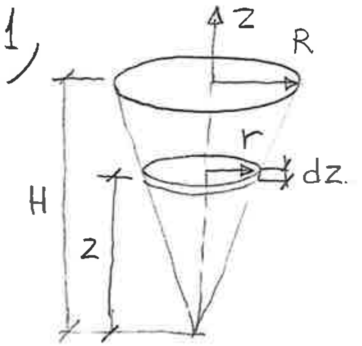
Sem. pass 9 lösning

1)



$$\bar{z} = \frac{\int z\,dV}{\int dV}, \quad dV = \pi r^2 dz$$

$$r = r(z) = z\frac{R}{H}$$

(koll: $z = H \Rightarrow r = R$)

*) $\int dV = \int \pi r^2 dz = \int_0^H \pi \left(z\frac{R}{H}\right)^2 dz =$

$$= \pi\frac{R^2}{H^2}\int_0^H z^2 dz = \pi\frac{R^2}{H^2}\frac{H^3}{3} = \frac{1}{3}\pi R^2 H$$

(Volymen av en kon eller pyramid är $\frac{1}{3}A\cdot H$ där A är bottenarean)

**) $\int z\,dV = \int z\pi r^2 dz = \int_0^H z\,\pi z^2 \frac{R^2}{H^2} dz =$

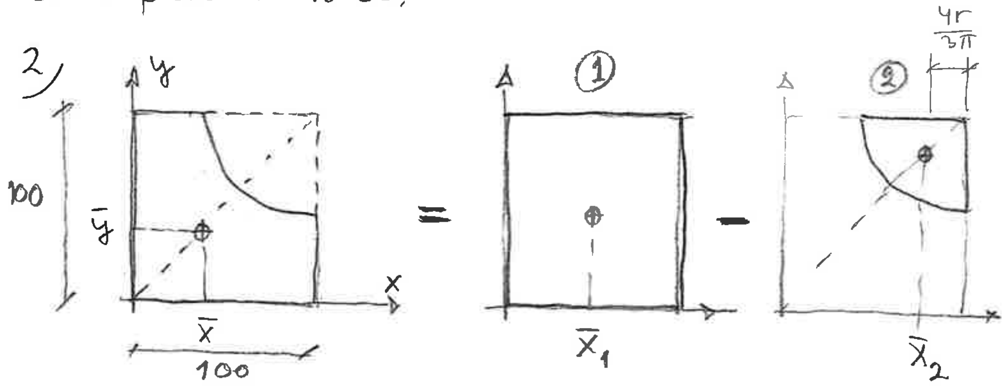
$$= \pi\frac{R^2}{H^2}\int_0^H z^3 dz = \pi\frac{R^2}{H^2}\frac{H^4}{4} = \frac{\pi R^2 H^2}{4}$$

Alltså $\bar{z} = \frac{\cancel{\pi R^2} H^{\cancel{2}}}{4}\cdot\frac{3}{\cancel{\pi R^2 H}} = \frac{3}{4}H$

(stämmer med App. AII i boken)

Sem. pass 9 forts.

2)



Symmetri $\Rightarrow \bar{x} = \bar{y}$

Kvartcirkelns TP ges av tabell:

$\bar{x}_1 = 50$ mm, $\bar{x}_2 = 100\text{ mm} - \frac{4\cdot 70}{3\pi} = 70.3$ mm

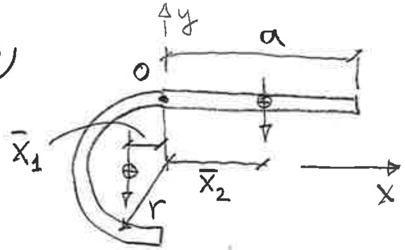
$A_1 = 10^4\ \text{mm}^2$, $A_2 = -\frac{\pi 70^2}{4} = -3.85\cdot 10^3\ \text{mm}^2$

El.	A	$\bar{x}$	$A\bar{x}$
1	10^4	50	$500\cdot 10^3$
2	$-3.85\cdot 10^3$	70.3	$-271\cdot 10^3$
	$6.15\cdot 10^3$	−	$229\cdot 10^3$

Tyngdpunkten: $\bar{x} = \frac{\Sigma A\bar{x}}{A} = \frac{229\cdot 10^3}{6.15\cdot 10^3}$

$= 37.2$ mm (< 50 mm)

3)



Se till att den gemensamma tyngdpunkten hamnar i O.

$$\bar{x}_2 = \frac{a}{2} \quad \text{och} \quad \bar{x}_1 = \frac{2r}{\pi} \quad \text{enligt App. AII}$$

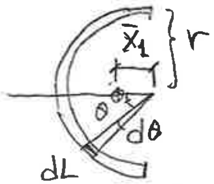
Stången betraktas som en tråd ($dm = \rho A dL$):

$$\bar{x} = \frac{\bar{x}_1 L_1 + \bar{x}_2 L_2}{L_1 + L_2} = 0 \quad \text{villkor}$$

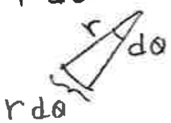
$$-\frac{2r}{\pi}\cdot \pi r + \frac{a}{2}\cdot a = 0 \; ; \quad \frac{a^2}{2} = 2r^2 \; ;$$

$$a^2 = 4r^2 \; ; \quad \underline{a = 2r}$$

<u>Extra</u>: Visa att $\bar{x}_1 = \frac{2r}{\pi}$



$$\bar{x}_1 = \frac{\int x\,dL}{\int dL} \qquad \begin{cases} x = r\cos\theta \\ dL = r\,d\theta \end{cases}$$



$$\int dL = \int_{-\pi/2}^{\pi/2} r\,d\theta = r\cdot\pi$$

$$\int x\,dL = \int_{-\pi/2}^{\pi/2} r\cos\theta\, r\,d\theta = r^2\Big[\sin\theta\Big]_{-\pi/2}^{\pi/2} =$$

$$= r^2(1-(-1)) = 2r^2$$

$$\Rightarrow \quad \bar{x}_1 = \frac{2r^2}{r\cdot\pi} = \underline{\underline{\frac{2r}{\pi}}}$$