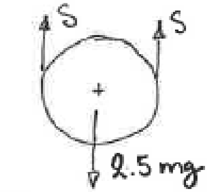
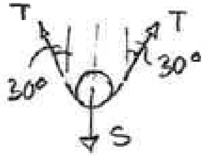
Lösningar Mekanik för V & Bi 190821 \P-E A

1)



(↑) $2S - 2.5\,mg = 0$; $S = 1.25\,mg$

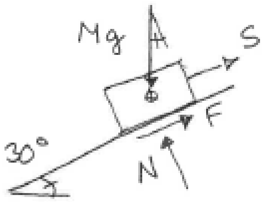


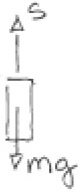
(↑) $2T\cos 30° - S = 0$; $T = \dfrac{S}{2\cdot\frac{\sqrt{3}}{2}}$;

$T = S/\sqrt{3}$

Alltså $T = 1.25\cdot 2\cdot 9.81/\sqrt{3} = 14.2\,N$

2)





Begynnande glidn.

Nedåt $F > 0$

Uppåt $F < 0$

Friktion : $F = \mu_s N$ ... (1)

(↖) $N - Mg\cos 30° = 0$; $N = \frac{\sqrt{3}}{2}Mg$ ...(2)

(↗) $S + F - Mg\sin 30° = 0$; $S = \frac{1}{2}Mg - F$ ...(3)

(↑) $S - mg = 0$; $S = mg$ ...(4)

(2) insatt i (1) ⇒ $F = \mu_s \frac{\sqrt{3}}{2}Mg$ ...(*)

(4) och (*) insatt i (3) ⇒

$mg = \frac{1}{2}Mg - \mu_s\frac{\sqrt{3}}{2}Mg$; $m = (1-\mu_s\sqrt{3})\frac{M}{2}$

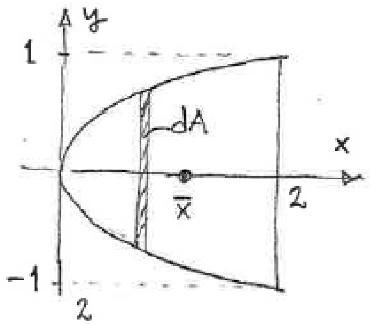
2 forts.

F byter riktning ⇒ $m = (1+\mu_s\sqrt{3})\frac{M}{2}$

$(1-0.12\cdot\sqrt{3})30 < m < (1+0.12\cdot\sqrt{3})30$;

$23.8\,kg < m < 36.2\,kg$

3)



$\bar{x} = \dfrac{\int x\,dA}{\int dA}$ med

$dA = 2y\,dx = 2\sqrt{\frac{x}{2}}\,dx$

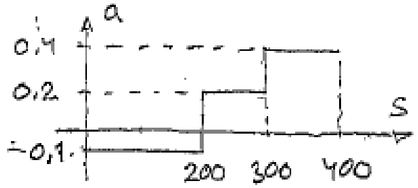
$\int x\,dA = \int_0^2 x\cdot 2\sqrt{\frac{x}{2}}\,dx = \ldots$

$= \int_0^2 \sqrt{2}\,x^{1,5}\,dx = \sqrt{2}\left[\frac{1}{2.5}x^{2,5}\right]_0^2 = \sqrt{2}\,\frac{2}{5}\,2^2\sqrt{2} = \frac{16}{5}$

$\int dA = \int_0^2 2\sqrt{\frac{x}{2}}\,dx = \int_0^2 \sqrt{2}\,x^{0,5}\,dx = \sqrt{2}\left[\frac{1}{1,5}x^{1,5}\right]_0^2 = \ldots = \frac{8}{3}$

Alltså $\bar{x} = \frac{16}{5}\cdot\frac{3}{8} = \frac{6}{5} = 1.2\,m$

4)



$v(0) = v_0 = 7\,m/s$

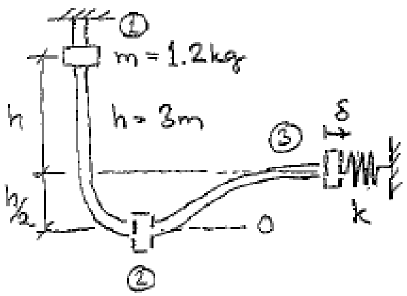
$a = v\frac{dv}{ds}$; $\int a\,ds = \int v\,dv$

Fart efter 400 m: $\int_0^{400} a\,ds = \left[\frac{v^2}{2}\right]_{v_0}^{v_{400}}$;

$-0.1\cdot 200 + 0.2\cdot 100 + 0.4\cdot 100 = \frac{1}{2}(v_{400}^2 - v_0^2)$;

$v_{400}^2 = v_0^2 + 80$; $v_{400} = \underline{11.4\,m/s}$

6)



Förlustfritt system ⇒ mekaniska energin bevaras dvs $E_1 = E_2 = E_3$

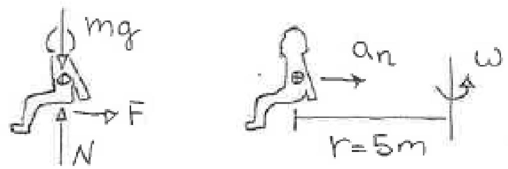
$E_1 = mg\frac{3h}{2}$, $E_2 = \frac{1}{2}mv^2$,

$E_3 = mg\frac{h}{2} + \frac{1}{2}k\delta^2$

$E_1 = E_2$ ⇒ $mg\frac{3h}{2} = \frac{1}{2}mv^2$; $v = \sqrt{3gh} = \underline{9.4\,m/s}$

$E_3 = E_1$ ⇒ $\frac{1}{2}k\delta^2 = mgh$; $\delta = \sqrt{\frac{2mgh}{k}} = \underline{54\,mm}$

5)



$a_n = \frac{v^2}{r}$

(↑) $N - mg = 0$, (→) NII: $F = m a_n$

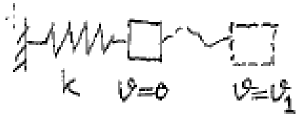
På gränsen till glidning: $F = \mu_s N = \mu_s mg$

⇒ $\mu_s mg = m\frac{v^2}{r}$; $v^2 = \mu_s rg$

$v = \sqrt{\mu_s rg} = \sqrt{0.2\cdot 5\cdot 9.81} = \underline{3.13\,m/s}$

7)

Hastigheten innan stöten ges av energibalans



$\frac{1}{2}ku^2 = \frac{1}{2}mv_1^2$; $v_1 = \sqrt{\frac{k}{m}}\,u$

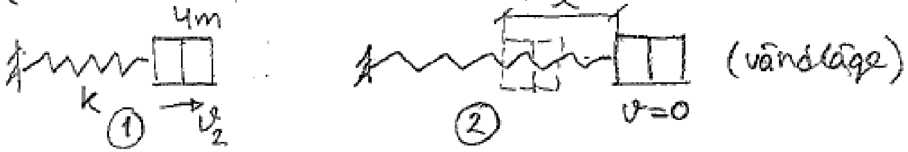
Stötförloppet; rörelsemängden bevaras:

(→) $mv_1 = 4mv_2$; $v_2 = \frac{v_1}{4}$

7 forts

Inbromsning; Rörelseenergi → fjäderenergi

(Fjädern ospänd i läge ①) ⇒



$\frac{1}{2}4m\left(\frac{v}{2}\right)^2 = \frac{1}{2}k\ell^2$; $\ell^2 = 4\frac{m}{k}\left(\frac{v}{2}\right)^2$; $\ell^2 = 4\frac{m}{k}\cdot\frac{v^2}{16}$;

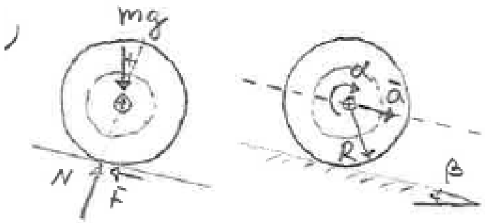
$\ell^2 = \frac{1}{4}\frac{m}{k}\frac{k}{m}u^2$; $\ell = \frac{u}{2} = 0.1\ m$

Anm. Om man inte tar hänsyn till stöten och räknar energi; startläge till slutläge:

$\frac{1}{2}ku^2 = \frac{1}{2}k\ell^2 \Rightarrow \ell = u$

Detta blir fel eftersom energi förloras i stöten

8)



Kinematik:

$\bar{a} = \text{konst} \Rightarrow s = \bar{a}\frac{t^2}{2}$;

$\bar{a} = \frac{2s}{t^2} = 0.49\ m/s^2$

Rullning utan glidning: $\bar{a} = \alpha R$; $\alpha = \frac{\bar{a}}{R} = 1.48\ rad/s^2$

(↘) $mg\sin\beta - F = m\bar{a}$; $F = m(g\sin\beta - \bar{a})$;

$F = 4.5(9.81\cdot\sin 15^\circ - 0.49) = 9.22\ N$

(↻) $F\cdot R = \bar{I}\alpha$; $\bar{I} = \frac{F\cdot R}{\alpha} = \frac{9.22\cdot 0.33}{1.48} = 2.06\ kgm^2$