Seminariepass 4

1) Lösning m. primitiv funktion:

$\frac{dv}{dt} = a_0$; $v(t) = a_0 t + c_1$

Beg. data $v(0) = v_0 \Rightarrow c_1 = v_0$

$v(t) = a_0 t + v_0$ ⇒

$\frac{ds}{dt} = a_0 t + v_0$; $s(t) = a_0 \frac{t^2}{2} + v_0 t + c_2$

Beg. data $s(0) = s_0 \Rightarrow c_2 = s_0$

$s(t) = a_0 \frac{t^2}{2} + v_0 t + s_0$

Alt. Lösning med integral:

$\frac{dv}{dt} = a_0$; $dv = a_0 dt$; $\int dv = \int a_0 dt$;

$\int_{v_0}^{v} dv' = a_0 \int_0^t dt'$; $v - v_0 = a_0 t$;

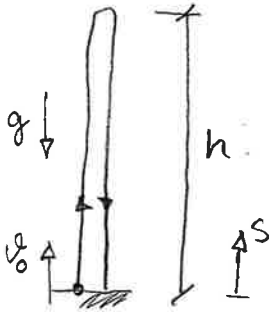
$v = v_0 + a_0 t$ ⇒

$\frac{ds}{dt} = v_0 + a_0 t$; $\int_{s_0}^{s} ds' = \int (v_0 + a_0 t') dt'$;

$s - s_0 = \left[v_0 t' + a_0 \frac{t'^2}{2} \right]_0^t$; $s - s_0 = v_0 t + a_0 \frac{t^2}{2}$;

$s = s_0 + v_0 t + a_0 \frac{t^2}{2}$

2) Bortse från luftmotstånd



Bestäm h och $t_{åter}$.

Jmf. 1) med

$s_0 = 0$, $v_0 = 200$ och $a_0 = -g$

$$\begin{cases} v = v_0 - gt \quad \dots (1) \\ s = v_0 t - g\frac{t^2}{2} \quad \dots (2) \end{cases}$$

(1) ⇒ t_{topp} ; $0 = v_0 - g t_{topp}$; $t_{topp} = \frac{v_0}{g}$

(2) ⇒ h ; $h = t_{topp}\left(v_0 - g\frac{t_{topp}}{2}\right)$;

$h = \frac{v_0}{g}\left(v_0 - \frac{g}{2}\frac{v_0}{g}\right)$; $h = \frac{v_0^2}{2g} = \frac{200^2}{2 \cdot 9.81} = 2039\,m$

(2) ⇒ $t_{åter}$; $0 = t_{åter}\left(v_0 - \frac{g}{2} t_{åter}\right)$;

$t_{åter} = \frac{2v_0}{g} = \frac{2 \cdot 200}{9.81} = 40.8\,s$

Alt.

$t_{åter} = 2 \cdot t_{topp}$ symmetri

$t_{åter} = 2 \cdot \left(\frac{v_0}{g}\right)$ samma

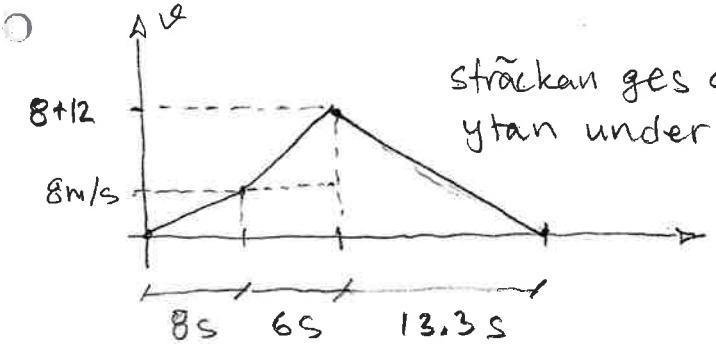
3) $\int dv = \int a\,dt$; $\int_0^{v} dv = \int_0^{t_{tot}} a(t)\,dt$;

$$0 = \int_0^{t_{tot}} a(t)\,dt$$

Ytor: $0 = 1\cdot 8 + 2\cdot 6 - 1{,}5\cdot \Delta t$;

$1{,}5\Delta t = 8+12$; $\Delta t = \frac{20}{1{,}5} = 13{,}3\ s$

Sträckan:



sträckan ges av ytan under grafen.

$$s = \frac{8\cdot 8}{2} + \left(6\cdot 8 + \frac{6\cdot 12}{2}\right) + \frac{13{,}3\cdot 20}{2} =$$

$$= 32 + 48 + 36 + 133 = 249\ m$$

4) $v_0 = 40\ km/h = 11{,}11\ m/s$ där $s=0$

$$\int v\,dv = \int a\,ds \ ; \ \int_{v_0}^{v} v'\,dv' = \int_0^{s_2} a\,ds \ ;$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = \int_0^{200} a\,ds \ ;$$

$$\frac{v^2}{2} = \frac{v_0^2}{2} + 100\cdot 0{,}8 + \left(100\cdot 0{,}4 + \frac{100\cdot 0{,}4}{2}\right) ;$$

$$\frac{v^2}{2} = \frac{11{,}11^2}{2} + (80 + 40 + 20)\cdot 2 \ ;$$

$$v = \sqrt{11{,}11^2 + 280} = \sqrt{403{,}4} = 20{,}1\ m/s \approx 72\ km/h$$