terms of a shearing deformation between atomic planes. Secondary creep will eventually lead to a failure. This is a typical behaviour for metallic materials subject to a constant stress.

For a material which will fail in a sudden and brittle manner when it is overstressed there is a fundamental problem. But this is not unique for glass. Other materials like concrete, steel and wood can fail in a similar manner under certain circumstances. The notion of a preferable ductile failure instead of a brittle failure is perhaps more emotional than connected to reality. Ductile failures can be experienced in deformation controlled laboratory experiments. Even an unreinforced concrete beam can exhibit large ductility in a very stiff testing machine. The final failure will be brittle. Steel subject to low temperatures or fast loading situations may also fail in a brittle manner. In reality when a load exceeds the load carrying capacity there will be an instantaneous failure where it often is not possible to distinguish between ductile and brittle characteristics.

It is common knowledge that when glass fails it will rupture in a number of pieces which can be a hazard for people in the vicinity. This has to be considered in a design strategy. The rupture strength of glass has a pronounced size and time dependence. This can be influenced by the environment where especially static fatigue play an important role. These dependencies are not unique for glass but they have a pronounced influence on the rupture strength. Therefore the influence need to be addressed.

From the examples in Chapter 1 it can be seen that several strategies have been utilized in applications. Lamination where two or three glass panes are glued together offers at least two advantages. When two glass panes are laminated together and a failure occurs in one pane it can be assumed that the other pane will take over the load carrying capacity while the broken pieces from the ruptured pane still will be glued to the unbroken pane. This will give time to make the necessary repair. If three panes are glued together the pane in the middle will to a large extent be protected from surface damage. No or little deterioration with time can be expected. If this is done for a beam the outer panes can have an overlap and the space in between can be filled a transparent sealant which will also protect the edge from deterioration.

In many applications annealed glass is substituted with heat treated glass. This introduces high compressive stresses in the surface region. As long as the externally introduced bending tensile stress is smaller than the compressive residual stress it is impossible for a crack growth from the surface area. But a breakage is still possible because of the existence of impurities in the region of high tensile stresses, (Jacob, 2003). Such impurities will act as stress racers and this can result in a delayed creep failure. Even if the most dangerous impurities are removed during the production there is no guarantee that these glass plates will last forever. So also in this case a lamination of two or three panes together is necessary.

It can be expected that structural units of glass will not exhibit any creep with visible deformation before a failure. The creep phenomenon static fatigue will reduce the height of a section locally and increase the local stress. It is possible that this can be measured by high performance fibre optical strain gauges. If this is possible a continuos surveillance could be used. There is no information available concerning this possibility.

The present design strategy for normal construction materials is that the strength of a material is characterized by a statistical distribution. A characteristic strength is defined as a lower 5% fractile value for a certain confidence level. The statistical distribution used is a normal statistical distribution out of convenience even if there is evidence that some other statistical distribution makes more sense. The confidence level reflects the influence of the sample size. In Figure 5.2 this is illustrated for a statistical distribution which is skew to the left. This is of similar type as a Weibull distribution with m = 8.

For glass the strength properties with a size and time dependence is well established. Because of this it is not equally important to use a confidence criteria. The lower 5% strength can be established with the methodology presented in Chapter 4.5 with great accuracy.

5.3 Material strength

The allowable strength of glass in structural applications is in general very low. This is understandable and it reflects a natural conservatism among designers. This should also be seen in the light of the lack of long term experience from the structural use of glass. In annealed glass there are residual compressive surface stresses of around 8 MPa. The mean rupture strength can be expected to be around 40 - 75 Mpa,(Compagno, EN1288). But this strength is easily reduced by a size and a time dependence. For a beam subject to a uniformly distributed load the height has to be doubled if the length is doubled .If a beam length is doubled its height also has to be doubled to give the same formal load carrying capacity according to the theory of elasticity. The width is kept constant. But the size dependence will decrease the real rupture load to approximately $R = R_o(1/4)^{1/8} \approx 0,84 \cdot R_o$.

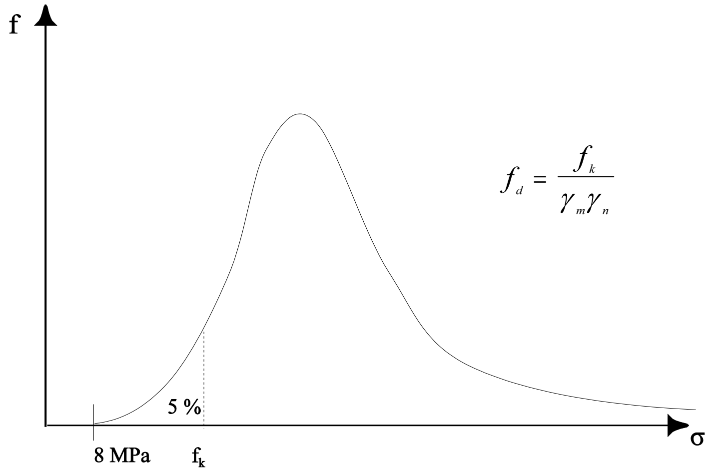

Figure 5.1 Definition of the lower 5% characteristic value.

Sustained stresses will introduce a time dependence of the rupture load. Such stresses will mainly occur for the dead load which can be a substantial part of the total load. If it is assumed that in an ordinary test of a glass beam where the load is increased continuously from zero to failure takes about 5 minutes the effective duration will be $D_o = 5/(16+1) \approx 0,3$ minutes or $2,0 \cdot 10^{-4}$ days. For a structure with a design life of 50 years the reduction of the rupture stress will be $R = R_o \{2,0 \cdot 10^{-4} /(50 \cdot 365)\}^{1/16} \approx 0,32 \cdot R_o$. This indicates that only one third of the mean rupture stress, which will be in the range of 13 - 15 MPa, can be utilized when the time dependence is considered.

In reality there should also be a safety margin. With the exponent m = 8 the coefficient of variability will be $cov \approx \pi/(m\sqrt{6}) \approx 0,16$. Based on the normal distribution a lower 5% fractile can be established as

$$R_k = (1 - k_5 \cdot cov)m = (1 - 1,64 \cdot 0,16)0,32 \cdot R_o = 0,24 \cdot R_o$$

where only the time dependence is considered. In reality it may be necessary to include the size dependence as well. With the example given previously this would reduce the original mean failure stress to about one fifth of the original mean failure stress. This will result in a design stress of 8 - 9 MPa which should be divided with appropriate partial safety factors.

The choice of safety factor, γ_R, depends on well R_k has been established and reliability class. In Sweden this is split up in γ_n which reflect the consequences of a failure with respect to human life and γ_m which reflect the confidence in establishing R_k. In view of the fact that R_k will come close to the level of residual stresses introduced during the manufacturing of annealed glass indicates that

$$R_d = R_k/(\gamma_n \, \gamma_m) \approx 6 - 7 \text{ MPa}$$

for the dead load. For other loads with shorter duration a higher stress level can be acceptable. This load level can be established in a similar manner as above. This is a similar strength characterization as is used for wood.

For heat strengthened glass it may seem reasonable to assume that R_d should be close to the compressive stress introduced in the surface region. But this is far from obvious. If foreign inclusions are present, which can be assumed, there is likely to be a delayed failure if the internal compressive stress is too high. The knowledge of such failures is at present very limited. There is definitely a need for more knowledge of a possible time dependence of heat strengthened glass.

It seems to be possible to modify the surface characteristics of glass and increase its initial strength without heat strengthening. Certain chemical compounds can react with the glass surface and reduce the flaw density. This may turn out to be an alternative to heat strengthening in the future.

5.4 Design methodology

Glass behaves basically as an elastic material. This means that the theory of elasticity is directly applicable for determining stresses and strains. It can be expected that all calculations also in the future will be based on the theory of elasticity. This theory is derived for small strains and corresponding stresses. Consequently this theory cannot be expected to give information about the failure stress. Information about the failure stress for structural materials show that the failure stress is size, time and temperature dependent. In some cases there will also be other dependancies like a humidity dependence. It is therefore natural to expect that the failure stress for glass exhibit dependencies. These dependencies need to be addressed if they have a pronounced influence in the failure stress.

Fracture mechanics and stochastic mechanics are methodologies which can be seen as "add on" theories to the theory of elasticity to explain the failure behaviour of glass. Both theories have their advantages as well as drawbacks. This is natural since it cannot be expected that there is a "universal" solution which is acceptable to everyone,

. In a comparison between fracture mechanics and stochastic fracture mechanics it will not be possible to present an absolute truth. The question is which theory gives adequate information about how dependencies can be accounted for in a proper way. Modern design philosophy is based on statistical principles where the goal is that the failure stress should reflect a certain failure probability. It is therefore an advantage if appropriate information concerning this issue also can be obtained.

Fracture mechanics is a deterministic method which gives the stress concentration at the tip of a crack as a function of geometry and loading conditions. This can be compared with a critical value to establish whether a crack will grow or not. The basic theory of fracture mechanics assumes that a critical crack exist perpendicular to a tensile stress field. No explicit consideration of a possible size and time dependence is included in the theory.

Cracks in glass are measurable down to a certain level with appropriate equipment. The size and orientation of these flaws can be assumed to be random. For increasing areas which are analysed it can be expected that there will be an increase in size of these defects. This can of course be accounted for by making flaw characteristics size dependent. It is very possible that this flaw information can be characterized with a statistical distribution, perhaps a Weibull distribution. With this technique it is possible to estimate the influence of a size dependence. The time dependence can be introduced in a similar way as the size dependence. This would reflect a growth of flaws with time which will alter the flaw statistics characterization. Alternatively as was done in Chapter 3 where the time dependence was introduced as an “add on” term. This should make it possible to establish estimates of a “characteristic” value which could be used for design purposes. In stochastic mechanics it is initially assumed that flaw characteristics are size and time dependent and the result is that the failure stress can be characterized with a Weibull distribution. Besides the influence of these flaws depend on the total stress field within a body or acting on surfaces. The result is directly applicable to body shape and loading conditions.

5.5 Research trends and development

During the last 25 years or so there has been an increasing demand for using glass in structural applications. This also include structural glazing where new techniques have changed the normal function. Therefore glass manufacturers and other parties involved with the use of glass have been forced to improve their knowledge. This is accompanied with the development of new technique to make the supporting structure to a glass surface as invisible as possible. Some implications are of this are presented below.

Float glass have properties which seem to be extremely good. At least when it is used as ordinary window panes. The possibility to heat strengthen glass makes it possible to use it in applications where fairly high tensile stresses are present. But the existence of impurities within a glass body can cause a delayed failure.

Glass can be surface treated. This is done on a regular basis to improve the u-value. But is also possible to apply surface deposits to improve the strength of glass. Experiments done at Philips in the Netherlands have revealed that the short term strength can be doubled with what seems to be fairly simple methods. This can be an alternative to heat strengthening. This might affect both the size and time dependence and improve the conditions for structural use of glass.

Glass panes with holes seem to be increasingly more popular. Theoretical solutions are available but it is not obvious that the basic assumptions are valid for a specific application. The available knowledge has primarily been obtained in connection with structural applications. The authorities have required full scale testing which has given information for certain conditions. There is a need for more general knowledge of the influence of bezels of different materials and its influence on the contact pressure.

Glass which is glued against a steel frame and glass panes with holes will sometimes be subject shear stresses in addition to bending stresses and membrane stresses. This is stress conditions which have not been addressed in the analysis of glass.

Many spectacular projects which have been undertaken over the last decade have in themselves given rise to several questions. It seems as if there is a desire to find new designs which require new solutions or the development of existing solutions. This will also create new research concerning the structural use of glass.

Above all there is a need for codes to address the question of how a proper design should be made. Such a code should preferably be a European Code (EC code) to harmonize designs within the common market. This may also require CEN standards to establish strength characteristics of glass. Such a code and CEN standards will require more knowledge of how glass behaves in different applications during both short term and long term loading conditions.

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Appendix A

A.1 General

Stress concentration will appear if a local change of dimension takes place for example when a notch or a crack is present. This means that the stress close to the notch is larger, and in some cases much larger, than the nominal stress. The stress intensity factor, K, give a measure of the stress singularities at the crack tips. The factor f is a function of geometry, (for example the crack length a) and the type of loading.

In a fracture mechanics experiment, a cracked structure is loaded with an increasing force until fracture occurs. The value of the stress intensity factor when the fracture occurs is supposed to be a material property, i.e. the critical value of the stress intensity factor is considered a material parameter. This parameter is called the fracture toughness of the material. Depending on the conditions at the crack tip (plane stress or plane strain) different fracture toughnesses will be obtained.
With a known fracture toughness of the material, a fracture criterion can be formulated. Under certain conditions , fracture is expected when

$$K_I = K_{IC}$$

where K_{IC} is the fracture toughness of the material. The above fracture criterion may be used only when plane deformation (plane strain), exists at the crack tip. Other criterion that must be fulfilled to ensure plane strain criterion are:

$$\left.\begin{cases} t \\ a \\ (w-a) \end{cases}\right\} \geq 2{,}5\left(\frac{K_{IC}}{\sigma_Y}\right)^2$$

where t is the thickness of the plate containing the crack, a is the crack length and (W-a) is the length of the area connecting the upper and lower part of the cracked structure. The stress σ_Y is the yield strength of the material.

The condition on plate thickness t is to ensure that the crack tip is loaded in plane strain. The plate must be thick enough so that stress σ_{zz} can be built up giving strain $\varepsilon_{zz}=0$ at the crack tip. The conditions on a and (W-a) are to ensure that only the singular stress terms dominate at the crack tip.

If the plate thickness t is very small, so that the crack is loaded under plane stress conditions (i.e. $\sigma_{zz}=0$, $\tau_{xx}=\tau_{yz}=0$), then a fracture criterion may be formulated as:

$$K_I = K_C$$

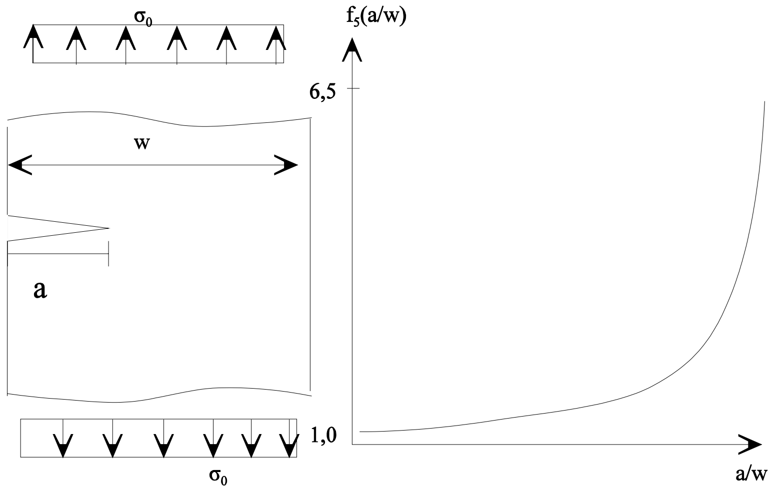
where K_C is the fracture toughness of the material under plane stress conditions. Note that $K_C \neq K_{IC}$.

A.2 Edge crack in a plate in uniaxial tension

The following relations are valid for uniaxial tension.

$$K_I = \sigma_0 \sqrt{\pi a} \cdot f_5\left(\frac{a}{w}\right)$$

$$f_5\left(\frac{a}{w}\right) = \frac{\sqrt{\frac{2w}{\pi a} \cdot \tan\left(\frac{\pi a}{w}\right)}}{\cos\left(\frac{\pi a}{w}\right)} \cdot \left[0{,}752 + 2{,}02\left(\frac{a}{w}\right) + 0{,}37\left(1 - \sin\left(\frac{\pi a}{2w}\right)\right)^3\right]$$



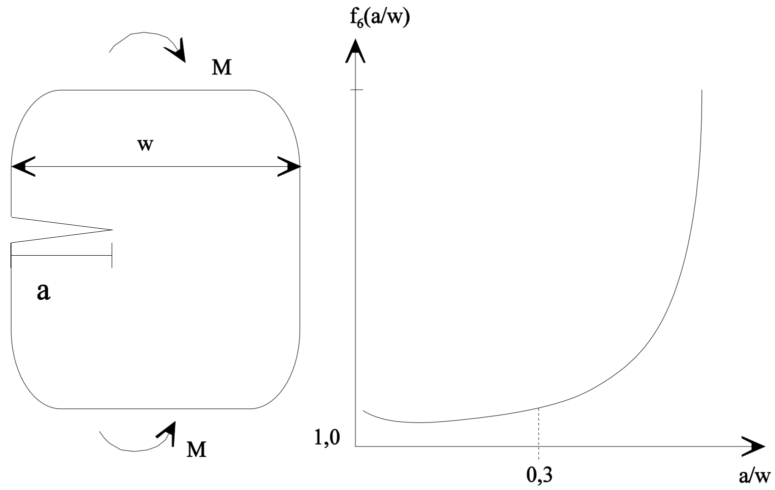
A.3 Edge crack in a plate in bending

The following relations are valid for bending.

$$K_I = \sigma_0 \sqrt{\pi a} \cdot f_6\left(\frac{a}{w}\right)$$

$$\sigma_0 = \frac{6M}{tw^2}$$ t is the thickness of the plate

$$f_6\left(\frac{a}{w}\right) = \frac{\sqrt{\frac{2w}{\pi a} \cdot \tan\left(\frac{\pi a}{2w}\right)}}{\cos\left(\frac{\pi a}{2w}\right)} \cdot \left[0{,}923 + 0{,}199\left(1 - \sin\left(\frac{\pi a}{2w}\right)\right)^3\right]$$



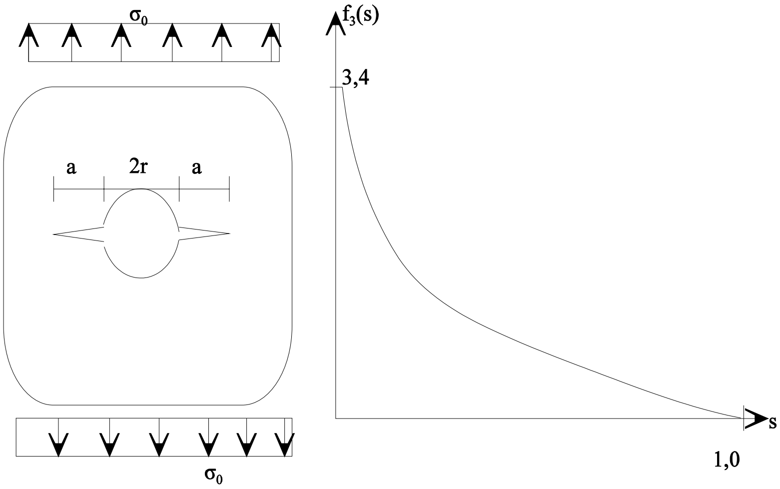
A.4 Cracks at a hole in an infinite plate

The following relations are valid for a hole in an infinite plate.

$$K_I = \sigma_0 \sqrt{\pi a} \cdot f_3(s)$$

$$s = \frac{a}{r + a}$$

$$f_3(s) = 0{,}5(3 - s)\left[1 + 1{,}243(1 - s)^3\right]$$



Appendix B

B.1 General

The “weakest link theory” of strength implies that the strength of a large member loaded in tension would be equal to the strength of the weakest of small pieces cut from the large member.

In the statistical strength theory, the cumulative frequency distribution of strength is assumed to be a special type - one which lends itself to mathematical development of equations for predicting average strengths and standard deviations of strengths. In equation form, the theoretical cumulative frequency curve is expressed as

$$S = 1 - e^{-B}$$

where S is the theoretical cumulative frequency and B is given by

$$B = \int f(\sigma)dV$$

which is the function of stress, f(σ) integrated over the volume element dV on which it acts. The function of stress f(σ) is a function expressing the local probability of failures. This is dependent on the local stress level and hence varies throughout the volume of a bending member.

B.2 Volume under stress in bending

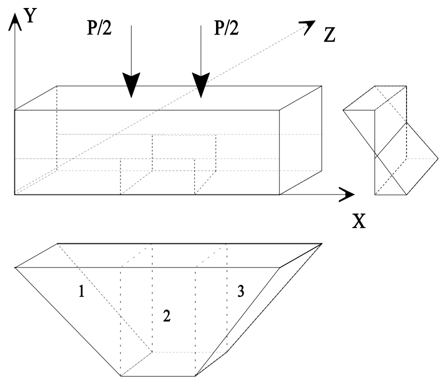


Figure B.1 The loading conditions for the beam.

The volume under stress is given by:

$$B_V = \iint \int \sigma(x,y,z)^m dxdydz = b\iint \sigma(x,y)^m dxdy = \iint\left(\frac{M(x,y)}{I}y\right)^m dxdy$$

For the section 1 and section 3 the volume under stress will be:

$$B_{V_1} = b\int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}}\left(\frac{Pxy}{2I}\right)^m dydx = b\int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}}\left(\frac{6Pxy}{bh^3}\right)^m dydx =$$

$$b\left(\frac{6P}{bh^3}\right)^m\int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}}(xy)^m dydx = b\left(\frac{6P}{bh^3}\right)^m\int_0^{\frac{l-e}{2}}\int_0 x^m\left[\frac{y^{m+1}}{m+1}\right]_0^{\frac{h}{2}}dx =$$

$$b\left(\frac{6P}{bh^3}\right)^m\frac{(h/2)^{m+1}}{m+1}\int_0^{\frac{l-e}{2}}x^m dx = b\left(\frac{6P}{bh^3}\right)^m\frac{(h/2)^{m+1}}{(m+1)^2}\frac{((l-e)/2)^{m+1}}{}$$

For section 2 the volume under stress will be:

$$B_{V_2} = b\int_{\frac{l-e}{2}}^{\frac{l+e}{2}}\int_0^{\frac{h}{2}}\left(\frac{3P(l-e)}{bh^3}y\right)^m dydx = b\left(\frac{3P(l-e)}{bh^3}\right)^m\int_{\frac{l-e}{2}}^{\frac{l+e}{2}}\int_0^{\frac{h}{2}}y^m dydx =$$

$$= b\left(\frac{3P(l-e)}{bh^3}\right)^m\left(\frac{(h/2)^{m+1}}{m+1}\right)^m\int_{\frac{l-e}{2}}^{\frac{l+e}{2}}1dx = b\left(\frac{3P(l-e)}{bh^3}\right)^m\left(\frac{(h/2)^{m+1}}{m+1}\right)e$$

$$B_{V_{TOT}} = 2\times B_{V_1} + B_{V_2} =$$

$$2b\left(\frac{6P}{bh^3}\right)^m\left(\frac{(h/2)^{m+1}}{(m+1)^2}\right)\left(\frac{L-e}{2}\right)^{m+1} + b\left(\frac{3P(L-e)}{bh^3}\right)^m\left(\frac{(h/2)^{m+1}}{m+1}\right)e =$$

$$2b\left(\frac{3P(L-e)}{bh^3}\right)^m\frac{(h/2)^{m+1}}{(m+1)^2}\frac{(L-e)}{2} + b\left(\frac{3P(L-e)}{bh^3}\right)^m\left(\frac{(h/2)^{m+1}}{m+1}\right)e =$$

$$b\left(\frac{3P(L-e)}{bh^3}\right)^m\left(\frac{h}{2}\right)^m\left(2\frac{h}{2}\frac{1}{(m+1)^2}\frac{(L-e)}{2} + \frac{h}{2}\frac{e}{m+1}\right) =$$

$$\left(\frac{3P(L-e)}{2bh^2}\right)^m \frac{b}{(m+1)^2}\left(h\frac{(L-e)}{2}+\frac{h(m+1)}{2}e\right)=$$

$$\left(\frac{3P(L-e)}{2bh^2}\right)^m \frac{b}{(m+1)^2}\frac{h}{2}(L-e+e\times m+e)=$$

$$\left(\frac{3P(L-e)}{2bh^2}\right)^m \frac{b}{(m+1)^2}\frac{hL}{2}\left(1-\frac{e}{L}+\frac{e\times m}{L}+\frac{e}{L}\right)=$$

$$\left(\frac{3P(L-e)}{2bh^2}\right)^m \frac{b}{(m+1)^2}\frac{hL}{2}\left(1+\frac{e}{L}m\right)$$

Further evaluation will simplify the obove expression to:

$$B=\frac{bhL}{2(m+1)^2}\left(1+\frac{e}{L}m\right)\left(\frac{3}{2}\frac{P(L-e)}{bh^2}\right)^m$$

$$\sigma=\frac{3}{2}\frac{P(L-e)}{bh^2}$$

$$\frac{1}{2(m+1)^2}=\frac{1}{c_0^m}$$

$$B=bhL\left(\frac{\sigma}{c_0}\right)^m\left(1+\frac{e}{L}m\right)$$

B.3 Evaluation of results from Carré (1996)

Test results from Helène Carré:

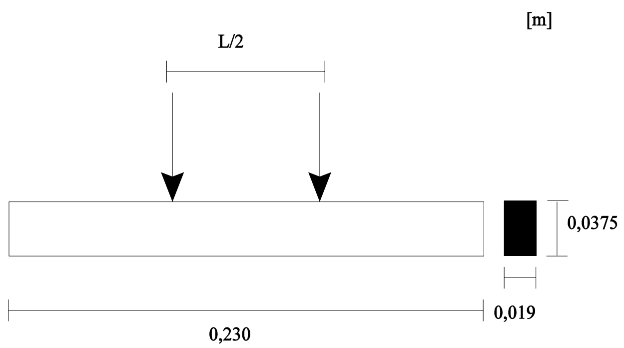


Figure B.2

$$P_f = 1 - e^{-B_{VTOT}} \quad \Rightarrow \quad 0{,}5 = 1 - e^{-B_{VTOT}} \quad \Rightarrow \quad e^{-B_{VTOT}} = 0{,}5$$

$$B_{VTOT} = -\ln 0{,}5 \quad \Rightarrow \quad B_{VTOT} = 0{,}230 \times 0{,}0375 \times 0{,}019 \times \left(\frac{41{,}8}{c_0}\right)^8 \left(1 + \frac{1}{2}8\right) = 0{,}695$$

$$c_0 = 17{,}74 \quad MPa$$

The example of beam in chapter 4:

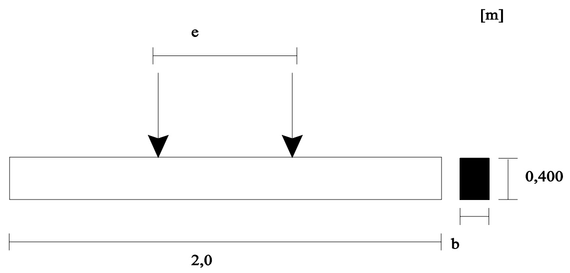


Figure B.3

Calculation techniques for failure probabilities:

$$B = 2{,}0 \times 0{,}4 \times 0{,}006 \left(\frac{14}{17{,}74}\right)^8 \left(1 + \frac{0{,}5}{2}8\right) = 0{,}002$$

$$P_f = 0{,}002$$

$$Load - stress, \; 2\times$$

$$B = 2{,}0 \times 0{,}4 \times 0{,}006 \left(\frac{28}{17{,}74}\right)^8 \left(1 + \frac{0{,}5}{2}8\right) \cong 0{,}55$$

$$P_f = 0{,}426$$

$$e = 1 \; m$$

$$B = 2{,}0 \times 0{,}4 \times 0{,}006 \left(\frac{14}{17{,}74}\right)^8 \left(1 + \frac{1}{2}8\right) = 0{,}0036$$

$$P_f = 0{,}0036$$

$$Load \; \times 3$$

$$B = 2{,}0 \times 0{,}4 \times 0{,}006 \left(\frac{42}{17{,}74}\right)^8 3 = 14{,}21$$

$$P_f = 1{,}000$$

$P_f = 0{,}05 \Rightarrow B = 0{,}0513$

$B = 2{,}0 \times 0{,}4 \times 0{,}006\left(\frac{\sigma}{17{,}74}\right)^8 \times 3$

$\sigma = 20{,}79$ MPa

B.4 The area under stress

The area under stress is evaluated for each section and area under tensile stress. For the sections 1 and 3 this can be expressed as

$$B_{A_1^*} = \int_0^{\frac{l-e}{2}}\int_0^b [\sigma(x,z)]^m dzdx = \int_0^{\frac{l-e}{2}}\int_0^b \left[\frac{M(x)}{I}\frac{h}{2}\right]^m dzdx = \int_0^{\frac{l-e}{2}}\int_0^b \left[\frac{Px}{2I}\frac{h}{2}\right]^m dzdx =$$

$$= \int_0^{\frac{l-e}{2}}\int_0^b \left[\frac{Px}{2}\frac{12}{bh^3}\frac{h}{2}\right]^m dzdx == \int_0^{\frac{l-e}{2}}\int_0^b \left[\frac{3Px}{bh^2}\right]^m dzdx == b\left(\frac{3P}{bh^2}\right)^m \left[\frac{x^{m+1}}{m+1}\right]_0^{\frac{l-e}{2}}$$

$$= b\left(\frac{3P}{bh^2}\right)^m \left[\frac{\left(\frac{l-e}{2}\right)^{m+1}}{m+1}\right] = b\left(\frac{3P}{bh^2}\right)^m \left(\frac{l-e}{2}\right)^{m+1} \frac{1}{m+1}$$

$$B_{A_1^{**}} = \int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}} \left(\sigma(x,y)\right)^m dydx = \int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}} \left(\frac{M(x)}{I}y\right)^m dydx = \int_0^{\frac{l-e}{2}}\int_0^{\frac{h}{2}} \left(\frac{Px}{2I}y\right)^m dydx =$$

$$\left(\frac{P}{2I}\right)^m \int_0^{\frac{l-e}{2}} x^m dx \int_0^{\frac{h}{2}} y^m dx = \left(\frac{P}{2I}\right)^m \left[\frac{x^{m+1}}{m+1}\right]_0^{\frac{l-e}{2}} \left[\frac{y^{m+1}}{m+1}\right]_0^{\frac{h}{2}} = \left(\frac{P}{2I}\right)^m \left[\frac{\left(\frac{l-e}{2}\right)^{m+1}}{m+1}\right]\left[\frac{\left(\frac{h}{2}\right)^{m+1}}{m+1}\right]$$

$$= \left(\frac{P}{2}\frac{12}{bh^3}\right)^m \frac{\left(\frac{l-e}{2}\right)^{m+1}}{m+1}\frac{\left(\frac{h}{2}\right)^{m+1}}{m+1} = \left(\frac{6P}{bh^2}\right)^m \frac{\left(\frac{l-e}{2}\frac{h}{2}\right)^{m+1}}{(m+1)^2} = \left(\frac{6P}{bh^2}\right)^m \frac{\left(\frac{l-e}{4}h\right)^{m+1}}{(m+1)^2}$$

where the notation * refers to the the bottom and ** refers one side. These expressions are valid for $0 \leq x \leq (l - e)/2$ and $(l + e)/2 \leq x \leq l$. The corresponding expressions for section 2 are

$$B_{A_2^*} = \int_{\frac{l-e}{2}}^{\frac{l+e}{2}} \int_0^b \left[\sigma(x,y,z)\left\{= \sigma_2\right\}\right]^m dxdz = ... = \left(\frac{3P(l-e)}{2bh^2}\right)^m e*b$$

$$B_{A_2^{**}} = \int_{\frac{l-e}{2}}^{\frac{l+e}{2}} \int_0^{\frac{h}{2}} \left[\sigma(x,y,z)\right]^m dxdy = ... = \left(\frac{3P(l-e)}{bh3}\right)^m \left(\frac{h}{2}\right)^{m+1} \frac{e}{m+1}$$

where * refers to the bottom and ** refers to a side. The expression is valid for (l - e)/2 ≤ x ≤ (l + e)/2. The total area under stress is derived from

$$B_{A_{TOT}^*} = 2 \times B_{A_1^*} + B_{A_2^*}$$

$$B_{A_{TOT}^{**}} = \left(2 \times B_{A_1^{**}} + B_{A_2^{**}}\right) \times 2$$

This results in the following expression for the total area under stress

$$B_{A_{TOT}} = B_{A_{TOT}^*} + B_{A_{TOT}^{**}}$$

$$B_{A_{TOT}} = \left(\frac{3P(l-e)}{2bh^2}\right)^m \frac{1}{(m+1)^2}\left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1}e(m+1)\right]$$

$$\left(\frac{1}{c_{0A}}\right)^m = \frac{1}{(m+1)^2}$$

$$B_{A_{TOT}} = \left(\frac{3P(l-e)}{2bh^2 c_{0A}}\right)^m \left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1}e(m+1)\right]$$

$$P_f = 1 - e^{-BATOT} \qquad \Rightarrow \qquad B_{ATOT} = -\ln\left(1 - P_f\right)$$

$$\left(\frac{3P(l-e)}{2bh^2 c_{0A}}\right)^m \left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1}e(m+1)\right] = -\ln(1-P_f)$$

$$................c_{0A} = 35{,}3\ MPa$$

$$B_{ATOT} = \left(\frac{3P(l-e)}{2bh^2 c_{0A}} \right)^m \left[b(l-e)(m+1) + eb(m+1)^2 + h(l-e) + h^{m+1} e(m+1) \right]$$

$$B_{ATOT} = \left(\frac{3 \cdot 6 \times 10^3 (2-0{,}5)}{2 \cdot 0{,}006 \cdot 0{,}400^2 \cdot 35{,}3 \times 10^6} \right)^8 \left[0{,}006(2-0{,}5)(8+1) + 0{,}50 \cdot 0{,}006(8+1)^2 + 0{,}400(2-0{,}5) + 0{,}400^{8+1} \cdot 0{,}5(8+1) \right]$$

$$B_{ATOT} = 6{,}79 \times 10^{-4}$$

$$P_f = 1 - e^{-BATOT} = 1 - e^{(-6{,}79 \times 10^{-4})} = 0{,}00063$$