Seminariepass 13

1) Naturliga svängnings frekvensen:

$f_n = \frac{46}{200} = 0{,}23\ \text{Hz} \Rightarrow T = \frac{1}{0{,}23} = 4{,}35\,\text{s}$

Vinkelfrekvensen: $\omega_n = 2\pi f_n = 1{,}45\ \text{rad/s}$

Utböjning?

Antag $x = A\sin\omega_n t$,

$\dot{x} = \omega_n A\cos\omega_n t$

$\ddot{x} = -\omega_n^2 A\sin\omega_n t$

Max acc. $a_{max} = \omega_n^2 A = 0.12\,\text{m/s}^2 \Rightarrow$

Max utböjn.: $A = \frac{0.12}{1{,}45^2} = 0{,}06\ \text{m}$

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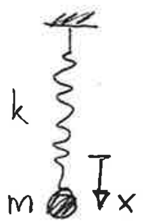
2)



Vid $t=0$, $x(0)=0$ & $\dot{x}(0)=0$

NII: (↓) $mg - kx = m\ddot{x}$; $\ddot{x} + \frac{k}{m}x = g$

Högerled $\neq 0 \Rightarrow$ partikulär lösn. krävs

$x = x_H + x_P$ homogen + part. lösn.

(jmf kursen i Analys)

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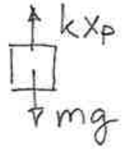
$x_H = A\cos\omega_n t + B\sin\omega_n t$, $\omega_n = \sqrt{\frac{k}{m}}$

↳ Lösning för högerled $= 0$

Part. lösn. prova $x_P = C = \text{konst.}$

$0 + \frac{k}{m}C = g$; $C = \frac{mg}{k}$

Jmf.  dvs statiska jmv. läget

Totalt $x = A\cos\omega_n t + B\sin\omega_n t + \frac{mg}{k}$

Beg. data $\Rightarrow$ A & B:

$x(0) = 0 \Rightarrow A + \frac{mg}{k} = 0$; $A = -\frac{mg}{k}$

$\dot{x}(0) = 0 \Rightarrow B$

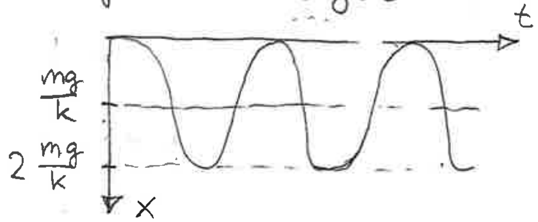
$\dot{x}(t) = -A\omega_n\sin\omega_n t + B\omega_n\cos\omega_n t$

$\dot{x}(0) = B\omega_n = 0$; $B = 0$

Alltså

$x(t) = \frac{mg}{k}(1 - \cos\omega_n t)$. $\left(0 < x < \frac{2mg}{k}\right)$

Svängningen sker kring statiska jämvikts läget:



(Frekvensen påverkas inte)

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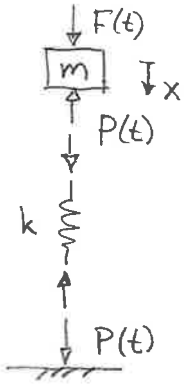
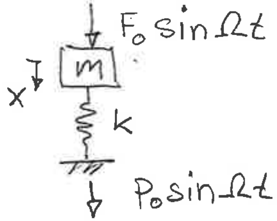
3) Massa - fjäder system :



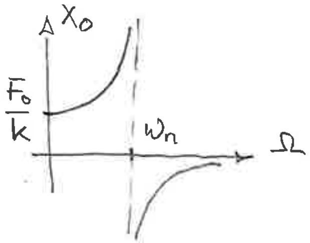
Försumma mg pga svängn. kring jmv. läget påverkar inte amplituder



kvoten $\frac{P_0}{F_0}$ intressant vid vibr. isolering

Enligt föreläsning :

$$x_0 = \frac{F_0}{k} \frac{1}{1 - \left(\frac{\Omega}{\omega_n}\right)^2}$$



Amplituden $P_0 = k x_0$

$$\frac{P_0}{F_0} = \frac{1}{F_0} \cdot k \cdot \frac{F_0}{k} \frac{1}{1 - \left(\frac{\Omega}{\omega_n}\right)^2} \quad ; \quad \frac{P_0}{F_0} = \frac{1}{1 - \left(\frac{\Omega}{\omega_n}\right)^2}$$

$\Omega = 50$ rad/s , $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10 \cdot 10^3}{100}} = 10$ rad/s

$$\frac{P_0}{F_0} = \frac{1}{1 - 5^2} = \frac{-1}{24} \approx 0.042$$

Ca 4% av störningen går igenom till underlaget - vibrationsisolering !