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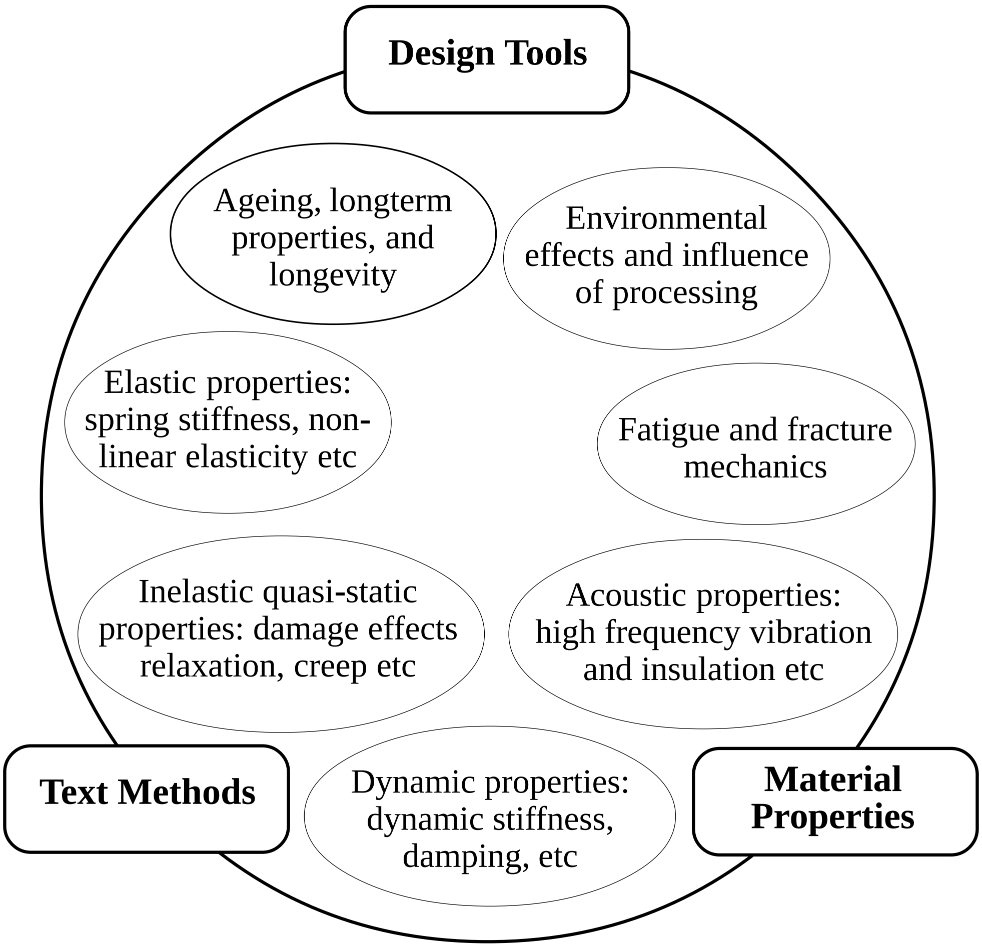

SURVEY OF DESIGN METHODS AND MATERIAL CHARACTERISTICS IN RUBBER ENGINEERING

A report in the NUTEK-VAMP research program

P.-E. AUSTRELL, M. BELLANDER, B. STENBERG,
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Survey of Design Methods and Material Characteristics in Rubber Engineering

a NUTEK - VAMP report

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July 24, 1998

Preface and overview

PER-ERIK AUSTRELL

This report is a part of the work encompassed by a research project financed by NUTEK - the Swedish National Board for Industrial and Technical development -, during the two-year period of June 1997 to June 1999, in the VAMP program (in swedish: Verkstadstekniskt materialprogram). The project concerns methodology for the development of rubber components. The aim is to investigate strategies for the design that the companies in this area currently are guided by and to improve and rationalize the procedures employed in order that better performance and better quality of products can be achieved.

A number of companies, research institutes and university departments participates. The companies are Adtranz, Alfa Laval Separation AB, Electrolux-Wascator AB, Hägglunds Vehicle AB, Svedala Skega AB, Trelleborg Industri AB, Volvo Articulated Haulers AB, and Volvo Wheel loaders AB.

The research institutes participating are IVF - the Swedish Institute of Production Engineering Research - (in swedish: Institutet för Verkstadsteknisk Forskning) and IFP -the Swedish Institute for Fiber and Polymer Research- (in swedish: Institutet för Fiber och Polymer teknologi).

The university departments involved, are Structural Mechanics LTH (Lunds Tekniska Högskola), Acoustics MWL (Marcus Wallenberg Laboratoriet), KTH (Kungliga Tekniska Högskolan), and Polymer Technology IPT (Institutionen för PolymerTeknologi) KTH.

The survey of material properties and design methods contained in this report represents a part of the initial phase of the project. It consists of contributions by several authors. The chapters concerning quasi-static and low frequency characteristics along with the chapter on fracture mechanics[1] are written by **Per-Erik Austrell** (Structural Mechanics LTH), who is also the editor of this report. The chapters concerning high frequency and acoustic properties are authored by **Ulf Carlsson** and **Leif Kari** MWL (Acoustics KTH) and the chapters on the choice of materials, on long-term properties, and on environmental effects are written by **Martin Bellander** and **Bengt Stenberg** IPT (Polymer Technology KTH). **Sture Persson** Svedala-Skega AB has examined the material thoroughly and given many valuable comments and suggestions.

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[1] Chapter 9 is based on the master thesis work of Jörgen Nilsson.

Motivation and background

The unique properties of elastomeric materials are taken advantage of in many engineering applications. Elastomeric units are used as couplings or mountings between stiff structures. Examples of these are shock absorbers, vibration insulators, flexible joints, seals and suspensions.

The development of computers and of analysis programs in this area has given engineers a new tool for the design of elastomeric components. Computer simulation by finite element analysis has become increasingly important, allowing the mechanical behavior of products with for complex geometries, as well as loading cases of different kinds to be evaluated. Computer simulations enable both static and dynamic aspects to be analyzed. These matters have been recognized by the manufacturers of rubber products and by their customers. The benefits are shorter time for product development and also quality improvements.

However, the possibilities available for finding less complicated technical solutions at lower cost with the use of elastomers, has not been fully utilized. Rubber components could be employed more frequently in design if engineers were more familiar with materials of this sort.

Part of the problem lies in education and in the dissemination of information. Engineers working in the design area tend to not be very familiar with elastomeric materials and their properties. The offerings of courses on the mechanics of polymers at schools and universities are very limited. Skillful engineers in this field have usually acquired their knowledge through many years of experience and not from formal education.

Moreover, the complicated nature of the material behavior involved makes it difficult to devise general design rules and design tools. Only recently have computers and analysis programs become powerful enough for the analysis of nonlinear elastic problems involving large strains.

It is essential, if one is to become competitive in high-tech applications, to possess a thorough knowledge of computer methods, material models and test methods available.

There has likewise been a lack of relevant data for the computer analysis of elastomeric materials. The design tools employed rely on the material models available and on the test data required for the calibration of these models. In many cases, the only information available for analysis is a value for the hardness of the rubber in question. The wide variety of rubber compounds is also a problem. The characterization of different materials is costly and time-consuming. There is thus a need for simple and reliable methods to characterize the different vulcanizates.

Overview

The basic topics covered in the report are shown in Figure 0.1. Questions concerning design tools, test methods, and material properties in connection with the topics encircled in the figure are discussed.

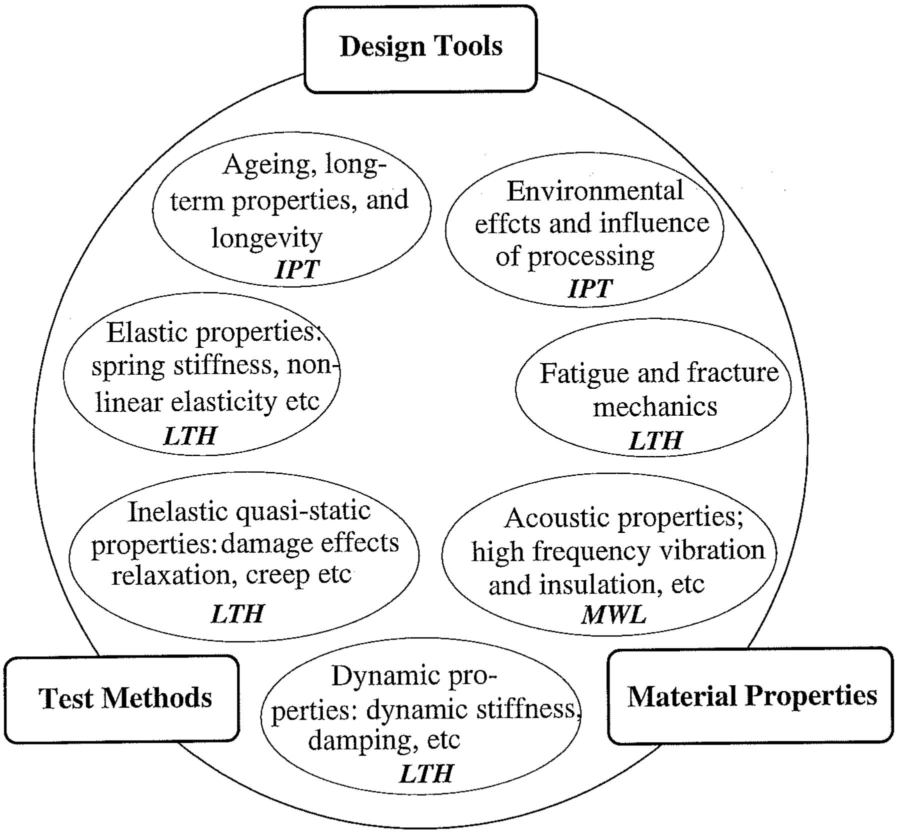

Figure 0.1: *Overview of areas covered in this survey.*

Contents

1 General aspects of rubber engineering

2 Basic considerations regarding choice of material and its specifications

3 Elastic properties and static design

Chapter 1

General aspects of rubber engineering

P.-E. AUSTRELL AND B. STENBERG

This chapter presents some general properties of rubber, including a brief description of the molecular structure of rubber vulcanizates and processes in manufacturing them, various mechanical properties of rubber, tools for the construction and design of rubber units, and a brief discussion of constitutive models. In addition, the chapter presents a survey of the literature and of other sources of information.

1.1 Molecular structure and manufacturing process

Although elastomers can be made of any of a wide variety of different organic substances, all of them are polymers consisting of very long molecular chains. The raw material having no chemical bonds between the separate molecular chains, is soft and plastic in its consistency.

The first elastomeric units were manufactured of natural rubber, which is still the most common material used for general-purpose applications. The most common synthetic rubber is made of styrene and butadiene its main application being in car tires, due to its good abrasion resistance.

The important process of *vulcanization* converts the plastic raw elastomeric material into a material of solid and elastic consistency. Vulcanization is a chemical process by which the long molecular chains are linked together, forming a stable and more solid molecular structure. The cross-linking is obtained using activators, accelerators, and curatives. The vulcanization process starts when the mixture is heated to $120 - 200^oC$, crosslinks connecting the molecular chains being formed.

Fillers such as *carbon black* are added so as to increase the stiffness of the material and, in some applications, the resistance to wear. Carbon black consists

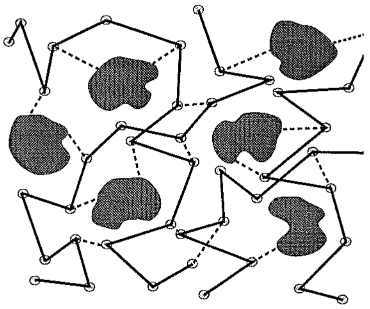

Figure 1.1: *Molecular structure for a carbon black-filled rubber vulcanizate. Carbon particles, polymer chains and crosslinks are schematically illustrated.*

of very small particles of carbon (20nm - 300nm) that are mixed into the raw rubber base prior to vulcanization. In the rubber phase, a continuous network is formed, the filler material being in the form of aggregates inside the rubber network. What is involved is thus a two-phase material of the two constituents which differ completely in their mechanical properties. Figure 1.1 shows schematically the structure on a molecular level of a carbon black-filled vulcanizate. The polymer chains are depicted as solid lines and the crosslinks as dashed lines.

Vulcanization and shaping are combined in the *moulding* process, the rubber-filler mix being inserted into the mould cavity and heated to the appropriate temperature, allowing vulcanization to start. The curing time is dependent on the temperature, of the size of the unit and on how well heat is transferred to the rubber unit.

In technical applications, elastomeric units are often composed of both rubber and steel. The attached steel parts, used to connect the rubber unit involved to other structures or to increase the units stiffness, can be attached to the rubber material in the moulding process, being bonded very efficiently then to the rubber. The bonding is stronger than the rubber material itself, in the sense that in a properly manufactured rubber-steel unit a rupture usually occurs in the rubber itself rather than at the bonding surface between rubber and steel.

The properties of different elastomer types, as well as the compounds and processing techniques employed, are described in references [2], [4], and [5].

1.2 Mechanical properties

The major properties of elastomeric materials taken advantage of in engineering applications are the ability to sustain large straining without permanent deformation, the vibration damping property and the resistance to wear.

The most prominent feature of vulcanized rubber is its *elastic property.* The ability to store large amounts of strain energy and to release most of it in unloading is a characteristic discussed further in Chapter 3. Although the molecular structure of vulcanized rubber enables it to undergo large deformations and recover almost completely in unloading, the material becomes harder, less elastic

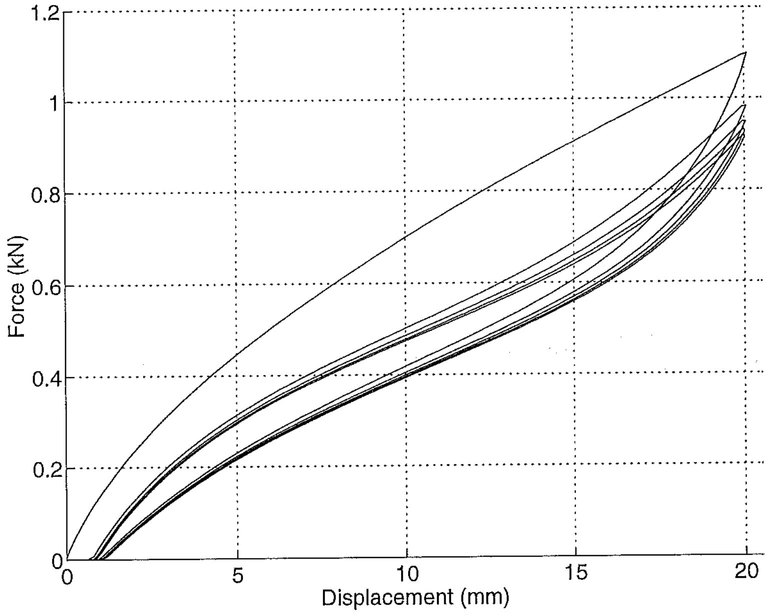

Figure 1.2: *Force-displacement relations for a rubber vulcanizate exposed to cyclic loading.*

and more leathery as the amount of filler mixed into it increases.

The elasticity of rubber is due to the long, tangled molecular chains and their ability to stretch and orient themselves in the direction of strain. This is possible due to the repeated molecular units in the polymer being able to rotate freely about the bonds joining the units. Elongations of several hundred percent are possible.

Another characteristic feature of rubber is the large difference between its shear modulus and its bulk modulus. A typical carbon black-filled rubber vulcanizate for technical applications has a shear modulus of about 1MPa and a bulk modulus of about 1000-2000 MPa (Freakley and Payne [2] p. 32). The large volumetric stiffness compared to the shear stiffness indicates the behavior being *nearly incompressible.* In many applications, in fact, complete incompressibility is a reasonable assumption.

Although rubber is highly elastic, it is not perfectly elastic. A difference is always observed between the loading and unloading curves in a stress-strain diagram. This phenomenon, referred to as *hysteresis*, is illustrated for a carbon black-filled rubber in planar tension in Figure 1.2. (Typical values of stress and strain for tension/compression and for shear are given below in Figure 1.4) In cyclic loading there is thus always a part of the energy that is not recoverable. The area enclosed by the loading and unloading curves represents energy that is dissipated, mainly as heat. In free vibration, this dissipation causes the amplitude of the vibrations to decrease over time, a material property termed *damping.* Adding fillers to the rubber compound increases the damping. This is discussed

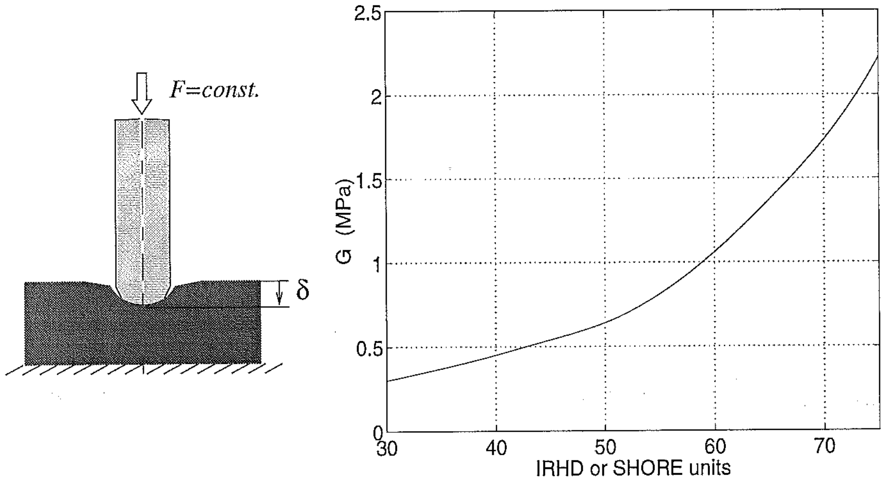

Figure 1.3: *Relationship between the shear modulus G and the hardness in IRHD or Shore A units.*

further in Chapter 4.

Stress softening, or Mullin's effect, is another phenomenon to be considered. The decrease in stiffness through straining is shown in Figure 1.2. If a previously unstrained rubber specimen is exposed to cyclic loading up to a specific strain level, the maximum stress and the distance between the loading and unloading curves will decrease in the first few load cycles. After some four to eight load cycles, a steady state is reached at this specific maximum strain level. If the specimen is exposed to a new set of cyclic strains at higher strain level, a new decrease in stress and hysteresis, occours until a new steady state is reached. The stress-softening behavior originates in configurational changes in the rubber network and a gradual breakdown of the molecular links (primarily the rubber-carbon links) as the strain increases (cf. Chapter 4).

In order to obtain stationary values in the testing of rubber specimens, it is thus necessary to pre-strain the specimens before recording the corresponding force-displacement values. This is called *mechanical conditioning.*

The filler phase has a very small stress-carrying capacity as compared with the rubber phase. The filler particles can be regarded as rigid inclusions embedded in the rubber matrix. Consequently, the stress and strain in the rubber phase reaches higher levels in elastomeric units to which filler has been added than in identical and unfilled units. The filler also affects the maximum elongation (at break), which is lowered. This effect which the filler has on the rubber phase is called *strain amplification.*

The stiffness of a rubber vulcanizate is classified in terms of a value of *hardness.* Two methods for measuring hardness are generally used: the IRHD test (International Rubber Hardness Degrees), which is also the ISO-standard test, and the Shore Hardness test. These involve the indentation by a ball or needle with a spherical tip (IRHD) or a truncated cone (Shore). A constant force is applied and the indentation depth is measured. The scales for the two methods

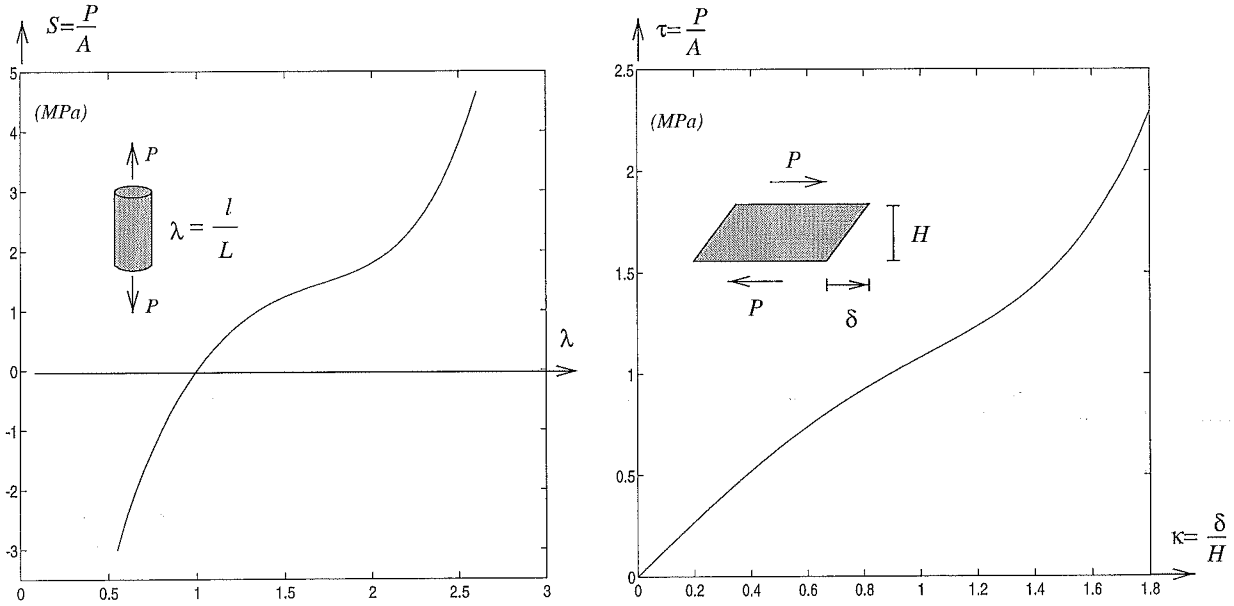

Figure 1.4: *Homogeneous deformations of a 60 IRHD rubber. Left: Uniaxial state of stress. Right: Simple shear.*

are almost identical for rubbers within the 30-80 IRHD range, in which most rubber mixes are found. Such a hardness test provides an indirect measure of the elastic modulus. This is sometimes the only value available for the modulus of the material. The relationship between the shear modulus G and hardness is indicated in Figure 1.3. The diagram is constructed from data reported by Lindley [10] (Table 3, p. 8).

Simple shear is more linear than other homogeneous modes of deformation. Since the shear modulus is relatively independent of the shear strain, it can be regarded, at least for moderate strains, as a material constant. This is not the case for Young's modulus, as can be seen in Figure 1.4, which shows loading curves in compression/tension and simple shear for a 60 IRHD carbon black-filled natural rubber vulcanizate. (The curve representing the uniaxial state of stress is based both on a tension test and a compression test.) The behavior of rubber in compression is progressive. For tension and simple shear in the case of large strains, the behavior is first digressive and then progressive. Modeling the behavior shown in Figure 1.4 by use of nonlinear elastic models will be discussed in Chapter 3.

Some dynamic properties will also be discussed briefly here. Dynamic tests in simple shear performed at small amplitudes for unfilled rubber vulcanizates, yield a *linear dynamic* response, a behavior characterized by a sinusoidal response to sinusoidal excitation. The response is of the same frequency as the excitation, with the response shifted by a phase angle δ.

Graphing the stress-strain loop yields an elliptic path, as shown in Figure 1.5. (The static load there being assumed to be applied slowly.) The elliptic hysteresis

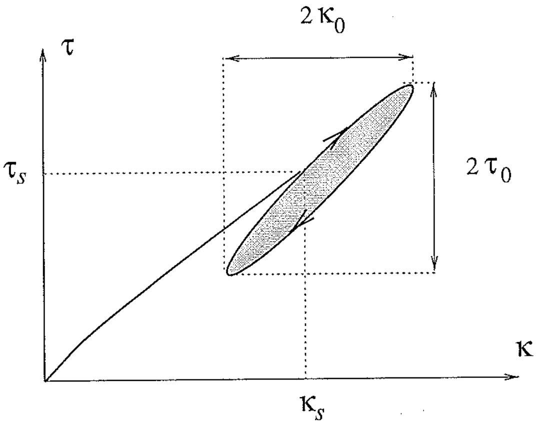

Figure 1.5: *Static and dynamic stress and strain.*

obtained is associated with the dissipation of energy. The energy loss for a strain cycle U_c is related to the phase angle δ according to $U_c = \pi\tau_0\kappa_0 sin\delta$, with τ_0 and κ_0 being the shear-stress and strain amplitudes, respectively.

The use of rubber components often involves both static and dynamic properties. A common load case is that of cyclic straining in combination with a static preload. The static (secant) and dynamic shear moduli are defined as

$$G_{stat} = \frac{\tau_s}{\kappa_s} \qquad G_{dyn} = \frac{\tau_0}{\kappa_0},$$

according to Figure 1.5. For simple shear, the static-secant and the static-tangential modulus are approximately equal. The *dynamic modulus* is always larger than the static (or the tangential) modulus, as indicated in the figure.

The dynamic properties of rubber change with temperature and frequency. The dynamic modulus and the phase angle are frequency- and temperature-dependent. An increase in temperature has a softening effect, and an increase in frequency a stiffening effect. For frequencies below 1000 Hz and temperatures between 0 and 50^oC, in the commonly used natural rubber vulcanizates, however the dependence is quite weak (Lindley [10] p. 15).

Nonlinearities in the dynamic behavior of rubber appear as a distortion in the hysteresis loop. Such nonlinearities are due to the nonlinear elasticity of the rubber network, and for filled rubbers due to the filler structure breakdown and reforming as well. Filler induced nonlinearity appears as a decrease of the dynamic shear modulus as the amplitude increases. The linear and nonlinear dynamic properties in rubber will be discussed further in Chapter 4 and the modeling of these properties in Chapter 5.

Considerations of the *fatigue* of components subjected to repeated loading is important in many engineering applications. Cracks may develop due to stress concentrations at defects or small flaws. Ultimately such cracks may enlarge and result in failure of the component. The *tearing energy* is the basis for fracture and fatigue predictions. This will be discussed in Chapter 9.

1.3 Tools for the design of rubber units

Static and dynamic design tools involve both closed-form expressions and computer-based techniques. A brief overview of these matters, which are discussed further in Chapter 3 and 5 will be presented here.

1.3.1 Closed-form expressions

Simple hand-calculation formulas for static and dynamic analysis are sufficient for the design of many rubber components. In the static analysis, linear elasticity involving only Hookes law in uniaxial stress or simple shear yields spring stiffness expressions and stress levels estimates useful in solving many problems.

Closed-form expressions for static and dynamic analysis are discussed in general treatises [2] [4] [5] [8] [10] on rubber engineering. An excellent introduction to the design of rubber components by the use of hand-calculation formulas, can be found in Gent [5], Chapter 8, which also presents examples.

1.3.2 Capabilities of FE-programs

In the following a brief summary is presented of capabilities with regards to analysis of rubber components in the well known and wide spread finite element codes ABAQUS, ANSYS, and MARC. The programs FLEXPAC and NISA are also included in this overview.

All programs discussed are capable of hyperelastic FE-analysis for large strains in two or three dimensions. The programs also include capabilities for handling damping on a structural level by defining a global damping matrix as a linear combination of the mass and stiffness matrices.

The two most versatile programs for the analysis of elastomers appear to be ABAQUS and MARC. The capabilities of these two programs are similar, the differences between them being indicated. A more thorough discussion of the terms dealt with here and the topics considered is presented in Chapters 3 and 5.

ABAQUS

ABAQUS allows both hyper-viscoelastic analysis for steady-state and transient analysis. A very comprehensive theory manual is a unique feature of the program's documentation in contrast to MARC and the other programs.

ANSYS

ANSYS provides no hyper-viscoelastic models for transient or steady state dynamic analysis involving large strains, although it allows for viscoelastic analysis for small strains to be carried out.

FLEXPAC

FLEXPAC has special features for fracture mechanics analysis and fatigue for elastomers. The program provides no hyper-viscoelastic analysis for large strains.

MARC

MARC possesses all the features regarding elastomers included in ABAQUS and is unique in providing a damage model capable of assessing Mullin's effect.

NISA

NISA provides a combination of hyperelastic analysis and modeling of damping on a structural level and also analysis of material damping in terms of viscous damping. These capabilities regarding damping can also be used in transient analysis.

1.4 Test methods

This section contains brief accounts of some ISO-standard mechanical tests. Other tests, some of them non-standard, are mentioned in Chapter 3 which concerns the determination of material parameters for static elastic FE-analysis.

Test methods are discussed in detail in Gent [5] Chapter 10 and in Friberg [4] Chapter 6.

1.4.1 ISO-standard mechanical tests

A brief description of some ISO-standard tests follows.

Hardness ISO48, ISO7619

Hardness is measured in degrees and is based on the indentation of a definined indentor under constant load (as briefly described in Section 1.2). Three scales are commonly used; IRHD (International Rubber Hardness degrees), Shore A and for hard materials Shore D. IRHD is preferred for most specifications, but Shore A is also in widespread use.

Tensile strength and elongation ISO37

A standard dumbbell-type test piece of known cross sectional area is stretched until it breaks. The force at break is recorded, the tensile strength obtained being expressed as force per unit of the original (unloaded) area. Elongation is defined as the engineering breaking strain, expressed in %.

Stress at a given elongation ISO37

The stress for a given elongation is measured as the force per unit of the original area, required to extend the dumbbell specimen by a specified percentage of its original length, for example at 100%, 200% or 300% strain.

Shear modulus ISO1827

The standard shear test is a quadruple shear test, using a specimen made of four rectangular blocks of rubber connected to metal. The specimen is loaded to 30% shear strain by applying a force at a constant rate of deformation. The shear modulus is determined from the force required for 25% shear strain.

Tension set ISO2285

A standard test piece of known length is stretched by a stated percentage for a certain period of time and is then released. After recovery, the length of the piece is measured and the tensile strain remaning is determined.

Compression set ISO815

A cylindrical specimen is compressed to a fixed height at a defined temperature for a specified period of time. The cylinder is then released and is allowed to recover, and its height being measured. The compression set that results is expressed in % as the deformation remaining divided by the initial deformation.

Stress relaxation ISO3384

A cylindrical specimen is compressed to a fixed height, the decrease in compressive force being measured.

Density ISO2781

Density, defined as the mass per unit volume, is measured by weighing the sample in air and in water.

Resilience ISO4662

Resilience is obtained by standard test equipment in which a test piece is struck with a pendulum and the bounce-back of the pendulum is measured.

Dynamic modulus and damping ISO4664

The dynamic modulus and the damping are determined from the complex modulus obtained in a forced vibration shear test at 10 Hz at a fixed amplitude.

Tear strength ISO34

Standard test pieces (of the trousers, angle, and crescent type), either nicked or unnicked, are used to measure the force required to tear the test piece.

Tension fatigue ISO6943

A standard dumbbell specimen is exposed to cyclic loading, the number of cycles required for failure being measured at different maximum stress levels, yielding a Wöhler type diagram.

Ozone resistance ISO1431

The test pieces are usually placed under a small degree of tension, for example by bending them round a mandril or stretching them by 5%. Samples are exposed under static conditions to a controlled atmosphere containing ozone.

The ozone causes the rubber to crack. The cracks are graded either by standard photographs, by measurements or by description, e.g. visible under "10 x magnification", "visible to the unaided eye" and the like. The results can be recorded either as the time required for a particular degree of cracking to be obtained or the grade of cracking apparent after a fixed period of time.

Accelerated aging ISO188

Heat aging is widely used as a method for evaluating long-term aging properties. Hardness and dumbbell-type tensile test pieces are placed in an air-circulating oven for a specified period of time at a given temperature (e.g. for 7 days at 70°C). The properties of the rubber are then tested and are compared with the properties prior to aging. For each of the properties the percentage retained is recorded. Change in hardness, for example, is recorded in degrees.

1.5 Mechanical data on materials

Tables of material properties are available in Friberg [4] Chapter 10, Table 10.3, p. 307 and in Freakley and Payne [2] Appendix I, p. 639. See the discussion in Section 1.7.1 as well. Properties corresponding to the ISO tests already discussed are given for a number of different rubbers in the references.

MRPRA Engineering Data Sheets is a series giving detailed information on the mechanical engineering properties of 50 different natural rubber vulcanizates.

Another source to mechanical data is RAPRA Abstracts database which includes the worlds largest collection of published literature on rubber and plastics.

The articles cited in the following chapters also contain information on materials in more dispersed form.

1.6 International research

Tun Abdul Razak Research Centre, TARRC (formerly MRPRA)

This is a part of the Malaysian Rubber Research and Development Board, located in England. It conducts research and support regarding all aspects of natural rubber and its use.

MRRDB, The Malaysian Rubber Research and Development Board, is a Government agency responsible for research, technical development and promotion in support of the Malaysian rubber industry. Extensive research and development activities are carried out at its research centres, the Rubber Research Institute of Malaysia (RRIM) and the Tun Abdul Razak Research Centre (TARRC) in the United Kingdom.

Address: Tun Abdul Razak Research Centre, Brickendonbury, Hertford SG13 8NL, United Kingdom. Tel: +44 (0)1992 584966, Fax: +44 (0)1992 554837, E-mail: general@tarrc.tcom.co.uk.

RAPRA Technology Ltd

RAPRA, Rubber And Plastics Research Association, conducts on research in the rubber and plastics area. Formerly the Research Association of British Rubber Manufacturers, it is now RAPRA Technology Ltd. is a multinational association of industrial firms found in some 30 countries involved in the manufacture, processing, and use of rubber and plastics materials and products. The RAPRA Abstracts database includes the world's largest collection of published literature on rubber and plastics.

Address: Rapra Technology Ltd, Shawbury, Shropshire, SY4, 4NR, UK. Tel: +44(0)1939 250383, Fax: +44 (0)1939 251118. Email: info@rapra.net.

DIK, Deutsches Institut für Kautschuktechnologie e.V.

A research institute that conducts research on all matters relating to the needs of the rubber industry.

Address: DIK e.V: Hannover, Eupener Strasse 33, 305 19 Hannover, Tyskland. Tel: +49 (0)511-842 01-0, Fax: +49 (0)511-8386826. Email: DIKautschuk@t-online.de.

Rubber Process Engineering Centre, RuPEC

A sub-department of IPTME (Institute for Polymer Technology and Materials Engineering), Loughborough University. Current research topics include rubber mixtures, the microstructure and characteristics of elastomer mixtures, and the use of finite element analysis in rubber manufacture.

Address: IPTME, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK, Tel: +44 (0) 1509 223171 or 223331, Fax: +44 (0) 1509 223949.

University of Akron, Institute of Polymer Engineering

One of the largest centers for rubber research in the world. The College of Polymer Science and Polymer Engineering consists of the Polymer Science and Polymer Engineering Departments, as well as two research units- the Institute of Polymer Science and the Institute of Polymer Engineering, and the Akron Polymer Training Center, a college devoted exclusively to polymer education and research. Alan N. Gent is Professor of Polymer Physics and Polymer Engineering at the University.

Adress: University of Akron, Institute of Polymer Engineering, Ohio 44325-0301.

MERL

MERL Materials Engineering Research Laboratory is an independent British company founded in 1986 by a small group of polymer scientists and engineers, providing research and development, finite element analysis, laboratory testing and consultancy services on polymeric materials for engineering systems and structures. MERL has a client base of several hundred companies worldwide.

Address: Tamworth Road, Hertford SG13 7DG, England, Tel: +44(0)1992 500120, Fax: +44(0)1992 586439

1.7 Literature, sources of information and education

The general textbooks on engineering with rubber cited frequently in this survey are Freakley and Payne [2] *Theory and Practise of Engineering with Rubber*, Friberg [4] *Konstruera i gummi*, Gent [5] *Engineering with Rubber*, and Lindley [10] *Engineering Design with Natural Rubber.* A review of these books and of other general texts in the area is presented below.

If only one book were to be recommended, the choice would fall on *Engineering with Rubber*, edited by Gent. It is very comprehensive and up-to-date and covers most matters relevant to the design of rubber units.

Nonlinear finite element analysis of elastomers [12] from MARC Analysis Research Corporation is a booklet (48 pages) providing a very satisfactory summary of topics in finite element analysis of elastomers, without mathematical derivations.

Scientific articles on computational methods in rubber engineering, on the other hand, are usually on a very high mathematical level. It is often easier to understand these subjects through use of manuals for finite element programs, see for example [9] *ABAQUS User's manual.*

1.7.1 General textbooks

Konstruera i gummi [4] (360 pages, Swedish)

(Designing in Rubber), G. Friberg (editor), PGI and Sveriges Mekanförbund, Ljungföretagen 1986.

Contents: Deals mainly with the selection and characteristics of different materials, as well as methods of testing, and simple designing examples. Includes many tables of data.
Data on materials: Characteristics of different types of rubber in terms both of relative merits and of absolute values. Includes mechanical characteristics, ozone resistence, aging characteristics, chemical resistance (highly detailed) and electrical characteristics.
Mathematical details: Very limited.

Elastguide för konstruktörer [11] (50 pages, Swedish)

(Guide to Elastic Materials for Constructors), H. Palmgren, Plast Nordica 197?

Contents: Although rubber materials are dealt with in only a very general way, readers content with being presented many facts but not desirous of going into matters in depth are provided a good basic description.
Data on materials: Limited.
Mathematical details: None.

Gummi som konstruktionsmaterial [3] (100 pages, Swedish)

(Rubber as a Construction Material), Frangeur, Friberg, Johansson, Levin, Lindeberg, Nilsson, and Nordenskjöld. Ingenjörsförlaget 1975.

Contents: General topics on rubber engineering.
Data on materials: Very limited. Vibration and noise insulation data; comparison with other types of material. A brief summary of the mechanical characteristics of different rubber types.
Mathematical details: Only few mathematical equations.

Engineering with Rubber [5] (320 pages)

A. N. Gent, Oxford University Press, 1992.

Contents: Introductory text describing principles of rubber science and rubber technology. Comprehensive and up-to-date. Covers most aspects relevant to the design of rubber units.
Data on materials: Has tables containing limited material data on the most usual rubber types. Gas permeability and solvent and interaction parameters for various rubber types are also included.

Mathematical details: To some extent. Equations are stated without derivations.

Theory and Practice of Engineering with Rubber [2] (650 pages)

P. K. Freakley, A. R. Payne, Applied Science 1978.

Contents: Introductory presentation containing a summary of the characteristics of rubber generally and a short review of different types of rubber and the specific characteristics of each.
Data on materials: Basic characteristics of the 17 most common types of rubber.
Mathematical details: Contains many simple formulas and computational methods.

Engineering design with natural rubber [10] (35 pages)

P. B. Lindley, MRPRA 1992

Contents: Mechanical characteristics, aging characteristics and the like, but only for natural rubber. Considerable material on the design of cushioning and insulating products. Elementary examples of product design and of the computations required.
Data on materials: Limited, presenting a number of brief tables.
Mathematical details: Various computational exercises.

Gummimaterial, Högre kurs i gummi- och plastteknologi 2G [6] (130 pages, Swedish)

(Rubber Material, Advanced Course in Rubber and Plastic Technology 2G), Sveriges Gummitekniska Förening 1996

Contents: Summary presentation of the most common types of rubber material. Detailed presentation of ingredients and mixtures.
Data on materials: Not directly, but dispersed through the text.
Mathematical details: None.

Gummi! Ett utbildningsmateriel för gummiindustrin. [7] (300 pages, Swedish)

(Rubber! Training -program material for the rubber industry.)

Contents: 9 booklets of about 30 pages each. Materials, processes, tests and control, products, etc. Descriptive review without going into depth.
Data on materials: Presented in a table showing mainly qualitative (relative) merits for a large number of rubbers.
Mathematical details: None.

Engineering in rubber [1] 24 pages

Haboro Rubber Company Ltd, England 1997.

Contents: A brief introductory account of different practical aspects of rubber engineering. Available free of charge from Harboro Rubber Company (E-mail admin@harboro.co.uk).

Data on materials: Presented in tables showing mainly the qualitative (relative) merits of a number of commonly-used rubbers.
Mathematical details: None.

1.7.2 Periodicals

Some well-known periodicals in the area are the following:

- Automotive Engineering
- Journal of Applied Polymer Science
- Journal of Natural Rubber Research
- Journal of Rheology
- Journal of Sound and Vibration
- Kautschuck Gummi und Kunststoffe
- Polymer Testing
- Rubber Chemistry and Technology
- Rubber Developments
- Rubber Plastics and Composites
- Rubber World

1.7.3 Web sites

Considerable information is available at various web sites. Here are some interesting sites to start with:

- MERL: www.merl-ltd.co.uk
- MRPRA: www.rubber.demon.co.uk
- RAPRA: www.rapra.net
- University of Akron: www.polymer.uakron.edu

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Chapter 2

Basic considerations regarding choice of material and its specifications

M. Bellander, S. Persson, and B. Stenberg

Summaries of the characteristics of different types of rubber material are available in the literature reviewed in Chapter 1 and in this chapter. Such characteristics are important for the selection of a suitable rubber material. Issues relevant to the selection of an appropriate type of rubber for the application at hand is the subject of section 2.2. A special section on characteristics of thermoplastic elastomers is also included in section 2.3.

2.1 Descriptions of some rubbers

Descriptions of properties of different rubbers are available in several sources for example in Friberg [3] chapter 3, in Gent [4] chapter 2, in Freakley and Payne [1] chapter 1, in Franta [5], and in Eirich et al. [6]. The text in this section is based on references [2] and [7].

Rubber materials can be classified into different categories in many different ways. In this text we will have the following classification: *Normal Rubber and Specialty Rubber types*, the latter which in turn can be classified into *Large Volume and Low Volume types.* The nomenclature and abbreviations of rubber materials may sometimes be a little confusing. Table 2.1 gives the common names in Swedish and English and the abbreviations according to ISO1629. The general properties of the rubbers to be described, and some more in addition, are given in tables 2.2 and 2.3. The Swedish Institution of Rubber Technology (SGF) has compiled the Rubber Handbook [8] which contains a lot of information about commercial materials, including both rubber materials and additives.

2.1.1 Normal rubber types

Natural rubber, NR (cis-polyisoprene):

Natural rubber is the original rubber material which has been in commercial use long before the turn of the century. It is one of the most widely developed rubbers and it also usually available at a low price.

Properties:

NR vulcanisates show a high tensile strength over a large hardness range. The high tensile strength is due to the self-reinforcing of the polymer obtained by crystallization (aligning of polymer chains when strecthed). This means that even unfilled NR has an unexpected high tensile strength. NR is, except for butadiene rubber (BR) the rubber material that shows the highest elasticity of all rubber.

Other conspiciuous properties of NR are low permanent set and stress relaxation, excellent electrial properties and high resistance to abrasion and fatigue.

The highest temperature of use for NR is around 90°C. NR vulcanisates are elastic down to -55°C. NR is like other unsaturated rubber materials sensitive to ozone. The ozone- and weather-resistance can be improved by addition of antiozonants in combination with waxes.

NR is not resistant to mineral oils or fuels, while the resistance to organic and inorganic chemicals such as non-mineral based brake fluids, silicon oils, greases, glycols, alcohols, water and non-oxidizing water-solutions (acid, bases, salts) is good.

Typical Applications:

NR is the best choice in a lot of applications where low heat build-up is a demand, such as large truck tires, tire frames, vibration dampers, springs and rubber bearings. Other important applications for NR are hoses, gaskets and seals, conveyor belts, rollers, rubber coated fabrics, elastic bands, medical applications and condoms.

Isoprene Rubber, IR (cis-polyisoprene):

IR is the synthetic variant of natural rubber. Even though they have identical chemical structure, they do not behave exactly the same. This difference is mainly due to different molecular weight and molecular weight distribution. IR tend to crystallize less than NR, thereby lowering the mechanical properties. On the other hand, the low temperature properties are somewhat better for IR. The long term properties and chemical resistance are almost identical as those of NR.

Typical Applications:

IR can be used pure or blended together with NR in most applications where NR is used.

Styrene Butadiene Rubber, SBR (polystyrene-co-butadiene):

SBR is a synthetic rubber which is easy to process in large quantities. Widely used in the footwear and tyre industries.

Properties:

In opposition to NR, unfilled SBR shows very low tensile strength (typically 10-15% of NR). This is due to the fact that SBR cannot crystallize upon elongation, as NR does. To obtain acceptable properties, SBR must be filled with reinforcing fillers, such as carbon black. The elasticity and low temperature properties are inferior compared to NR, while the heat resistance is better. The temperature range of SBR is between -50 and +100°C. As most other highly unsaturated elastomers, SBR shows bad ozone resistance. However, this can be improved by blending with EPDM and by addition of antiozonants together with protecting waxes. The chemical resistance of SB is very much like that of NR.

Typical Applications:

SBR was developed during the Second World War, to replace NR. Generally, SBR can replace NR in most applications, except for demanding dynamic applications such as truck tires and certain dampers and bearings.

2.1.2 Specialty rubber types - large volume

Ethylene Propylene Rubber, EPDM (Polyethylene-co-propylene-codiene monomer) and EPM (Polyethylene-co-propylene monomer):

Properties:

EPDM was originally developed in 1950s for tyre applications. It became more widely used because of its suitability for outdoor use. Ethylene Propylene Rubbers are the most water resistant type of rubber - also very resistant to most water based chemicals. It has a very inert structure (low degree of unsaturations), and remains stable over long periods of time. Very good weathering resistance.

Ethylene Propylene Rubbers can withstand temperatures of up to 120°C for extended periods of time (months). Drawbacks are that they do not resist oil or oil based products. Compression set is not as good as some other rubbers, but can largely be improved by careful compounding.

Typical Applications:

Ethylene Propylene Rubbers is used in many different applications, such as O-rings, gaskets, door- and window seals, cable insulation, roller-coating, conveyor belts, hoses, bellows and clothing.

Chloroprene Rubber, CR (Polychloroprene):

Properties:

CR is one of the first synthetic rubbers developed in the search for oil resistant rubber. It is widely used due to its combination of useful properties and comparatively low price. One common trademark of CR is Neoprene which is a registered trade mark of Du Pont.

CR is resistant to a wide range of hostile environments including oils and chemicals, and is very weather and water resistant. It can withstand temperatures from -30°C to 95°C. Easy to process and compound, offering cost benefits. It has a tendency to tear once there is initial damage. CR may crystallize during storage or use, causing temporary stiffening (increase in modulus/hardness). If parts are deformed during crystallisation, they may take on a set. However, crystallisation is a readily reversible phenomenon and can be removed by warming to temperatures higher than 80°C and can be prevented by the use of special grades.

Typical Applications:

The most important applications of CR includes adhesives, hoses, v-belts, coated fabrics, cable mantles, tire sidewalls, seals and gaskets in contact with cooling media. CR is not particularly resistant to oil, but it is inherently flame retardant to some degree (due to the presence of the halogen atom - chlorine - in the polymer chain). CR is particularly useful in marine environments due to its good ozone resistance.

Nitrile Rubber, NBR (Polyacrylonitrile-co-butadiene):

Properties:

NBR is another early development in the search for an oil resistant rubber. It is the most suitable rubber for applications requiring resistance to petroleum based fluids. Rubbers with higher degrees of resistance are available but these are much more expensive. It shows good high temperature resistance - up to 100°C (120°C with special cure systems), and is economical to compound and produce. It has low permeability to gases. Drawbacks are, poor resistance to outdoor weathering without special compounding (antiozonantz and waxes) and flammability with toxic fumes.

Typical Applications:

NBR is the most common rubber for oil and fuel resistant components such as gaskets and seals, membranes, hoses, and cable mantles.

Butyl Rubber, IIR (Polyisobutylene-co-isoprene):

IIR was first introduced during the Second World War largely for use in inner tubes. IIR and its derivatives have retained an important role as special-purpose rubbers with production similar to ethylene propylene rubbers and it is only exceeded by SBR, NR, and BR.

Typical Applications:

The main use of IIR is in applications which demand a low gas permeability, such as inner tubes for vehicles, football bellows, vacuum seals, and membranes. Other applications are vulcanisation bellows, steam hoses, shoch absorbers, gaskets, cable insulation, and encapsulation of medical bottles.

2.1.3 Specialty Rubber types - low volume

Acrylic Rubber, ACM:

Properties:

ACM is a synthetic rubber which is particularly resistant to hot oils. Further it shows excellent resistance to oxidation and weathering. Low temperature applications are limited to temperatures above -20°C. It has poor chemical resistance to acids and bases. The resilience below 70°C is very low.

Typical Applications:

ACM is used preferably in applications where the combination of resistance to oils, oil additives and heat is a demand. Examples are O-rings and gaskets.

Ethylene Acrylic Rubber, AEM (Polyethylene-co-acrylate):

Properties:

Trademarks of AEM are for example Vamac (Du Pont) and Hycar (BF Goodrich). AEM provides good oil resistance over a wide temperature range. It is more expensive than nitrile rubber but cheaper than silicone rubber. Moreover, it shows good resistance to oxidation and weathering, and good oil resistance. It is strong and abrasion resistant.

Typical Applications:

The field of use is quite similar to that of ACM. AEM is used preferably over ACM when low temperature properties are important. Examples of applications are seals, gaskets, hoses, and ignition-cable mantles.

Silicone Rubber, Q (Polysiloxane):

Properties:

Silicone rubber is a synthetic rubber with a wide temperature range and outstanding resistance to weathering. It is exceptional among the synthetic rubbers, as it contains both an organic and an inorganic (silicon) part.

The silicon rubbers can be divided into three groups: 1) Heat vulcanised non-liquid rubbers 2) Heat vulcanised liquid rubbers 3) Room temperature vulcanised (RTV) rubbers, supplied "ready-to-use" in one- or two-component systems.

Silicone rubbers can be used within a wide temperature range, with exceptional properties at low temperatures. The resistance to weathering is extremely good and the electrical properties are excellent. Other favorable properties are good resistance to oil and low level of toxicity. Furthermore, silicone rubbers are easily coloured.

Drawbacks are, low strength and poor resistance to fuels. It is also expensive compared to other rubbers.

Typical Applications:

The main use is in electrical insulation, ignition cables, O-rings, static seals, breathing masks, hoses and roller coating in the medical and food industry. The RTV-types are used mainly in the building industry, for encapsulation of electronic components and for production of flexible moulds.

Fluorocarbon Rubber, FPM (copolymer of different fluoro-containing monomers):

Properties:

All fluoro-containing polymers are very stable and have an exceptional resistance to oxidation, weather, fire, chemical attack, and swelling in many different liquids, like water, fuels, oils and most chemicals. Some of the limitations may be use at low temperatures, -20°C being the limit for flexibility, and their high price.

Typical Applications:

FPM is used mainly in critical applications where the demands are extreme in terms of resistance to oils, fuels, chemicals and elevated temperatures. FPM can be used continuously at +200°C and intermittently, for shorter times, up to +300°C. Main products include O-rings, gaskets, shaft-seals, fuel hoses, membranes, and cable mantles under extreme conditions. FPM is approximately 30-40 times more expensive than natural rubber.

Fluorosilicone Rubber, FMQ:

Properties:

Fluorosilicone rubbers are compounded in the same way as conventional silicone rubbers. The properties of FMQ are also similar to conventional silicone rubbers. But, in addition, they show a better resistance to many aggressive oils and fuels. However, the temperature resistance is somewhat limited as compared to conventional silicone and fluorocarbon rubbers. FMC performs excellently in the temperature range -60°C to 175°C.

Typical Applications:

FMQ is more expensive than conventional silicone rubber, and is used mainly in applications where low temperature properties in combination with low swell are important.

Chlorosulphonated Polyethylene, CSM:

Properties:

CSM, also known under the trademark Hypalon, was developed in the 1950s as a speciality rubber for rugged applications. It is resistant to oil and fluids, especially at higher temperatures, above 125°C. It is extremely resistant to ozone and weathering, being able to withstand harsh outdoor conditions for up to 15 years. The fuel resistance, however, is poor.

Typical Applications:

Situations where there is likely to be heavy weather conditions or exposure to hot liquids and/or gases.

Hydrogenated Nitrile Rubber, HNBR (Hydrogenated Polyacrylonitrile-cobutadiene):

Hydrogenated nitrile rubber provides good all round performance at a compound cost between Nitrile and Fluoroelastomer. Its highly saturated main chain provides good resistance against thermal oxidation and chemical attack.

Properties:

HNBR has good physical properties, including abrasion resistance, at high temperatures and good dynamic behaviour and flex cracking resistance at elevated temperatures. It has excellent heat, ageing and ozone resistance and outstanding resistance to steam and hot water.

Limitations are the absence of inherent flame retardency and the poor electrical properties. Furthermore, it is unsuitable in contact with aromatic and polar organic solvents.

Typical Applications:

HNBR is used in diaphragms requiring chemical and heat resistance, chain tensioners, seals in vehicle engines, oil exploration and production.

English	Svenska	Designation ISO1629
Acrylonitrile butadiene rubber	Nitrilgummi	NBR
Alcylated CSM	Alkylerat CSM	ACSM
Brominated butyl rubber	Brombutylgummi	BIIR
Butadiene rubber	Butadiengummi	BR
Butyl rubber	Butylgummi	IIR
Carboxylated NBR	Karboxylerad NBR	XNBR
Chlorinated butyl rubber	Klorbutylgummi	CIIR
Chloropolyethylene	Kloretengummi	CM
Chlorosulphonated polyethylene	Klorsulfonerad polyeten	CSM
Epichlorohydrine copolymer rubber	Epiklorhydringummi (sampolymer)	ECO
Epichlorohydrine-ethylene oxide	Epiklorhydringummi (sampolymer)	GECO
Epichlorohydrine homopolymer rubber	Epiklorhydringummi (homopolymer)	CO
Epoxidised natural rubber	Epoxiderat naturgummi	ENR
Ester polyurethane	Uretangummi	EU
Ethylene acrylate copolymer	Akrylgummi	ACM
Ethylene acrylic terpolymer	Eten-akrylgummi	AEM
Ethylene copolymer	Etenpropengummi	EPM
Ethylene propylene diene terpolymer	Etenpropen-diengummi	EPDM
Ethylvinyl acetate	Eten-vinylacetatgummi	EVM
Fluoro, fluoroalkyl or fluoroalkoxy	Fluorgummi	FPM
Fluoromethyl silicone	Fluorsilikongummi	FMQ
Hydrogenated NBR	Hydrerat nitrilgummi	HNBR
Isoprene rubber	Isoprengummi	IR
Methyl silicone with vinyl groups	Silikongummi	VMQ
Natural rubber	Naturgummi	NR
Perfluoro rubber	Perfluorgummi	FFKM
Phenyl/vinyl methyl silicone	Silikongummi	PVMQ
Polychloroprene	Kloroprengummi	CR
Polyester polyurethane	Uretangummi	AU
Polyfluoralkoxyphosphazene	Polyfosfazengummi	FZ/PZ
Polynorbornene	Norbornengummi	PNR
Polysulphide rubber	Polysulfidgummi	OT/EOT
Propylene oxide/allyl glycidyl ether	Propengummi	GPO
PVC/NBR blends	Nitril-PVC-blandning	PNBR
Styrene butadiene rubber	Styrenbutadiengummi	SBR
Tetrafluoroethylene propylene rubber	Tetrafluoretenpropengummi	FEPM

Table 2.1: *Names and abbreviations for common rubber materials.*

2.2 Selecting the most suitable rubber

To select the most suitable rubber the following information is required:

a) The highest temperature likely to be encountered in service.

b) The highest temperature at which continuous service will be required.

c) The lowest temperature at which the component must remain operable.

d) What fluids will be encountered in service and at what temperature?

e) Is the frequency of contact with the fluid continuous, intermittent, or very occasional (e.g. accidental contamination)?

f) Is long-term weather or ozone resistance an important factor?

Tables 2.2 and 2.3 below shows properties of the rubbers described in section 2.1 as related to these questions. The cost factor is also indicated in the table.

	NR/ IR	SBR	BR	EPM/ EPDM	PNR	GPO	IIR	BIIR/ CIIR	NBR	NBR/ PVC	HNBR	XNBR	CR	CM
Max temp (^{o}) 1000h	80	90	80	120	60	125	90	90	90	80	125	90	85	125
Min temp (^{o}), *)	-50	-40	-75	-35	-50	-55	-45	-45	-30	-25	-25	-25	-35	-10
Tear resistance	4	3	3	3	2	3	3	3	3	3	3	4	3	3
Permanent set at -30 - +70^{o}C	4	4	4	3	2	2-3	2	2	3	2	2-3	3	3	3
Permanent set at +70 - +150^{o}C	1	1	1	2-3	1	2-3	2	2	3	2	4	3	2-3	3-4
Permanent set after 1000 h	50a)			75b)					50b)				60b)	
Abrasion	4-5	4-5	5	3	3	2	3	3	3	3	3	4	3-4	3
Oil and petrol resistance	1	1	1	1	1	2	1	1	3-4	4	3-4	3-4	2-3	2
Gas diffusion inert gas	2	2-3	2	2	2	2	5	5	4	4	4	4	3-4	4
Oxidation ability	2	2	2	5	2	3-4	3-4	3-4	2-3	3	5	2-3	3-4	5
Weather and ozon resistance	1-2	1-2	1-2	5	1-2	4-5	4	4	1-2	3-4	5	1-2	3	4
Heat ageing	1-2	1-2	1-2	3-4	1-2	3-4	3-4	3-4	3	3	4	3	3	3-4
Cold stiffening	4-5	4	5	4	3-4	4	3-4	3-4	2-3	2	2-3	2	2-3	2
Water resistance	4	4	4	5	4	3-4	4	3	3-4	3	3-4	3-4	2-3	2-3
Fire resistance	1	1	1	1	1	2	1	1	1	2-3	1	1	3	3
Rebound elasticity, cold	5	3	5	3	3	5	1	1	3	2	3	3	3	2
Rebound elasticity, warm	5	3	5	3	3	5	3	3	3	2-3	3	3	4	3
Dynamic fatigue	4	4	4	3	3	3-4	3	3	2	2-3	4	2	4	2
High friction	5	4	3	3	4	3-4	4	4	3-4	3	3-4	3-4	3-4	3
Adhesion to metal	4-5	4-5	4-5	3	4-5	3	2	2	4	3	3	4	4	3
Adhesion to textile	4-5	4-5	4-5	3	4-5	3	2	2	3-4	3	3	3-4	4-5	3
Price indication, volume index	1	1	1	1,2	1,3	3,3	1,3	1,8	1,4	1,5	11,7	2,1	2,1	1,8

Table 2.2: *General properties for different rubber materials. 5 - Excellent, 4 - Very good, 3 - Good, 2 - Less good, 1 - Bad. *) 10 times cold stiffening.*

	CSM	ACSM	CO/ ECO/ GECO	ACM	AEM	EVM	AU/ EU	FPM	FFKM	FEPM	PMQ/ VMQ	FMQ/ FVMQ	FZ/ PZ	OT/ EOT
Max temp (°) 1000h	125	125	120	140	150	150	70	200	250	200	200	175	160	80
Min temp (°), *)	-40	-45	-40	-20	-30	-30	-10	-15	-15	-5	-80	-55	-55	-40
Tear resistance	3	3	2-3	2-3	2	3	4-5	1-2	2	4	1	1	2	1
Permanent set at -30 - +70°C	2	2	2-3	3	4	2	3	2-3	2-3	2	4	3	3	1-2
Permanent set at +70 - +150°C	2	2	3	3	4	2	1-2	5	3-4	3-4	4	4	4	1-2
Permanent set after 1000 h					50c)			50e)			50d)			
Abrasion	3	3	3	2-3	3	2	5	3	3	3	1-2	1	2	1
Oil and petrol resistance	2-3	2-3	4	4	3	2	4-5	5	4	4	2-3	4	3-4	4-5
Gas diffusion inert gas	3	3	4-5	2	2-3	1	4	3	4	-	1-2	3	4	3
Oxidation ability	5	5	4	4	4	5	3	5	5	5	5	5	4	1-2
Weather and ozon resistance	4	4	4	3	4	5	5	5	5	5	5	5	4	4
Heat ageing	3-4	4	3	4	4	2	3	5	5	4-5	4-5	4-5	4	2-3
Cold stiffening	2-3	2-3	3-4	2	3	3	2	1-2	1-2	1-2	5	4	4	3-4
Water resistance	3	3	4	2	2	3	2	3	3	4	2	2	2-3	3
Fire resistance	2	2	2	1	2	1	2-3	4	4	-	2	3	5	-
Rebound elasticity, cold	3	3	3-4	2	1	-	1	2	2	1	5	4	4	2
Rebound elasticity, warm	3	3	5	4	2	-	3	4	4	2	5	4	4	1-2
Dynamic fatigue	3	4	3	2-3	3	2	4	1-2	1-2	3	1-2	2	3	1
High friction	3	3	3	3	3	3	1	2	2	-	3-4	3	-	-
Adhesion to metal	4	4	3	3	3	4	5	2	2	2-3	2	2	3	3
Adhesion to textile	3	3	3	3	3	4	5	2	3	-	2	2	-	-
Price indication, volume index	3,3	4,3	5	4,2	3,8	2,5	4,2	33	1650	45	5,8	90	225	3

Table 2.3: *General properties for different rubber materials, continued. 5 - Excellent, 4 - Very good, 3 - Good, 2 - Less good, 1 - Bad. *) 10 times cold stiffening.*

2.3 TPE-materials

Although the first TPE materials appeared on the market already during the 1950s, the variety of different TPE materials that have become available has increased explosively with the introduction of new manufacturing techniques. The development of increasingly specialized materials with improved characteristics, however, can sometimes make it more difficult to find the right material for a particular application. A comprehensive review on thermoplastic elastomers has been given by Holden et al. [9].

What is TPE?

TPE stands for ThermoPlastic Elastomers. These represent polymer chains consisting partly of soft and partly of hard molecule segments. The soft segments are responsible for the rubberlike behavior of the material, whereas the hard segments, which correspond to the crosslinks hold it together to form an elastic material. What makes TPE material so special is the fact that, when the hard segments are warmed up, they assume a fluid character, allowing the material to be moulded freely into shape, just as a usual thermoplastic such as polyethylene can be. This is the major difference between TPE and usual rubber material. When cooled, the material becomes solid again, and rubberlike in its characteristics. Since such a process can be repeated, TPE material can be recycled far more easily than vulcanized rubber can. A limitation is, however, that each warming up results in a thermal stress on the polymer, which leads to a certain breakdown of the material and thereby lower properties. Thus, the material cannot be recycled indefinetely without a deterioration in its characteristics.

2.3.1 Temperature resistance

The highest temperature at which a TPE material can be used is a function of the temperature at which the hard segments in the polymer chains soften and begin to flow. The polymer chains become readily movable so that they are free to separate from each other. This results in the material's losing its mechanical characteristics. Even at a temperature below that at which the hard segments soften, however, the mechanical characteristics of the material begin to deteriorate. Particularly when TPE material is placed under strain for long periods of time, its deformation resistance becomes inferior to that of vulcanized rubber material, i.e. rubber material in which crosslinks are present.

2.3.2 Resistance to solvents

In order for a TPE material to be resistant to a particular solvent, the hard segments must be resistant to it. Since there are no chemical crosslinks between the polymer chains, the material would otherwise break down completely. Advances in the TPE area during the past few years, however, have led to the development of increasing numbers of TPE materials that are resistant to both oil and solvents. It is unlikely, on the other hand, that TPE materials will come to replace vulcanized rubber in all applications in which resistance to solvents is called for.

2.3.3 Commercial names

Since an intensive development of new TPE materials is underway, new materials, new quality levels and new commercial names for such materials appear all the time. This sometimes makes it difficult to choose a suitable material within the jungle of different names that flourish. A list of the different TPE materials presently found and of their commercial names, as well as of the companies manufacturing them, is contained in Table 2.4. Explanations of abbreviations used in the Table are given below:

Alcryn	Chlorinated double-bonded ethylene interpolymer mixture
EPDM	Ethylene/propylene diene-monomer
PP	Polypropene
EVA	Ethylene vinylacetate
TPU	Termoplastic polyurethane
SBS	Styrene butadiene styrene
SEBS	Styren ethylene butadiene styrene
TPO	Thermoplastic olefin

Commercial name	**Type**	**Manufacturer**
Acsium	Alcryn*	DuPont Dow
Adiprene	Polyether TPU	Dupont
Alcryn	Alcryn*	DuPont Dow
C Flex	SEBS	Notdome
Calprene	SBS	Repsol
Deflex	TPO	RPI/Dexter
Desmopan	TPU	Bayer
Diorez	TPU	Hyperlast
Diprane	TPU	Hyperlast
Dryflex	SEBS	Elastoteknik
Dynathane	TPU	Hyperlast
Dytron	Alcryn*	AES
Elvax	EVA	DuPont Dow
Escorene Ultra	EVA	Exxon
ET- semicon	Alcryn*	Borealis
Ferroflex	TPO	Ferro Corp
ET-polymer	Alcryn*	Borealis
Finaclear	SBS	PetroFina
Finaprene	SBS	PetroFina
Forflex	Modif. PP	So.F.Ter.
Forprene	PP+EPDM	So.F.Ter.
Geolast	Alcryn*	AES
Hytrel	Polyether-ester	Dupont
Kelburou	TPO	DSM
Laprene	SEBS	So.F.Ter.
Laripur	TPU	COIM
Levaprene	EVA	Bayer
Pellethane	TPU	Dow Chemicals
Polathane	TPU	Polaroid
Poytrope	TPO	A. Schulman
Santoprene	EPDM/PP	AES
SEBS Compound	SEBS	Borealis
Sofprene	SBS	So.F.Ter.
Solprene	SBS	Housmex
Telcar	TPO	Teknor Apex
Texin	TPU	Bayer
TPR	TPE	Uniroyal
Ureflex	TPU	Baul
Vector	SBS	Dexco
Vestolen	TPO	Chemische Werke Huls
Vistaflex	TPO	AES
Vitacom TPO	TPO	British Vita
Vyram	EPDM/PP	AES

Table 2.4: *TPE: Commercial name, type and manufacturer.*

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Chapter 3

Elastic properties and static design

P.-E. AUSTRELL

This chapter, which concerns static elastic analysis methods, is divided into two sections, the first dealing with spring stiffness expressions and the second with finite element analysis. A good description of the elastic behavior of rubber is essential for the designing of rubber units, which usually begins with some form of static elastic analysis with the aim of deriving a particular load-deflection curve or of determining the stress levels in a unit. This is achieved either by closed-form expressions or by computer calculation methods such as the finite element method.

The first section focus on spring stiffness expressions and closed form solutions and the second section in this chapter concerns nonlinear finite element analysis.

3.1 Spring stiffness expressions

Spring stiffness expressions and closed form solutions are treated in most general rubber engineering texts, such as in [6], [7], [8], and [13] which also presents formulas for simple geometries and load cases. A short overview of spring stiffness expressions is provided by Muhr [16].

Gent [8] (chapter 3) discusses these matters from a linear elastic point of view, making use of linear elasticity theory to estimate the stress levels in bonded rubber blocks subjected to tension or compression.

Matters within this area are also discussed by Grafström in [7] (chapter 8.1) who present semi empirical diagrams concerning the design of rubber springs of differing hardness. Recommended stress levels for different shape factors are shown in diagrams.

Linear elastic relations are used in hand calculation methods for rubber springs. Although rubber is known for its ability to undergo large elastic deformations, in practice the strains to which many rubber springs are subjected to are rela-

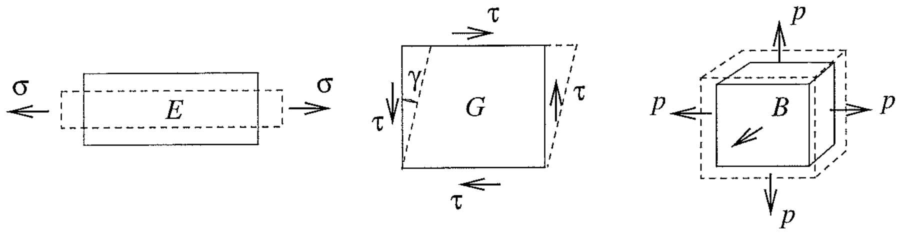

Figure 3.1: *Basic load cases and corresponding elastic constants.*

tively small. The spring stiffness expressions discussed in this section are valid for strains of up to about 25% in compression or extension and of up to about 75% in shear (according to Gent [8] Chapter 3 p.35). Suitable modifications of the formulas are required for taking nonlinearities and inhomogenities into account.

3.1.1 Basic linear elastic relations

The basic assumtions are that the material can be regarded as a linear elastic, isotropic, and homogeneous. With these assumptions it is sufficient with two parameters to characterize the material. The use of Young's modulus E and Poisson's ratio ν as material parameters is not suitable here. Young's modulus varies considerably with strain.

The shear modulus G and the bulk modulus B are more suitable as characteristic material constants since rubber is quite linear in shear and its volumetric strains are small so that G and B can be regarded as basically constant material parameters (c.f. Figure 1.4).

Uniaxial stress

The first linear elastic relationship (c.f. Figure 3.1 at the left) used for deriving spring stiffness expressions is

$$\sigma = E\epsilon, \tag{3.1}$$

which is the conventional Hookes law valid for uniaxial stress, where $\sigma = P/A$ is the stress, represented by the force P, divided by the cross section area A, and where $\epsilon = \delta/L$ is the strain in the direction of the load. Young's modulus can be expressed then in terms of the bulk and shear modulus as

$$E = \frac{9BG}{3B+G} \tag{3.2}$$

since only two constants are required for caracterizing the material. Note that (3.2) implies that for large values of the bulk modulus B, i.e. when incompressibility is approached, $E = 3G$.

Simple shear

The second relationship used is the expression for simple shear

$$\tau = G\gamma \tag{3.3}$$

where the loading is that shown in Figure 3.1 (middle).

Hydrostatic stress

The third relationship is valid when the elastic response is due to the bulk modulus of the material. This is the case for very confined rubber springs, for example springs consisting of thin layers of rubber as in some laminated rubber metal components. The connection between hydrostatic stress p and volumetric strain $\Delta V/V$ is given by

$$p = B\frac{\Delta V}{V} \tag{3.4}$$

where B is the bulk modulus. Hydrostatic loading is illustrated in Figure 3.1 (to the right).

These linear elastic expressions are the basis for the spring stiffness expressions taken up in the next section.

3.1.2 Compression and shear springs

Expressions for the stiffness of compression, shear, and combined shear and compression springs will be discussed.

Compression springs

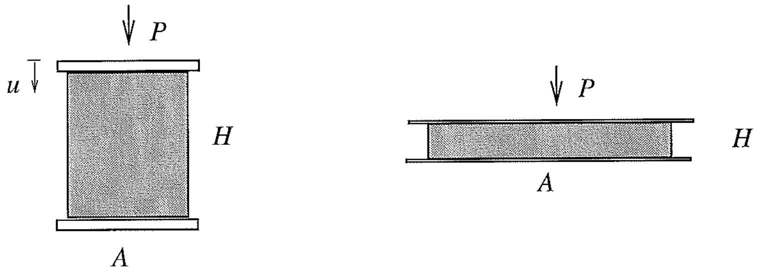

Figure 3.2: *Compression springs with different shape factors. Cross sectional area A and height H.*

Figure 3.2 shows compression springs of different width-to-height ratios. The lateral expansion is not particularly restrained in the compression spring at the left, whereas the spring at the right is highly confined between the bonding surfaces. It is reasonable to assume, therefore, that Hooke's law (3.1) is approximately valid (for moderate strains) for the spring to the left and that the volumetric stiffness expression given by (3.4) is important for the spring at the right in the figure. Using (3.1) and inserting it into $\sigma = -P/A$ and $\epsilon = -u/H$ yields the following stiffness expression valid for the spring at the left

$$P = K_c u \quad \longrightarrow \quad K_c = \frac{3GA}{H} \tag{3.5}$$

Hardness IRHD	E_0 (MPa)	G (MPa)	k	B (MPa)
30	0.92	0.30	0.93	1000
35	1.18	0.37	0.89	1000
40	1.50	0.45	0.85	1000
45	1.80	0.54	0.80	1000
50	2.20	0.64	0.73	1030
55	3.25	0.81	0.64	1090
60	4.45	1.06	0.57	1150
65	5.85	1.37	0.54	1210
70	7.35	1.73	0.53	1270
75	9.40	2.22	0.52	1330

Table 3.1: *The factor k related to the stiffness of carbon-black-filled natural rubbers, according to Lindley*

the spring stiffness K_c depending on Young's modulus, which is equal to $3G$. In the highly confined case, the bulk modulus equation (3.4), with $p = -P/A$ and $\Delta V/V = -Au/AH = -u/H$, yields

$$P = K_c u \quad \longrightarrow \quad K_c = \frac{B\,A}{H}\,. \tag{3.6}$$

There is thus a considerable difference between the compression stiffness obtained in the two cases since $B \approx 1500G$.

The transition between the influence of Young's modulus to that of the bulk modulus can be described by the so called shape factor S, defined as

$$S = \frac{loaded\ area}{force\ free\ area}. \tag{3.7}$$

By definition, the loaded area is restricted to the upper bonded surface exposed to the force P, the force free surface being the vertical surrounding (bulging) surface shown in Figure 3.2. By defining the so called compression modulus E_c, which is dependent upon the shape factor, the stiffness of the compression springs obtained for various shape factors can be described in a single expression

$$K_c = E_c(S)\frac{A}{H}\,. \tag{3.8}$$

According to Lindley [13] p.33, the dependence of the compression modulus on the shape factor can be expressed as

$$E_c(S) = E\,(1 + 2k\,S^2) \tag{3.9}$$

where k is a factor between 0.9 and 0.5 which decreases with increasing rubber hardness, as given in [13] Table 3 p.8., reproduced here as Table 3.1. A similar table is given in [8] Chapter 8 p.215 of Sheridan et. al.

Lindley [13] also presents a nonlinear expression for compression springs

$$P = E_c A e(1 + e) \tag{3.10}$$

where $e = u/H$ is the compression strain. For small values of e, the slope of the force displacement curve given by expression (3.10) coincides with the linear compression stiffness as given by (3.8).

Shear springs

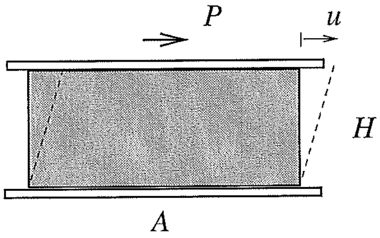

Figure 3.3: *Shear spring.*

The basic expression used for the shear spring is Equation (3.3). Inserting $\tau = P/A$ and $\gamma = u/H$ yields the stiffness expression valid for the shear spring

$$P = K_s u \quad \longrightarrow \quad K_s = \frac{G\,A}{H}\,. \tag{3.11}$$

Expression (3.11) is valid as long as bending does not substantially influence the mode of deformation, requiring that the height is small compared with the width.

For heigths that are sufficiently small, the forces on the vertical surfaces that are present in the ideal state of simple shear, shown in Figure 3.1, are insignificant.

The transition from bending deformation in which the shear spring is relatively high to the simple shear state in which the height is small is considered by Gent p.40 and by Sheridan et. al. [8] p.213.

……………………………………………………

Case 1

Determine the compression stiffness K_c of a rubber cylinder of radius R and height H bonded by steel plates. The initial Young's modulus E_0 is assumed to correspond to approximately 50 IRHD material. Data for the cylinder are shown in Figure 3.4

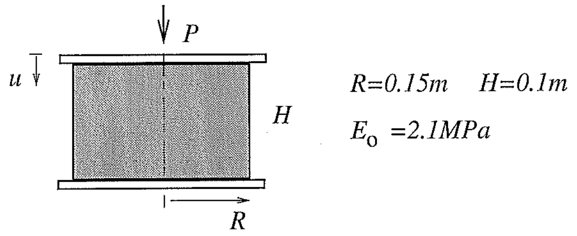

Figure 3.4: *Cylindrical compression spring.*

Solution:

The spring compression stiffness K_c is given by (3.8)

$$K_c = E_c(S)\frac{A}{H} \quad with \quad E_c(S) = E\,(1 + 2k\,S^2) \tag{3.12}$$

where according to Table 3.1 the factor $k = 0.75$ for $E_0 = 2.1MPa$. For the cylindrical component, the shape factor is

$$S = \frac{\pi R^2}{2\pi RH} = 0.75$$

The compression stiffness can now be calculated:

$$K_c = 2.1(1 + 2 \cdot 0.75 \cdot 0.75^2)\frac{\pi \cdot 15^2}{10} = 273N/mm\ .$$

This linear stiffness expression is compared with the nonlinear finite element solution presented in Case 2 in Section 3.2.

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Combined shear and compression springs

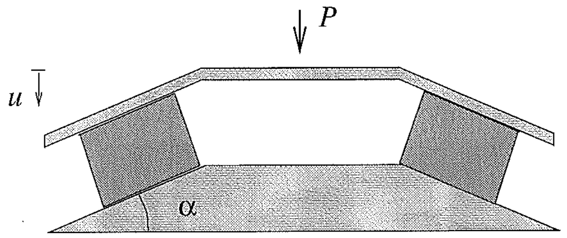


Figure 3.5: *Combined compression and shear spring.*

The combined shear and compression spring will be illustrated here by an example. Figure 3.5 shows two rubber blocks subjected to combined shear and compression due to the inclined mounting by an angle α.

Compression and shear are considered independently, giving the force components

$$P_c = K_c u_c \quad P_s = K_s u_s \tag{3.13}$$

in the compression and shear direction, respectively, dependent on the corresponding displacements. In these components, the external load P can be expressed on the basis of equilibrium considerations as

$$P = 2P_c cos\alpha + 2P_s sin\alpha\ . \tag{3.14}$$

The displacements in the compression and shear directions are given by

$$u_c = ucos\alpha \qquad u_s = usin\alpha \tag{3.15}$$

where u is the displacement in the direction of the load P. Combining the equations yields the spring stiffness expression K for this combined load case

$$K = 2(K_c cos^2\alpha + K_s sin^2\alpha) \tag{3.16}$$

where it can be readily seen that the stiffness depends on the angle α.

3.1.3 Other stiffness expressions

Several other cases are considered in the references cited earlier, including many axisymmetric cases. In the following, various formulas taken from Muhr [16] which also involve the torsional stiffness of some axisymmetric components are given. In the case of torsional stiffness the stiffness expression correlates a torsional moment to a torsional angle, the spring stiffness being denoted by T.

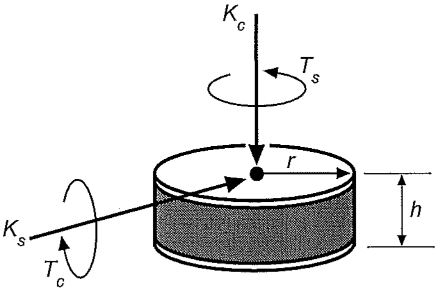


Figure 3.6: *Cylindrical bearing.*

As shown in Figure 3.6, the cylindrical bearings have "compressive" stiffnesses K_c and T_c, which are affected by the shape factor, and "shearing" stiffnesses K_s and T_s which are not.

The expressions involved are

$$K_c = \frac{3\pi r^2 G(1+2S^2)}{h} \qquad T_c = \frac{3\pi r^4 G(1+\frac{2}{3}S^2)}{4h}$$

and

$$K_s = \frac{\pi r^2 G}{h} \qquad T_s = \frac{\pi r^4 G}{2h}$$

where r is the radius and h is the height of the cylindrical component and the shape factor is $S = r/2h$.

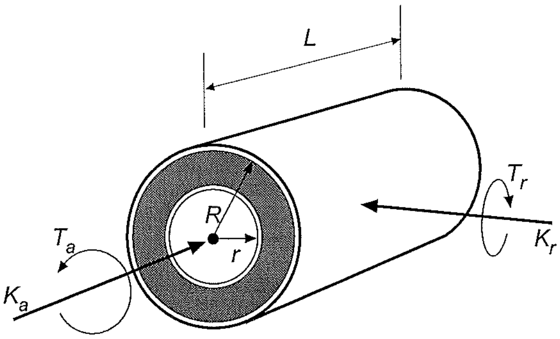


Figure 3.7: *Bush mounting.*

A hollow cylindrical bush mounting with the outer radius R, the inner radius r, and the length L is shown Figure 3.7. It has the axial stiffnesses K_a and T_a

which are not affected by the shape factor and the radial stiffnesses K_r and T_r which are affected by the shape factor.

The expressions for the axial stiffnesses are

$$K_a = \frac{2\pi \cdot GL}{ln(R/r)} \qquad T_a = \frac{4\pi GL}{\frac{1}{r^2} - \frac{1}{R^2}} .$$

The expressions for the radial stiffnesses are

$$K_r \approx \beta LG \qquad T_r \approx \frac{1}{12}\beta L^3 G$$

where it is assumed that $L > 2R$ and the factor β is given by the smallest of the following expressions

$$\beta = \beta_l \quad or \quad \beta = \beta_s + (\beta_l - \beta_s)\frac{0.08L}{R - r}$$

with

$$\beta_s = \frac{80\pi(R^2 + r^2)}{25(R^2 + r^2)ln(R/r) - 9(R^2 - r^2)} \qquad \beta_l = \frac{4\pi(R^2 + r^2)}{(R^2 + r^2)ln(R/r) - (R^2 - r^2)} .$$

3.2 Finite element analysis

In this section finite element analysis, as well as material models describing the nonlinear elastic behavior of rubber materials related to the derivation of material constants for finite element analysis, are discussed.

For rubber units with complex geometry and loading, hand calculation methods are of limited value, computer methods neding to be used. The insufficiencies of closed form solutions as compared with computer methods in more complicated cases are discussed by Finney [5].

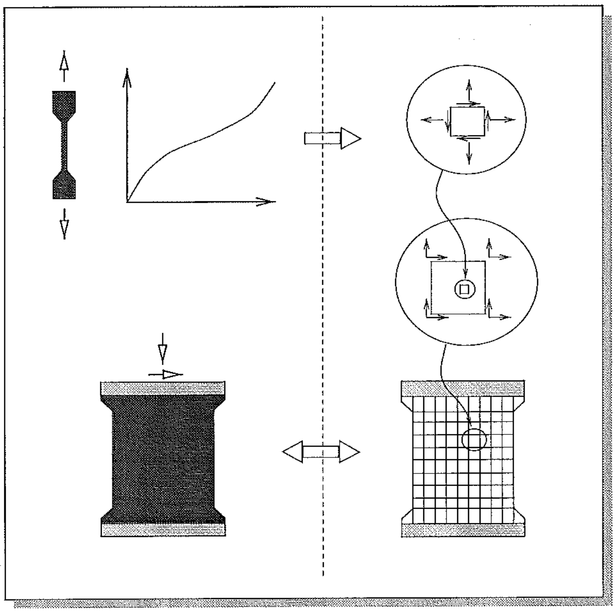


Figure 3.8: *Material characterization, finite element analysis, and comparative testing.*

The finite-element method is the most versatile and successful computational method used in solid mechanics. It provides a systematic procedure for analysing structures of different types of material and arbitrary geometric form. The structure to be analyzed is divided into smaller parts, or elements, connected at nodal points. The force-displacement relations for a particular element, i.e. the connection between nodal forces and displacements, can only be obtained if the relation between stress and strain in the element is known. This relationship is provided by the constitutive model (cf. Figure 3.8).

The steps involved in the finite element analysis of rubber units are shown in Figure 3.8 and listed here. These are:

- Laboratory tests of rubber specimens.
- Determination of the parameters of the mathematical model for the elastomeric material in question.
- Establishment of the finite-element model through geometric modeling, material data input, and load specification.

The first step concerns the experimental stress-strain values for the elastomeric material in question. Test specimens made of the same material as the rubber unit to be designed are manufactured and are tested in the laboratory.

In the second step the pairs of measured stress-strain values obtained are used to calibrate the hyperelastic model, i.e. the mathematical model connecting stress and strain. The hyperelastic model is fitted to the experimentally obtained stress-strain values, and the parameters in the model are determined by some fitting procedure, such as *the method of least squares.* Well-established hyperelastic constitutive models, derived from a strain energy function, are implemented in many commercial finite element codes.

The first two items above result in a set of parameters defining the constitutive model of the elastomeric material. These parameters, together with a geometric description and a specification of the loads, provide the necessary input for the finite element analysis in step three.

A simple introduction to rubber elasticity and to fitting to test data, relevant to the first two steps listed above, can be found in [1] Chapter 2.

An example of the capabilities finite element analysis provides is given in Figure 3.9. The figure shows a simulation of the mounting and compression of a seal used for concrete pipes. Rigid elements are employed in the mounting and compression steps. The analysis involves several intricate features of finite element analysis, such as large strains, nonlinear material behavior, contact conditions, and nearly incompressible behavior. The analysis can among other things, reveal whether the contact pressure between the pipe and the seal is sufficiently large.

The handling of incompressible material behavior is a specific problem in finite element analysis. A large difference between deviatoric (shear) and volumetric (bulk) stiffness tends to make ordinary elements far too stiff, and even complete "locking" can be encountered. Special "hybrid" elements have been developed that can circumvent the locking phenomena.

Several general-purpose finite element codes are available for nonlinear elastic analysis. Among the best known are ABAQUS, ANSYS, MARC and NASTRAN, to mention some of the best known.

The well-known texts of Zienkiewicz and Taylor (vol.1 and 2) [24] provide a comprehensive treatment of different aspects of finite element analysis. An introduction to the finite element method can be found in [18].

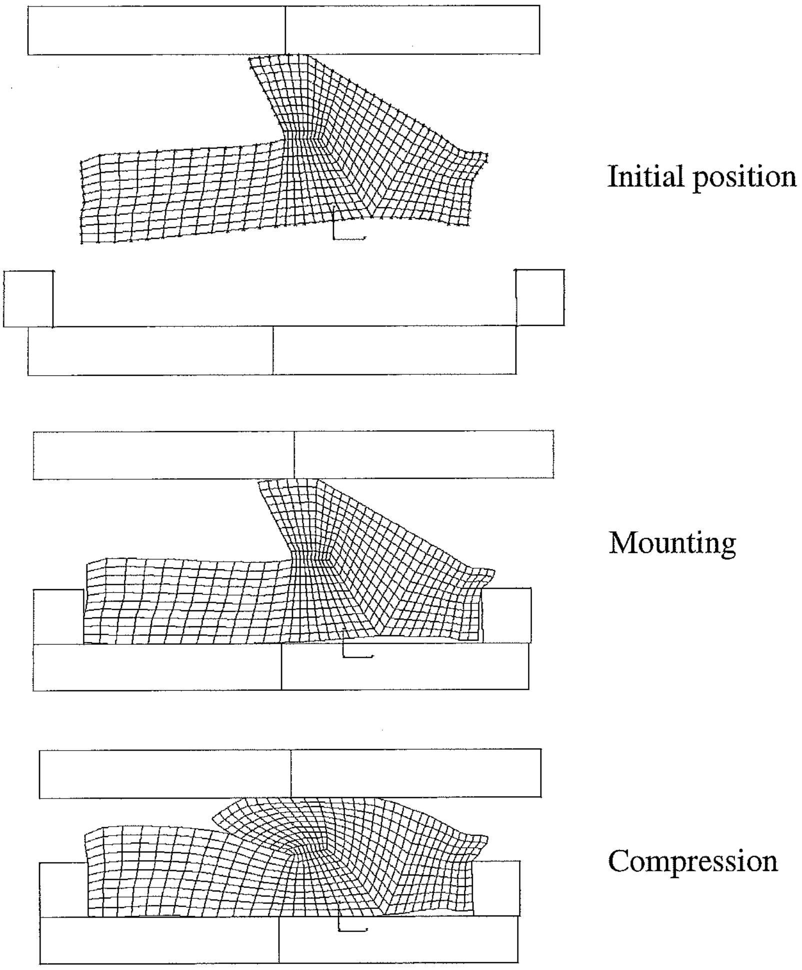


Figure 3.9: *Example: Analysis of a seal by means of computer simulation using the finite element method.*

3.2.1 Nonlinear elastic material models

A theory of rubber elasticity, termed statistical theory based on the molecular structure of the chain network, was developed by Treloar [21] and by others. This theory resulted in a strain energy expression containing only one material parameter, which can be identified as the shear modulus. Mooney [15] and Rivlin [19] developed the phenomenological theory of rubber elasticity. Mooney initiated this work, Rivlin developed a general theory then on the basis of a strain energy density expression, postulated without molecular considerations. The strain energy density function is regarded as a potential function for the stresses, the elastic properties being determined on the basis of this function in these so called *hyperelastic materials* developed by Rivlin.

3.2.2 Strain energy density expressions

A function W, representing the *strain energy per unit volume in the reference geometry* usually the undeformed geometry is introduced. Two general strain energy function formats will be discussed here: the polynomial form and the Ogden form. These functions are expressed in terms of strain invariants and principal stretches, respectively.

The general polynomial form of the strain energy density function W, implemented in most of the general finite element programs capable of handling hyperelastic materials, is given by the series expansion

$$W = \sum_{i=0,j=0}^{\infty} C_{ij}(I_1 - 3)^i (I_2 - 3)^j \tag{3.17}$$

where C_{ij} are constants and I_1 and I_2 represent the so called strain invariants, which are generalized measures of strain. (The sum is formally written as a sum to infinity, although normally only a few terms are used.)

The general form of the Ogden model [17] is also expressed by a series expansion, although the strain measure, as mentioned, is replaced by the principal stretches λ_i according to

$$W = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3)\,, \tag{3.18}$$

with the material constants α_i and μ_i . The Ogden model can in general not be compared with the polynomial form, except for certain specific choices of the constants.

In the polynomial form, the elastic constants are linear dependent, whereas the Ogden form, have the constants as exponents, yielding a non-linear dependence of the constants. Although this makes the fitting of the Ogden model more complicated, the fit to experimental data obtained tends to be better and more stable. However, for natural rubber vulcanizates, the special choice of constants by Yeoh [23], disscussed below, with the three parameters C_{10}, C_{20} and C_{30} in the polynomial form, gives an accurate and stable fit to experimental data.

3.2.3 Specific models

A large number of strain energy density functions applying to different types of rubbers have been proposed, their validity being dependent upon the range of strain involved. Several of these choices with differing number of constants are discussed by Charlton and Young [3], Finney and Kumar [4], and Treloar [21]. Some of these hyperelastic models are also discussed in [1] Chapter 3.

The Mooney-Rivlin model, having two parameters, C_{10} and C_{01}, is a special case of the polynomial form. It is widely used in modeling of rubber elasticity, and it has been successfully fitted to experimental data for unfilled rubber vulcanizates. In engineering problems, however, rubbers are often carbon-black-filled vulcanizates of hardness 40-80 IRHD and for these rubbers, the agreement with experiments obtained for this simple model is poor. Figure 3.10 shows a comparison of the two-parameter Mooney-Rivlin model with the actual behavior of a carbon black-filled rubber having a hardness of about 65 IRHD. For large stretch values, the Mooney-Rivlin model is linear in tension whereas the actual behavior of a carbon-black-filled rubber specimen is progressive. There is also a deviation in simple shear, the Mooney-Rivlin model showing a linear behavior in simple shear.

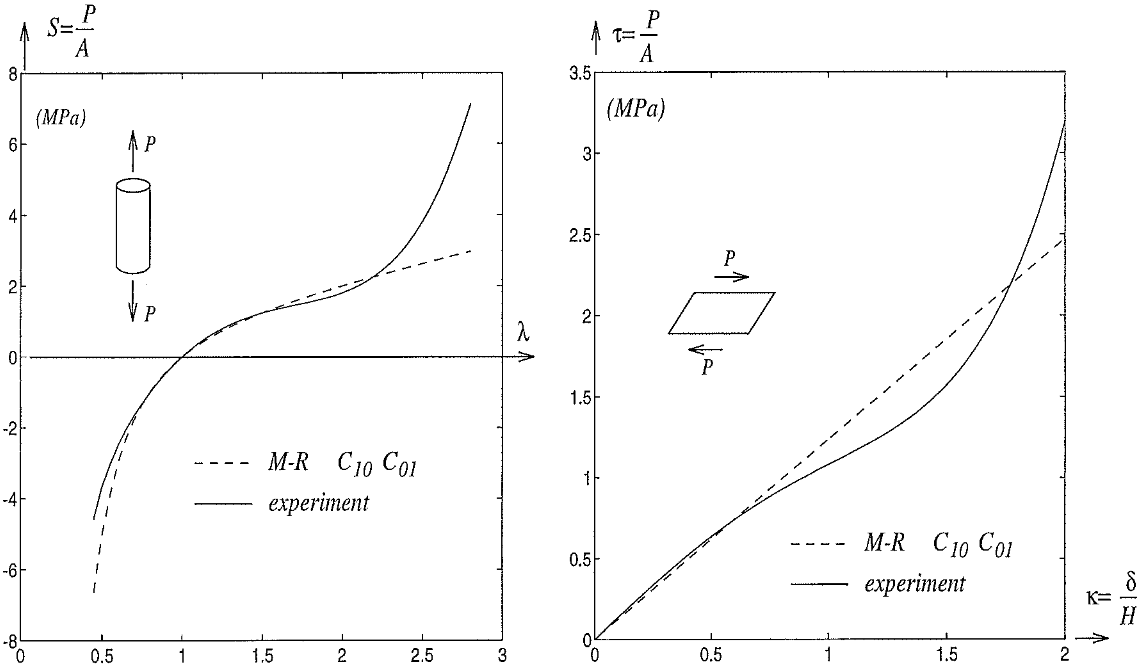


Figure 3.10: *The Mooney-Rivlin model compared to actual behavior of carbon-black-filled rubber in compression/tension and simple shear.*

The three-parameter model proposed by O.H Yeoh [23], defined by

$$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 \tag{3.19}$$

yields the qualitative behavior of a carbon-black-filled rubber vulcanizate shown in Figure 3.10. This model yields correct physical behavior and is quite simple having only three constants, which are linearly dependent.

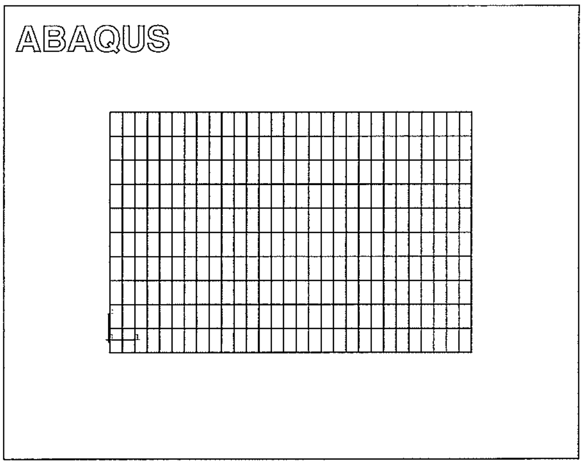
……

Case 2

The ABAQUS input file required to determine the force-displacement relation for the cylindrical damper in Case 1 Section 3.1 is given here:

```
*HEADING
  Cylinder R=15 H=10,
  Yeoh material (50 IRHD)
**
** Corner nodes
**
*NODE
  1,0.,0.
  30,15.,0.
  1001,0.,10.
  1030,15.,10.
**
** Boundary node sets
**
*NGEN,NSET=BOT
  1,30,1
*NGEN,NSET=SYM
  1,1001,100
*NGEN,NSET=TOP
  1001,1030,1
*NGEN,NSET=OUT
  30,1030,100
**
** Internal nodes.
**
*NFILL,NSET=ALL
  SYM,OUT,29,1
**
** Element generation
**
*ELEMENT,TYPE=CAX4H
  1,1,2,102,101
*ELGEN,ELSET=RUB
  1,29,1,1,10,100,100
**
** Material definition
**
*MATERIAL,NAME=YEOH
*HYPERELASTIC,N=3
  0.3493,0.0,-0.0363,0.0,0.0,0.0069
  0.0,0.0,0.0,0.00,0.0,0.0
*SOLID SECTION,ELSET=RUB,MATERIAL=YEOH
**
** Fixed boundary conditions
**
*BOUNDARY
  BOT,1,2
  TOP,1
  SYM,XSYMM
```

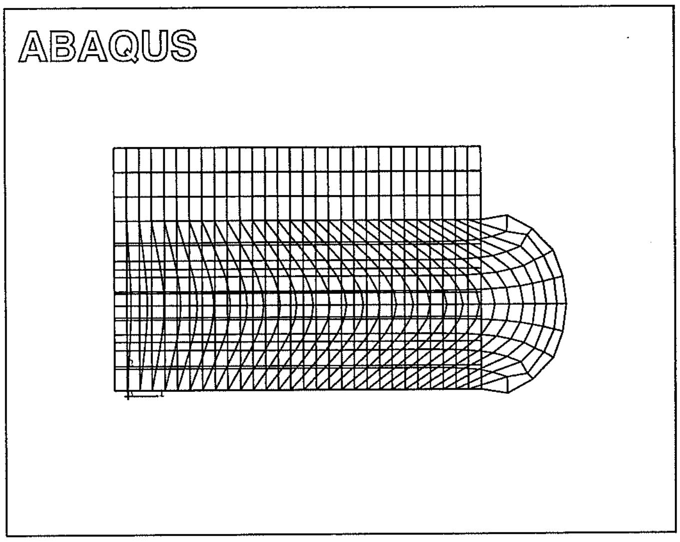
The axisymmetric mesh (radius 15 mm and height 10 mm) is defined *NODE, *NGEN, *NFILL, *ELEMENT, and *ELGEN instructions, giving the mesh below:



The material parameters C_{10}, C_{20}, and C_{30} are supplied in the *HYPERELASTIC instruction.

```
**
** Calculation step
**
*STEP,NLGEOM
*STATIC
  0.1,1.,1e-10,0.1
*BOUNDARY
  TOP,2,,-5.
**
** Results: force and displacement
**
*NODE PRINT,NSET=TOP
  U
*NODE PRINT,NSET=BOT,TOTALS=YES
  RF
*END STEP
```

The calculation step yields the deformed mesh below (30% compression):



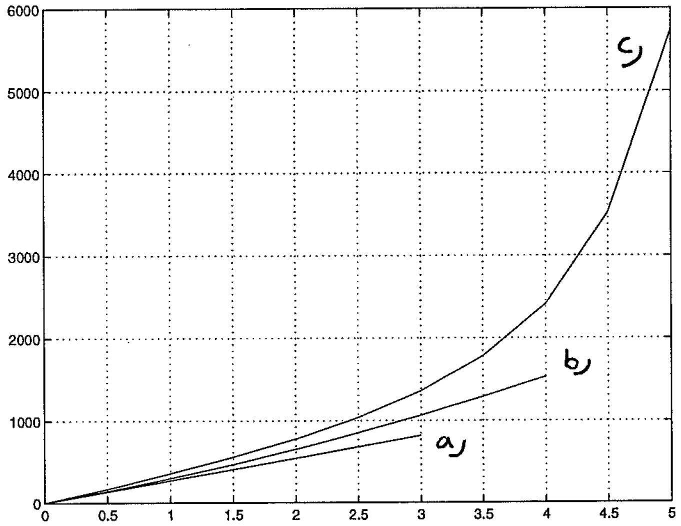


Figure 3.11: *Cylindrical damper. Comparison of force(N)-displacement(mm) relations a) linear Eq. (3.8) b) nonlinear Eq. (3.10) c) present finite element analysis. (Increasing stiffness a) to c))*

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3.3 Hyperelastic characterization

Hyperelastic characterization involves the choice of a strain energy density function, the type of test specimens to use, and the test procedure to employ. A review of methods for characterizing the elastic behavior of rubber is given by Charlton and Young [3]. A short but comprehensive description of the steps involved in hyperelastic characterization is provided by Holst [10]. Discussions of the characterizing of rubber can also be found in the documentation of finite-element programs, for example [9] and [14].

As indicated, for static analysis the models discussed here concern only the elastic properties. Laboratory tests designed to determine the elastic properties thus must also account for inelastic effects and in particular, for stress softening. It is desirable to perform some kind of pre-straining, or mechanical conditioning, of the test specimens, since in engineering applications most rubber units are exposed to some form of periodic loading which results in stress softening under operating conditions. Procedures for mechanical conditioning are discussed in James and Green [12], Yeoh [23], and in [1] Chapter 5.

Test specimens and analytical strain-stress expressions applicable for the fitting procedure can be found in several sources for example in [3] and [9]. A number of test specimens and the analytical expressions applicable to them are presented in [1] Chapter 4. Test specimen accuracy as compared with the ideal analytical expressions is also discussed in [1] Chapter 6. The topics of model stability and of checking on stability are also addressed.

The stress-stretch relationship in compression and tension and the least square fitting to experimental data are discussed in the following sections. The fitting procedure will be illustrated by an example, where a carbon-black-filled natural rubber vulcanizate is characterized in terms of the Yeoh model mentioned earlier.

3.3.1 Stress-stretch relationship

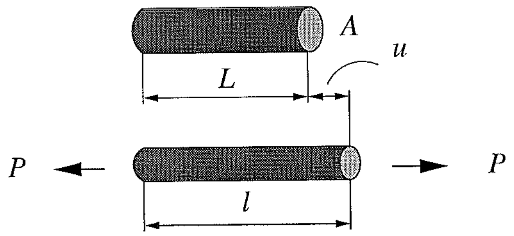

Figure 3.12: *Nonlinear elastic bar loaded by force P.*

The uniaxial stress-stretch relationship is derived here for a nonlinearly elastic bar, shown in Figure 3.12. It's original length is L and the cross-section area is A. The force P causes the displacement u at the end of the bar. Strain energy is stored up in the bar as the length increases to l giving the stretch $\lambda = l/L$. The strain energy density, denoted by W, is calculated as strain energy per unit volume of the undeformed bar, and assumed to be a function of the stretch λ.

The total strain energy U in the bar is computed by multiplying the strain energy density W with the volume, giving

$$U = AL\,W(\lambda)\ .$$

The incremental work done by the external force P should be equal to the increment in total strain energy. Hence, the energy balance is stated as

$$Pdu = dU\ , \tag{3.20}$$

and the increment in total strain energy can be expressed by use of W as

$$dU = AL\,dW = AL\,\frac{dW}{d\lambda}\,d\lambda\ . \tag{3.21}$$

The displacement increment can also be written in terms of stretch by using $u = l - L = (\lambda - 1)L$. Differentiation yields

$$du = Ld\lambda. \tag{3.22}$$

Inserting (3.22) and (3.21) into the energy balance equation (3.20) yields

$$PLd\lambda = AL\,\frac{dW}{d\lambda}\,d\lambda\ .$$

Simplifying the expression gives

$$\frac{P}{A} = \frac{dW}{d\lambda}\ , \tag{3.23}$$

where P/A is the nominal stress, i.e. force per original cross section area, derived from the strain energy function.

The stress can be obtained from the strain energy density function (3.17) by introducing the strain invariants for this uniaxial case

$$W = W(\lambda) = W(I_1(\lambda), I_2(\lambda))\ . \tag{3.24}$$

The generalized strain measures, the strain invariants I_1 and I_2, mentioned earlier can be expressed in terms of λ for compression and tension [1] as

$$\begin{cases} I_1 = \frac{2}{\lambda} + \lambda^2 \\ I_2 = \frac{1}{\lambda^2} + 2\lambda \end{cases}\ . \tag{3.25}$$

Taking partial derivatives of (3.24) yields

$$\frac{dW}{d\lambda} = \frac{\partial W}{\partial I_1}\frac{dI_1}{d\lambda} + \frac{\partial W}{\partial I_2}\frac{dI_2}{d\lambda} \tag{3.26}$$

By using the expressions for the invariants (3.25) in (3.26), the expression for the nominal stress (3.23) becomes

$$\frac{P}{A} = 2(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W}{\partial I_2})(\lambda - \frac{1}{\lambda^2}) \tag{3.27}$$

The nominal uniaxial stress expression for the Yeoh model is obtained by inserting the strain energy density W (3.19) and the expressions for the first invariant (3.25) into (3.27), giving

$$\frac{P}{A} = 2(C_{10} + 2C_{20}(\frac{2}{\lambda} + \lambda^2 - 3) + 3C_{30}(\frac{2}{\lambda} + \lambda^2 - 3)^2)(\lambda - \frac{1}{\lambda^2})\,. \tag{3.28}$$

The material parameters C_{10}, C_{20} and C_{30} can be determined by a least squares procedure as will be described next.

3.3.2 Least squares fit to test data

The parameters in the hyperelastic model are obtained from experimental data by a fitting procedure, described in this section.

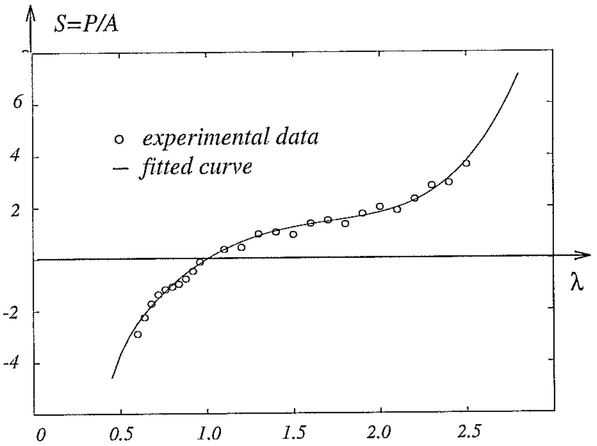

Figure 3.13: *Fit of the model to test data.*

Consider Figure 3.13 where the experimental data points and the stress-stretch relation from theory are shown schematically. The small circles correspond to the (λ_i, S_i) values $i = 1, ..., n$, where λ_i is the stretch and S_i is the nominal stress, obtained from experiments and n is the number of data points. The stress-stretch relation obtained from the constitutive model should closely fit the experimental data points, and the conditions to be fulfilled as close as possible, for every data point are

$$S_i^{teor} \approx S_i^{exp}\,. \tag{3.29}$$

This "closest fit" has to be defined in some way. In the method of least squares the "closest fit" is defined as the minimum of the sum, over all data points, of the square of the errors between theory and experiments i.e.

$$\Psi = \sum_{i=1}^{n} (S_i^{teor} - S_i^{exp})^2\,. \tag{3.30}$$

Relation (3.29) can be written alternatively as

$$S_i^{teor} / S_i^{exp} \approx 1 \tag{3.31}$$

and the corresponding sum of squares is written as

$$\Phi = \sum_{i=1}^{n} (S_i^{teor}/S_i^{exp} - 1)^2 \tag{3.32}$$

where the sum is expressed by the relative error. The form (3.32) is a normalized version of (3.30). The absolute error puts more emphasis on the higher stress values where the absolute error is higher, while the relative error gives equal weight to the data points.

The nominal stress obtained from theory S_i^{teor} depends on the unknown elastic parameters in the strain energy function $W(I_1, I_2)$. Minimizing Φ with respect to the unknown parameters yields a system of equations. The solution to the system is the elastic parameters giving the best fit in the least squares sense. The equation system is linear, if the polynomial form of strain energy function is used.

The procedure will be discussed here specifically in terms of a fit to compression and tension data. The nominal stress values S_i^{exp} and corresponding stretches λ_i are shown as circles in Figure 3.13.

The tension and a compression stress-stretch relation S^{teor} is defined by (3.27) giving

$$S^{teor} = 2(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda}\frac{\partial W}{\partial I_2})(\lambda - \frac{1}{\lambda^2}) \, , \tag{3.33}$$

where λ is the stretch in the loaded direction.

Setting up the (approximative) equalities at every experimental point yields a linear system of equations. Normally the number of experimental points exceeds the number of coefficients, yielding an overdetermined linear system of equations. This overdetermined system of equations can be written in matrix form as

$$\mathbf{Ac} = \mathbf{b} \, . \tag{3.34}$$

To be more specific, consider the three parameter Yeoh model where (3.33) is given by (3.28). In this case $\mathbf{A}$ is a nx3 matrix corresponding to the n experimental points, $\mathbf{c}$ is a 3x1 matrix with $c_1 = C_{10}$, $c_2 = C_{20}$, $c_3 = C_{30}$ and $\mathbf{b}$ is a nx1 matrix.

The overdetermined system (3.37) can not be solved in the ordinary way because there is no unique solution to this system of equations.

Inserting a trial solution $\mathbf{c}^*$ in (3.37) will always yield a difference between the left- and the right-hand side. This difference is expressed by the residual

$$\mathbf{e} = \mathbf{Ac}^* - \mathbf{b}$$

which is a vector containing the relative error in each data point. We want to find a solution $\mathbf{c}^*$ that minimizes the residual. The size of the residual $\mathbf{e}$ must be measured in some way. Using

$$||\mathbf{e}||_2^2 = \mathbf{e}^T\mathbf{e}$$

yields an expression precisely equal to the sum of the squares of the relative error between theory and experiments, i.e

$$\Phi = ||\mathbf{e}||_2^2 = (\mathbf{Ac} - \mathbf{b})^T(\mathbf{Ac} - \mathbf{b}) = \sum_{i=1}^{n} (S_i^{teor}/S_i^{exp} - 1)^2 \, .$$

Minimizing Φ is equivalent to finding the solution to the equations

$$\frac{\partial \Phi}{\partial c_i} = 0 \quad i = 1, 2, ...n \ . \tag{3.35}$$

which can be written conveniently [1] in matrix form as

$$\mathbf{A}^T\mathbf{A}\mathbf{c} = \mathbf{A}^T\mathbf{b} \ . \tag{3.36}$$

Observe that (3.36) is an ordinary linear system of equations with the same number of unknowns as the number of equations.

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Case 3

The cubic I1 model (3.19) according to Yeoh, is used here to illustrate the fitting procedure. The strain energy expression is

$$W = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$

with three hyperelastic constants. The model is fitted to experimental data in compression/tension for a 65 IRHD carbon-black-filled rubber vulcanizate. The experimental data points, shown as circles in Figure 3.14 are listed in the table below. These values are used in the relative error format of (3.31), which was stated as $S_i^{teor} / S_i^{exp} \approx 1$.

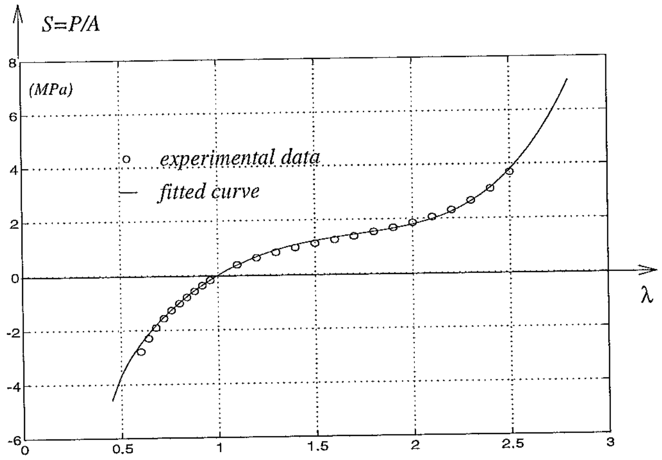

Figure 3.14: *Fitting of the Yeoh model (solid line) to experimental data (circles).*

The compression stress-stretch relationship (3.33) with three constants to be determined

$$S^{teor} = 2(C_{10} + 2C_{20}(\frac{2}{\lambda} + \lambda^2 - 3) + 3C_{30}(\frac{2}{\lambda} + \lambda^2 - 3)^2)(\lambda - \frac{1}{\lambda^2})$$

is applied in the quotient S_i^{teor} / S_i^{exp} for each of the 25 experimental stress-stretch values, yielding

λ	S^{exp} (MPa)
0.6	-2.788
0.64	-2.30
0.68	-1.904
0.72	-1.564
0.76	-1.276
0.8	-1.02
0.84	-0.792
0.88	-0.576
0.92	-0.376
0.96	-0.18
1.1	0.378
1.2	0.630
1.3	0.814
1.4	0.98
1.5	1.130
1.6	1.258
1.7	1.380
1.8	1.536
1.9	1.684
2.0	1.852
2.1	2.056
2.2	2.306
2.3	2.648
2.4	3.092
2.5	3.694

$$\Longrightarrow \quad \begin{bmatrix} 1.5623 & 2.1663 & 2.2530 \\ 1.5664 & 1.6748 & 1.3431 \\ 1.5574 & 1.2570 & 0.7610 \\ 1.5461 & 0.9158 & 0.4069 \\ 1.5224 & 0.6369 & 0.1998 \\ 1.4951 & 0.4186 & 0.0879 \\ 1.4577 & 0.2523 & 0.0328 \\ 1.4282 & 0.1346 & 0.0095 \\ 1.3908 & 0.0565 & 0.0017 \\ 1.3897 & 0.0137 & 0.0001 \\ 1.4474 & 0.0816 & 0.0034 \\ 1.6049 & 0.3424 & 0.0548 \\ 1.7403 & 0.7952 & 0.2725 \\ 1.8159 & 1.4112 & 0.8225 \\ 1.8682 & 2.1796 & 1.9072 \\ 1.9227 & 3.1148 & 3.7844 \\ 1.9623 & 4.1854 & 6.6955 \\ 1.9419 & 5.2474 & 10.6347 \\ 1.9275 & 6.4096 & 15.9852 \\ 1.8898 & 7.5594 & 22.6782 \\ 1.8222 & 8.6096 & 30.5086 \\ 1.7289 & 9.5056 & 39.1978 \\ 1.5944 & 10.0751 & 47.7495 \\ 1.4401 & 10.3495 & 55.7838 \\ 1.2669 & 10.2620 & 62.3419 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The overdetermined system of equations above is written in matrix form as

$$\mathbf{Ac} = \mathbf{b} \,. \tag{3.37}$$

where $\mathbf{A}$ is a 25x3 matrix corresponding to the 25 experimental points, $\mathbf{c}$ is a 3x1 matrix with $c_1 = C_{10}$, $c_2 = C_{20}$, $c_3 = C_{30}$ and $\mathbf{b}$ is a 25x1 matrix.

Applying (3.36) to the overdetermined system of equations yields an ordinary system of equations with three equations and three unknowns i.e.

$$\begin{bmatrix} 67.92 & 147.44 & 487.11 \\ 147.44 & 649.47 & 2717.13 \\ 487.11 & 2717.13 & 12699.74 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} 40.8894 \\ 87.6551 \\ 303.5157 \end{bmatrix} . \tag{3.38}$$

Solving (3.38) yields

$$C_{10} = 0.6803\,MPa \quad C_{20} = -0.0982\,MPa \quad C_{30} = 0.0188\,MPa \,.$$

We can conclude that in order to obtain the unknown elastic parameters we have to set up the matrices $\mathbf{A}$ and $\mathbf{b}$ and solve (3.36). The solution obtained in this way is the "closest fit" to experiments in the sense that the sum of the squares of the errors for all experimental points is minimized.

The experimental data points and the fitted curve are shown in Figure 3.14. The fitting to the experimental points is good, and the fitted curve is progressive outside the interval of experimental data. For this choice of parameters the least squares procedure generates a model with a good fit and reasonable behavior outside the interval of fitted points.

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Chapter 4

Inelastic quasi-static and dynamic properties

P.-E. AUSTRELL

This chapter contains a discussion of inelastic and dynamic properties and the microstructure involved. Creep, relaxation, stress softening, and steady state dynamic behavior are dealt with.

4.1 Microstructure and damping properties

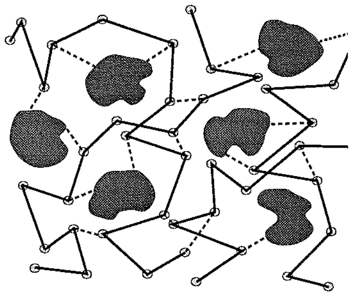

Figure 4.1: *Molecular structure for a carbon-black-filled rubber vulcanizate. Carbon particles, polymer chains (solid lines) and crosslinks (dashed lines) are shown schematically.*

The origin of the damping property of rubber can be understood on the basis of the molecular structure of the material. Vulcanized rubber is made up of very long cross-linked polymer molecules. The damping is increased when filler, usually carbon-black, is added to the rubber compound. Carbon-black consists of small particles of carbon (20nm - 300nm). These are mixed into the raw rubber base before vulcanization. The material is thus of two-phase character, containing constituents differing completely in their mechanical properties. Figure 4.1 shows schematically the molecular level structure of a carbon-black-filled vulcanizate.

The cross-links are indicated as dashed lines and the rubber network is shown with solid lines inside which the agglomerates of carbon particles are formed.

The damping property of filled rubbers, i.e. the ability to dissipate mechanical energy into heat, is mainly due to two kinds of mechanisms. One is of viscous character, the viscous damping originating in the resistance in the rubber phase to reorganize the molecular chains. The fact that reorganization of the long molecular chains cannot occur instantaneously results in a rate dependent resistance of viscous character. The other mechanism is due to the filler, damping being increased when fillers are added to the rubber compound. The filler particles can be regarded as rigid compared with the relative flexibility of the rubber matrix. As mentioned above, these form agglomerates which result in a filler structure [12] inside the rubber network. When the compounded rubber material is deformed forces develop at the filler interfaces, causing the filler structure to break. The increase in damping which the filler causes must therefore be attributed to a resistance in the rubber-carbon and carbon-carbon interfaces. This part of the damping is of a rate-independent character. It is responsible for the nonlinear dynamic behavior of filled rubbers that appears as amplitude dependence of the dynamic stiffness and of the phase angle (as discussed in Section 4.3).

4.2 Inelastic quasi-static properties

In this section, creep and relaxation, as well as strain softening or Mullin's effect, are dealt with.

4.2.1 Creep and relaxation

Both rate and time dependence and dissipative mechanisms can be identified in such material behavior as creep and relaxation phenomena. Creep is an increase in strain in response to a step-stress loading, and relaxation is a decrease in stress in response to a step-strain loading.

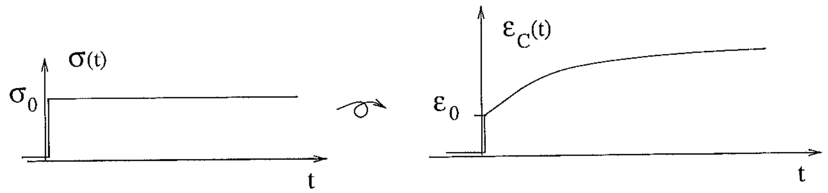

Figure 4.2: *Creep behavior, increase in strain in response to a stress step.*

A creep test is performed by applying a stress step, creep compliance being defined as $J_C(t) = \epsilon_C(t)/\sigma_0$ (cf. Figure 4.2).

A relaxation test is performed by applying a strain step, the relaxation modulus being defined as $E_R(t) = \sigma_R(t)/\epsilon_0$ (cf. Figure 4.3).

Freakley and Payne ([3] p.35) discuss creep and relaxation. Gent ([5] p.172) describes a logaritmic relationship for creep and relaxation with respect to time.

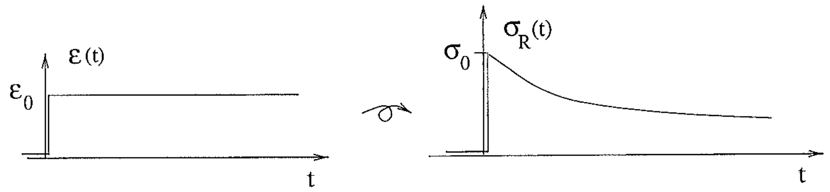

Figure 4.3: *Relaxation behavior, decrease in stress in response to a strain step.*

Related matters are discussed by Stenberg [10] in "Compression relaxation of rubbers".

Linear viscoelasticity is characterized by the ability to define a single relaxation function independent of the strain step amplitude.

For unfilled rubbers a linear viscoelastic behavior is observed in simple shear, which is a linear mode of deformation if the strains are not too large. In the case of filled rubbers, however, nonlinear behavior is present even in the case of simple shear and small strains, due to breakdown and reforming of the carbon-black structure. In relaxation tests a step size dependence is observed, the smallest strain step yielding the largest relaxation modulus. This is discussed in [1] Chapter 8.

4.2.2 Mullins' effect

Cyclic straining of a rubber specimen causes a successive decrease both in stiffness and in the distance between the loading and unloading curves, the hysteresis. The breakdown is limited, however, a stationary state being reached after about four to eight cycles. If the stretch is increased then further breakdown occurs and a new stationary curve is obtained, one which lies below the previous stationary curve, the stiffness of the material having decreased, cf. Figure 1.2.

Mullins [7] observed that, if strain amplification (cf. Chapter 1) is taken into account, the differences in softening between gum and filled vulcanizates are very small. He concluded, therefore that for strains larger than 0.1% the softening is mainly due to the effects the strain has on the rubber network and not to interaction with the filler. Recovery from softening was, however found to be slower in filled vulcanizates. Mullins attributed this to the hindering effect of the filler. He also concluded that only the unrecoverable part of the softening is due to breakdown of the cross-links, wheras the part of the softening that is recoverable (after about 24 hours) is due to configurational changes in the rubber network.

For strains of below 0.1%, softening was attributed to rupture of the filler aggregates. According to Payne [8] this softening at very small strains due to effects of the filler is immediately recoverable to a large extent.

The proper treatment of strain softening is important in the testing of rubber specimens aimed at hyperelastic characterization. The usefulness of the test data obtained depends on how the pre-stretching or mechanical conditioning has been performed.

4.3 Dynamic properties

This section concerns the dynamic properties of rubber, special emphasis being placed on the dynamic modulus and its dependence on frequency, temperature, and amplitude.

4.3.1 Definition of dynamic modulus and damping

Steady state harmonic excitation is an important type of loading. When nonlinearities are present, the response to a sinusoidal excitation is not perfectly sinusoidal, even for small vibrations. Figure 4.4 shows two hysteresis loops for a sample subjected to a pure sinusoidal strain. The left-hand path is elliptic, corresponding to linear dynamic (viscoelastic) behavior. The right-hand path is a more general hysteresis in which nonlinearities are present. The stress response contains higher harmonic components, i.e. multiples of the frequency of the strain input.

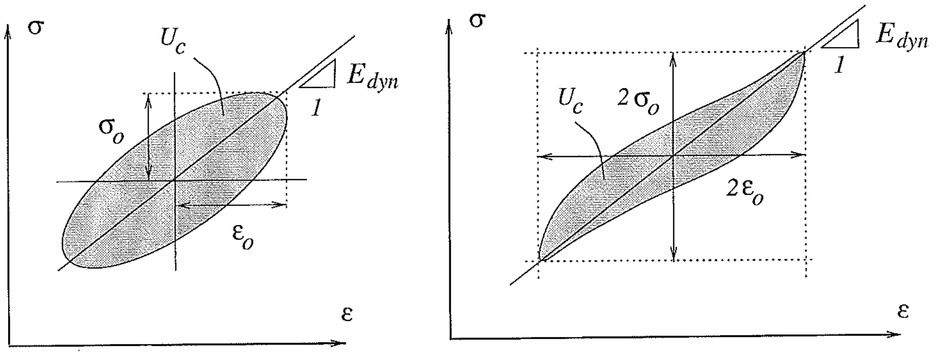

Figure 4.4: *Linear viscoelastic and general hysteresis loop for harmonic excitation.*

A general definition of the dynamic modulus valid also for the nonlinear case is required. Definitions consistent with linear viscoelasticity for the dynamic modulus E_{dyn} and normalized damping d are

$$E_{dyn} = \frac{\sigma_0}{\epsilon_0} \qquad d = \frac{U_c}{\pi \sigma_0 \epsilon_0} \,, \tag{4.1}$$

where U_c is the hysteresis work done during a single cycle. The dynamic modulus is defined in the same way as for linear viscoelasticity. At moderate damping, when $d \approx \delta$, normalized damping coincides with the phase angle.

If the nonlinearities are not too severe, so that the deviation from a sinusoidal shape is only slight, then a least squares fit to the response of an harmonic function with the same fundamental frequency as the input yields approximately the same dynamic modulus and phase angle as the definition above provides. This can be regarded as an equivalent viscoelastic damping. The dynamic modulus and damping are discussed further in Chapter 5 in connection with linear viscoelasticity.

4.3.2 Frequency and temperature dependence of the dynamic modulus

The dynamic properties of rubber are dependent upon frequency, temperature, static load and amplitude.

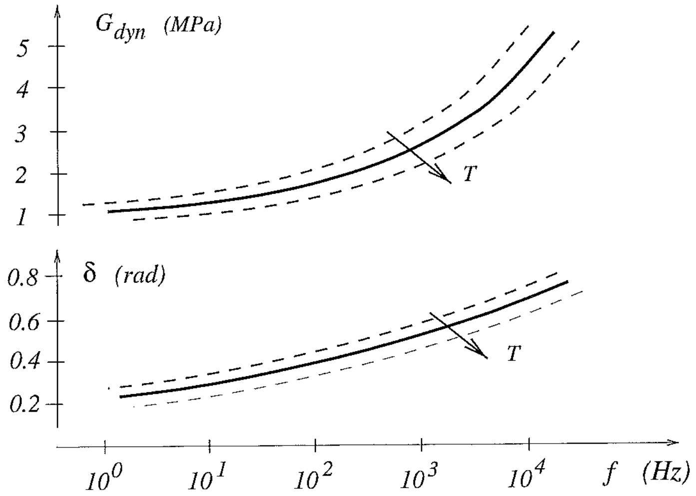

Figure 4.5: *Frequency dependence of dynamic shear modulus and phase angle for a filled natural rubber. Influence of temperature is also shown.*

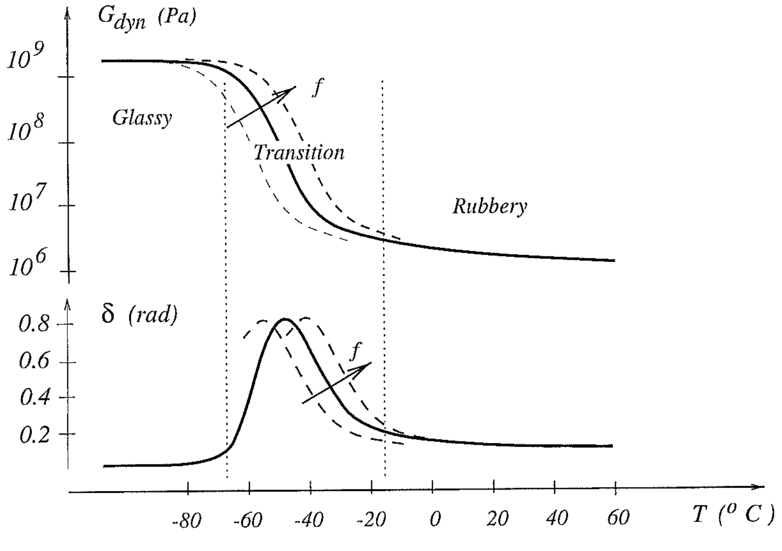

Figure 4.6: *Temperature dependence of dynamic shear modulus and phase angle for a filled natural rubber. Influence of frequency is also shown.*

Figure 4.5 shows the frequency dependence of the dynamic modulus and the phase angle for a filled rubber. The dynamic shear modulus and the phase angle are shown as functions of frequency. The effect of changing the temperature is also shown. The values given are approximate and are also strongly dependent upon the amplitude, as will be discussed below.

Figure 4.6 shows the temperature dependence of the dynamic shear modulus and the phase angle for a filled rubber. The values given are approximate and are also dependent upon the amplitude. The dynamic modulus decreases with an increase in temperature. The effect of changing the frequency is also shown. The general behavior shown in Figure 4.6 is the same for all rubbers, although the temperature scale can be shifted due to additives in the rubber compound.

Three temperature regions are indicated. The first is the glassy region in which the thermal motion of the polymer molecules is reduced. In this range almost no inter-molecular motion is possible, the material being glass-like and brittle. The mechanical behavior of rubber in this temperature range is dependent on the interatomic bonds. The modulus in this region is thus very high (≈ 1 GPa) , whereas the damping represented by the phase angle is low. The second region is the transition region, in which there is a drastic drop in the dynamic modulus, the maximum damping appearing as a peak in the phase angle. The third temperature region is the rubbery region in which the dynamic modulus is low (≈ 1 MPa). Under functional conditions rubbers need to be near the end of the transition region or in the rubbery region in order to have rubber-like properties.

The shift in the modulus and the phase curves shown in Figure 4.6 is a general property valid for a variety of polymers over a wide frequency and temperature range. The frequency-temperature correspondence has been summarized by Williams, Landel, and Ferry [14] in an empirical universal function valid for amorphous polymers at temperatures above the glassy region. An important consequence of this correspondence is that over many decades of frequency, modulus and phase curves can be obtained at different temperatures by performing experiments within a limited frequency range. The entire frequency range is covered by shifting the limited frequency data along the frequency axis. Low temperature measurements correspond to high frequencies and high temperature measurements to low frequencies. The combined curves give the appearance of being mirror images of the temperature curves shown in Figure 4.6.

4.3.3 Amplitude dependence of the dynamic modulus

Special emphasis will be placed here on the strain amplitude dependence of the dynamic modulus. The modulus is seen to decrease with increasing strain amplitude. This effect is sometimes denoted the *Payne effect*, due to investigations of reinforced elastomers made by Payne [8]. He interpreted the decline in modulus with increasing strain amplitude as being a result of filler structure rupture. The structure is composed of aggregates held together by van der Waals bonds. Payne found the modulus to be almost recoverable upon the return to small amplitudes, which means that the filler structure largely reforms within an amplitude cycle. The mechanisms involved should not be confused with Mullin's effect, which is mainly attributable to reorganizations within the rubber network [7], with recovery times of about 24 hours.

Payne also observed that for low frequencies the shape of the decline in modulus with increasing strain amplitude was nearly independent of frequency. He also refered to Warnaka [13], who observed this same type of frequency independence at higher frequencies, up to 1500 Hz.

A comprehensive review of the dynamic properties and the amplitude dependence of carbon-black-filled rubbers can be found in the often-cited article by Medalia [6].

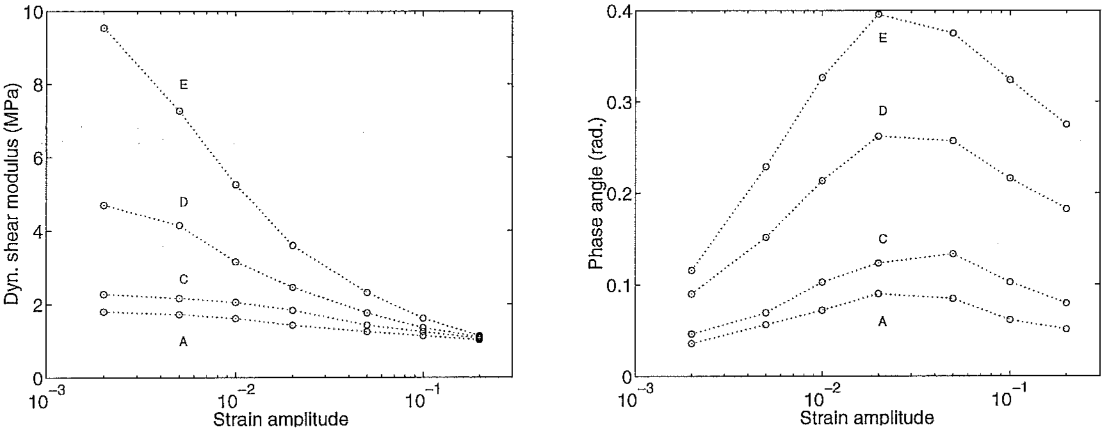

Figure 4.7: *Amplitude dependence of dynamic shear modulus and phase angle for some filled natural rubbers of various filler contents. A-E represent increasing filler contents. (Reproduced from Harris and Stevenson.)*

Harris and Stevenson [2] have carried out experimental studies of various nonlinear aspects of the dynamic behavior of filled rubbers in particular, investigating the effects of frequency, amplitude, and elastic nonlinearity for filled and unfilled rubbers. For unfilled rubbers the hysteresis loop was found to have an elliptic shape for small amplitudes and small preloads, the behavior being viscoelastic. For filled rubbers, a significant effect of the strain amplitude on the dynamic modulus was obtained. Figure 4.7 shows the equivalent complex modulus, reproduced from [2], for natural rubbers (NR) with different carbon-black loading. The rubbers were all of approximately the same hardness (about 55 IRHD). These conditions were obtained by using three different types of carbon-black and by balancing the reinforcing effect through the addition of a high-viscosity aromatic oil. Figure 4.7 shows the influence of the strain amplitude in simple shear. The vulcanizates are denoted in [2] as A,B,C,D,E in the order of increasing reinforcement of carbon-black, the carbon-black content varying between 30 and 75 phr, (parts per hundred of rubber by weight) with 75 phr for vulcanizate E. B was omitted in the reproduction shown in Figure 4.7) because vulcanizates B and C behaved similarly. The vulcanizate with the highest damping effect and the greatest variation in dynamic modulus is the one with the highest filler content.

Harris and Stevenson also reported that for harmonic loading in simple shear the stress response is influenced by higher harmonic components, resulting in a distorted elliptic shape which tends towards a parallelogram when graphing stress versus strain. The most significant contribution is from the third harmonic component. The ratios of the third to the first harmonic component for a 1 Hz test of vulcanizate E at different amplitudes were given. The maximum ratio was 0.035, obtained for a strain amplitude of 5%.

The combined influence of amplitude and temperature for a filled rubber similar to vulcanizate E above is shown in [5] Figure 10.11 p 304.

4.4 Summary of deviations from linear dynamic behavior

Deviations from linear dynamic behavior are based on the underlying nonlinear elastic properties and on the damping mechanisms introduced by the filler.

For unfilled rubbers, linear viscoelastic behavior can be observed in simple shear, which is a linear mode of deformation if the strains are not too large. In the case of filled rubbers, nonlinear dynamic behavior is found even in the case of simple shear, due to the breakdown and reforming of the carbon-black structure.

The dynamic properties of carbon-black-filled rubbers can be summarized summarized as follows:

- In quasi-static loading a difference between loading and unloading curves can be observed irrespective of how slow the loading rate is. It can also be observed that in quasi-static loading the hysteresis loop takes on the approximate shape of a parallelogram.
- The shape of the periodic strain history function does not influence the shape of the hysteresis loop appreciably.
- In relaxation tests, a step size dependence can be observed, the smallest strain step yielding the largest relaxation modulus. The relaxation modulus falls towards an asymptotic value with increasing step size.
- In harmonic loading the dynamic modulus shows strong amplitude dependence. For large amplitudes, the modulus declines with amplitude towards an asymptotic value. The damping represented by the equivalent phase angle reaches a maximum, where the decline in modulus is greatest.
- In harmonic testing, the dynamic modulus has been shown to be almost recoverable during a strain cycle. The mechanisms involved at the microstructural level, namely the breakdown and reforming of the carbon-black structure, are thus different from these involved in the Mullins effect.
- The shape of the decline in the dynamic modulus with increasing amplitude is insensitive to frequency. Experiments indicate that the amplitude and the frequency dependence are independent.

4.5 Test methods

Descriptions of the test methods can be found for example, in Friberg [4] Chapter 6 and Gent [5] chapter 10.

4.5.1 Quasi-static testing

Relaxation and creep tests are examples of quasi-static tests. The compression set and tension set tests mentioned in Chapter 1 belong to this category. Mechanical conditioning procedures represent quasi-static tests.

4.5.2 Dynamic testing

The resilience test, a simple dynamic test method described in Chapter 1, gives a rough estimate of material damping properties.

More accurate dynamic tests and equipment are described in Gent [5] pp. 301-303, which deals in particular with resonant-beam, servohydraulic, and electrodynamic testers.

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Chapter 5

Modeling of inelastic and dynamic properties

P.-E. Austrell

This chapter concerns viscoelasticity, various fundamental concepts of linear dynamics, and models for highly filled rubbers. Deviations from viscoelasticity as well as aspects of nonlinear dynamic behavior introduced by the adding of filler are also discussed.

Linear dynamics and viscoelastic material models are the basis for the prediction of various dynamic properties of rubber components, including such important aspects as resonance frequency, transmissibility and dynamic stiffness.

However, as was discussed in Chapter 4 several aspects of the dynamic behavior of rubber are not explainable by linear dynamics. Examples mentioned were the amplitude dependence of the dynamic stiffness, the presence of multiple frequencies in harmonic responses, and distorsion of the hysteresis loop. Models dealing with these aspects of rubber behavior will be disscussed in Section 5.5. The section that follows will focus on linear dynamic models of rubber units.

5.1 Linear dynamics

Linear dynamic systems responds to sinusoidal excitation with a delayed sinusoidal output of the same frequency as the input as shown in Figure 5.1. This property enables the use of complex quantities and permits a definition of a complex stiffness that depends upon frequency alone.

5.1.1 A one-dimensional model system

The basic features of linear dynamics can be derived from a study of a simple one-dimensional system consisting a linear spring, and a viscous damper connected in parallel to a mass as shown in Figure 5.1.

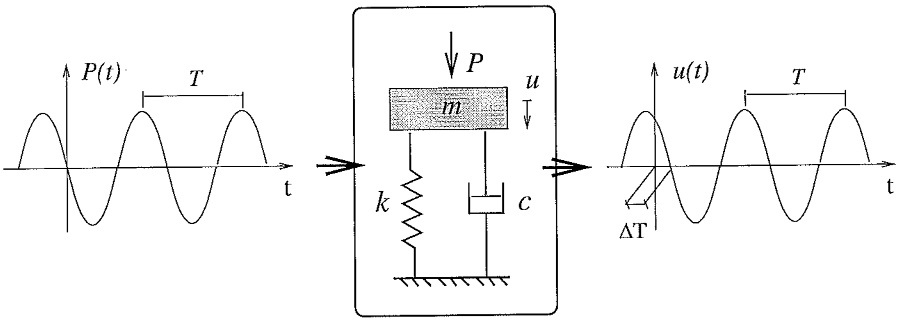

Figure 5.1: *Linear dynamic system exposed to steady state sinusoidal loading. The response is a delayed sinusoidal function of the same frequency.*

The dynamics of the system is described by the second-order differential equation

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = P(t) \tag{5.1}$$

where the viscous damping force is proportional to the displacement rate such that $F_v = c\,\dot{u}$ with the proportionality constant c and the elastic force being proportional to the displacement $F_e = ku$ containing the proportionality constant k.

Free vibrations

The vibration frequency attained by the system (5.1) in force-free vibrations (i.e. for $P = 0$), is called the natural frequency or eigenfrequency. The free response is an exponentially decaying harmonic function.

The undamped system for which $c = 0$, has a natural angular frequency ω_0 as given by

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}} \tag{5.2}$$

where f_0 is the *natural frequency.* The response of the system (5.1) to free vibrations for which there is slight to moderate damping results in a natural vibration frequency that deviates only slightly from the natural frequency of the undamped system according to (5.2).

Forced vibrations

The most interesting case technically is that of forced vibrations, when the system is driven by a sinusoidal force. The response in this case is a sinusoidal displacement of the same angular frequency as the force but with a delay. The undamped ($c = 0$) displacement solution for the case of forced vibration in which $P = P_0 sin(\omega t)$ is

$$u_0 = \frac{P_0}{k}\frac{1}{1-(\frac{\omega}{\omega_0})^2} \tag{5.3}$$

where ω is the angular frequency of the driving force. The expression (5.3) can be viewed as the static solution P_0/k multiplied by an amplification function. The solution illustrates an important phenomena. Expression (5.3) shows that the vibration amplitude can increase to an unlimited extent when the driving force hits the system's natural frequency. At this frequency the system is in resonance, and an alternative designation for the natural frequency is therefore *resonance frequency*. Introducing damping limits the response at resonance and produces a slight shift in resonance frequency. Hence, in a moderately damped system the maximum displacement amplitude is attained at approximately $\omega \approx \omega_0$.

Transmissibility

An important question in vibration damping applications is how much of the driving force is transmitted through the damper and into the supporting structure. The ratio of the transmitted force to the driving force is called *transmissibility*. For the undamped system this ratio can be expressed as

$$T = |\frac{ku_0}{P_0}| = |\frac{1}{1-(\frac{\omega}{\omega_0})^2}| \tag{5.4}$$

i.e. as the amplitude of the force in the spring divided by the amplitude of the driving force. The expression (5.4) is shown graphically in Figure 5.2. This expression is approximately valid for a moderately damped system also if the frequency of the driving force is well above the resonance frequency. This is not a restriction since in order for a reduction in the transmitted force to be obtained the frequency has to be well above the resonance frequency.

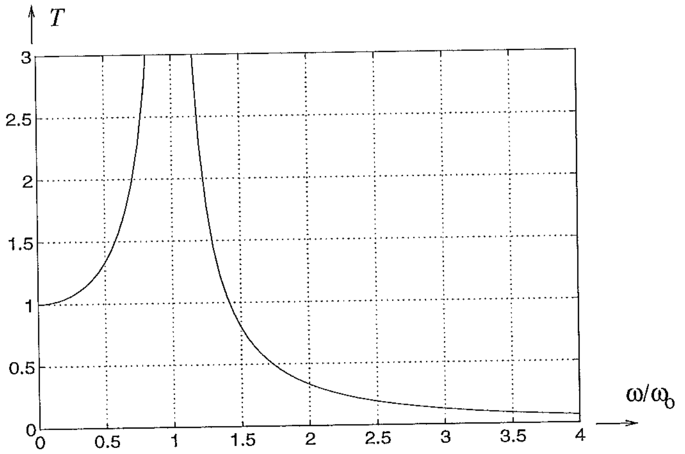

Figure 5.2: *Transmissibility versus normalized frequency for an undamped system.*

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Case 1

Determine the resonance frequency ω_0 and the transmissibility T at $\omega = 3\omega_0$ for the cylindrical component with the spring stiffness K_c=273N/mm analyzed in cases 1 and 2 in Chapter 3. The cylinder is connected to a mass m=20kg, as shown in Figure 5.3.

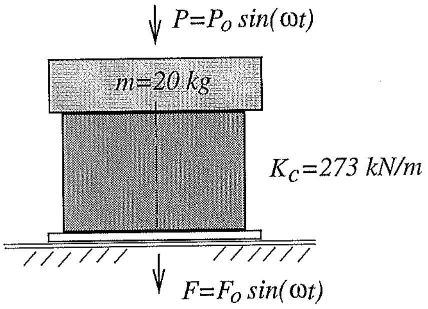

Figure 5.3: *Cylindrical compression spring attached to a mass.*

Solution:

The resonance frequency f_0 is calculated from expression (5.2),

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{273 \cdot 10^3}{20}} = 18.6 \ \ Hz.$$

Transmissibility is calculated from expression (5.4),

$$T = |\frac{ku_0}{P_0}| = |\frac{1}{1-3^2}| = 0.125,$$

indicating that 12.5% of the exitation force amplitude is transmitted through the damper and into the support at a frequency $\omega = 3\omega_0$.

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5.2 Linear viscoelasticity

The previous section concerned dynamic behavior at a structural level using a simple model of the damper consisting of a linear spring and a viscous damper connected in parallel. This section discusses more sophisticated modeling at a material level in terms of linear viscoelasticity. Viscoelasticity provides a way to take into account time- and frequency- dependent properties of rubber behavior. Certain fundamental concepts such as those of complex modulus and relaxation modulus are discussed in this section.

5.2.1 Creep and relaxation modulus

Material behavior explainable by linear viscoelastic models includes creep and relaxation phenomena. Creep is an increase in strain in response to a step-stress loading, and relaxation is a decreasing stress as response to a step-strain loading.

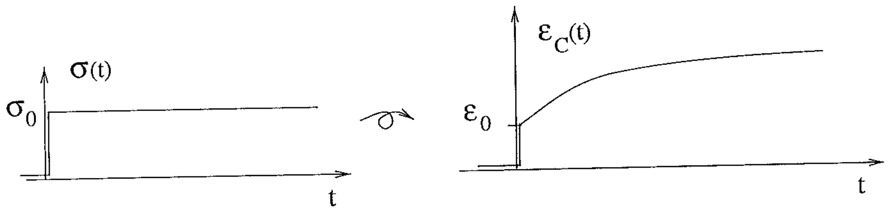

Figure 5.4: *Creep behavior: increasing strain in response to a stress step.*

For a linear viscoelastic material the *creep compliance* $J_C(t) = \epsilon(t)/\sigma_0$ (cf. Figure 5.4) is a characteristic function independent of the stress step σ_0. For a given linear viscoelastic material, this function is unique.

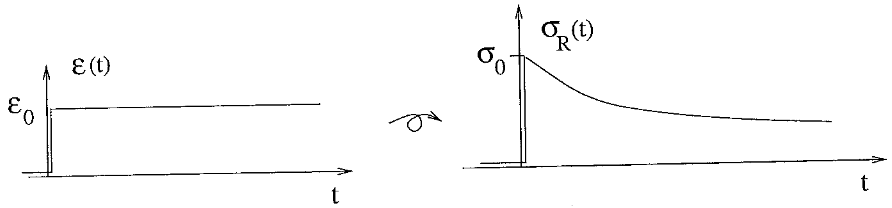

Figure 5.5: *Relaxation behavior; decreasing stress as response to a strain step.*

For a linear viscoelastic material, the *relaxation modulus* $E_R(t) = \sigma(t)/\epsilon_0$ (cf. Figure 5.5) is a characteristic function independent of the strain step ϵ_0. This function is unique for a given linear viscoelastic material.

It can be shown that the instantaneous elasticity, i.e. the relationship between the creep compliance and the relaxation modulus at $t = 0$, is $E_R(0) = 1/J_C(0)$.

The behavior of a linear viscoelastic material can be defined on the basis of this single-step response function. Linearity and superposition (Boltzmann's superposition principle) leads to a constitutive equation defined as a convolution integral (also called the hereditary integral). The stress history $\sigma(t)$ corresponding to a particular strain history $\epsilon(t)$ can be obtained from this integral, given by

$$\sigma(t) = \int_{-\infty}^{t} E_R(t-s)\,\frac{d\epsilon}{ds}ds$$

the constitutive model thus being defined by the relaxation modulus. For the inverse relationship, the creep compliance takes the place of the relaxation modulus in a similar expression.

5.2.2 Harmonic excitation and the complex modulus

The response to a stationary sinusoidal strain history is of interest in many engineering applications. Harmonic excitation of a linear viscoelastic material produces a harmonic response of a frequency equal to the excitation frequency. The response is however, phase shifted compared to the excitation. Consequently, if the strain $\epsilon = \epsilon_0 \sin(\omega t)$ and the stress $\sigma = \sigma_0 \sin(\omega t + \delta)$ are plotted in the (σ, ϵ) plane, an elliptic path is obtained, as shown in Figure 5.6.

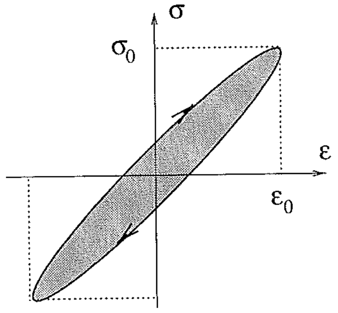

Figure 5.6: *The stress* $\sigma = \sigma_0 sin(\omega t + \delta)$ *and the strain* $\epsilon = \epsilon_0 sin(\omega t)$ *plotted in the* (σ, ϵ) *plane, giving an elliptical hysteresis loop.*

The phase relationships are conveniently shown in a rotating-vector representation and it is convenient to use complex notation. The strain ϵ^* and the stress response σ^* can be written as the complex quantities

$$\epsilon^* = \epsilon_0 e^{i\omega t} \quad \text{and} \quad \sigma^* = \sigma_0 e^{i(\omega t+\delta)} \tag{5.5}$$

Behavior in stationary harmonic loading can be characterized by a complex modulus that depends on the frequency but is independent of the amplitude

$$\sigma^* = E^*(\omega)\epsilon^* \; .$$

An interpretation of the complex modulus in terms of measurable quantities is given by

$$\mid E^* \mid = \frac{\sigma_0}{\epsilon_0} \quad \text{and} \quad arg(E^*) = \delta \; .$$

The absolute value $\mid E^*(\omega) \mid$, here termed the *dynamic modulus*, is thus the amplitude ratio of stress and strain and the phase angle $arg(E^*(\omega))$ is the phase shift between stress and strain.

Note that the dynamic modulus and the phase angle are functions of the angular frequency ω only. Hence, for a linear viscoelastic material, the dynamic modulus is independent of the strain amplitude ϵ_0.

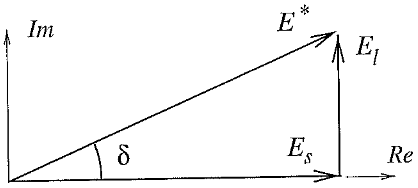

Figure 5.7: *Relation between the polar and the rectangular form of the complex modulus.*

Alternatively, the complex modulus can be expressed in rectangular form as

$$E^* = \frac{\sigma_0}{\epsilon_0} cos\delta + i\frac{\sigma_0}{\epsilon_0} sin\delta = E_s + iE_l \tag{5.6}$$

where the real part E_s is termed the *storage modulus* and the imaginary part E_l the *loss modulus*. The storage modulus represents an in-phase response and the loss modulus represents an out-of-phase response, as shown in Figure 5.7. An alternative representation of the rectangular form is

$$E^* = E_s(1 + i\, tan\, \delta) \ ,$$

where $tan\, \delta$ is called the *loss factor*.

For small values of δ, the relation of the polar to the rectangular form of the complex modulus can be simplified. The approximate equalities $sin\, \delta \approx tan\, \delta \approx \delta$ and $cos\, \delta \approx 1$ yield

$$E_s \approx \mid E^* \mid \qquad \text{and} \qquad E_l \approx \mid E^* \mid \delta \ .$$

For example, $\delta = 0.2\, rad$ yields $sin\, \delta = 0.199$, $cos\, \delta = 0.980$, and $tan\, \delta = 0.203$.

In cyclic loading, viscoelastic materials dissipate energy, which for example results in damping of free vibrations. The area enclosed by the loading and the unloading curves (cf. Figure 5.6) represents the energy dissipated as heat. The energy dissipated during a single cycle is

$$U_c = \pi \sigma_0 \epsilon_0 sin\, \delta \ .$$

The dissipated energy increases with the phase difference δ.

5.2.3 Rheological models

An illustrative way to describe and interpret the nature of viscoelastic behavior is to use rheological models. These are mechanical analogue assemblages of linear springs and dashpots. The mechanical behavior of a specific viscoelastic material is analogous to the behavior of a proper combination of springs and dashpots.

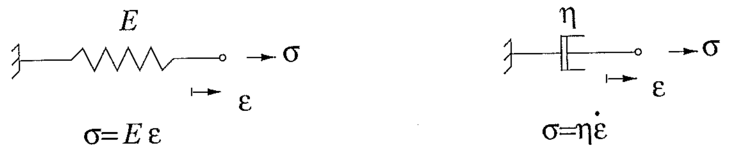

Figure 5.8: *Basic elements in rheological models: the linear spring and the dashpot.*

The elastic behavior is due to the linear springs, whereas the viscous behavior is due to the dashpots, as shown in Figure 5.8. The stress in the linear spring is proportional to the strain giving $\sigma = E\epsilon$. The dashpot stress is proportional to the strain rate giving $\sigma = \eta\dot{\epsilon}$, where the dot denotes the time derivative and η is the viscosity coefficient (Ns/m^2).

Simple viscoelastic models can be obtained using linear springs and dashpots coupled in series and in parallel. The spring and dashpot combination in series yields the Maxwell model, which is a viscoelastic fluid model. The spring and dashpot components coupled in parallel yield the Kelvin model, which is a crude viscoelastic solid model. The Kelvin model is not particularly physical. For example, a sudden application of strain yields a discontinuous stress response.

The simplest viscoelastic solid model displaying reasonable physical behavior is obtained by a spring and a Maxwell element coupled in parallel. This is the standard linear solid model, also called the Zener model.

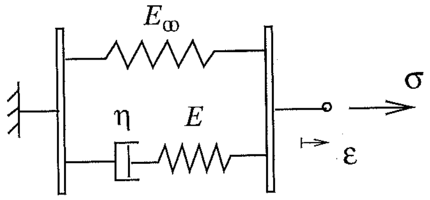

Figure 5.9: *The Zener model.*

The Zener model is the simplest viscoelastic model with solid properties that reflect the behavior of real solid materials in the sense of the relaxation behavior being reasonable and the creep response limited.

5.2.4 Properties of linear viscoelastic solids

To understand the behavior and restrictions of the rheological models for different strain histories, the behavior for different loading rates (frequencies) will be examined. The Zener model responds in an approximately elastic way to both very slow and very fast loading. This is because a dashpot behaves like a rigid element at fast loading rates and because the stress in it is being relaxed at slow loading rates.

Behavior is similar in the case of harmonic excitation. Consider three cases, those of low, medium, and high frequency:

$$1)\quad \omega t_r \ll 1 \qquad 2)\quad \omega t_r \approx 1 \qquad 3)\quad \omega t_r \gg 1$$

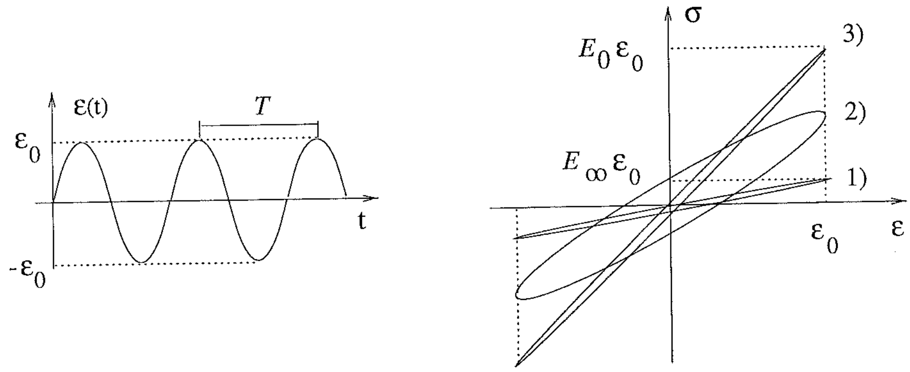

Figure 5.10: *Response of a linear viscoelastic solid to sinusoidal excitation.*

with $t_r = \eta/E$ being a characteristic relaxation time for the Zener model. The elliptic paths in the (σ, ϵ) plane for the three cases are shown in Figure 5.10. Low and high frequencies yield approximately elastic behavior. Alternatively, this can be shown as in Figure 5.11, where the frequency dependence of the complex modulus is indicated.

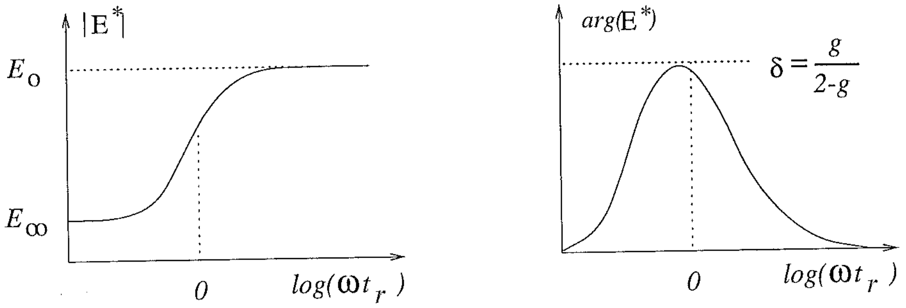

Figure 5.11: *Zener model; frequency dependence of the complex modulus.* $\delta_{max} \approx g/(2-g)$ *for* $g \leq 0.5$, *with* $g = E/(E_\infty + E)$.

The Zener model is a simple example of a linear viscoelastic solid model having physically reasonable properties. Figure 5.11 shows the frequency dependence schematically.

Some properties of the linear dynamic behavior of viscoelastic solids are summarized here:

- The dynamic modulus is independent of the amplitude.
- Harmonic excitation yields a phase-shifted harmonic response of the same frequency.
- The hysteresis loop has an elliptic shape.
- The dynamic modulus increases with frequency.
- Elastic behavior (no damping) for both low and high strain rates or frequencies.

5.2.5 Hysteretic damping models

The observation that the complex modulus is approximately constant over a limited range of frequency and temperatures is the basis for the so-called hysteretic damping model. The assumtion is thus that the modulus can be set to a constant complex number, independent of frequency.

The hysteretic damping model cannot be derived from linear viscoelasticity and it has some theoretical anomalities. However, in practice it is often used for frequency domain calculations.

5.3 Dynamic spring stiffness expressions

The static spring stiffness expressions can be converted to dynamic stiffness expressions by introducing a complex modulus into the spring stiffness expressions presented in Chapter 3.

For example, the complex shear modulus

$$G^* = G_s(1 + i\, tan\delta)$$

inserted into the expression of the shear spring (3.11) of Chapter 3, yields the complex stiffness

$$K_s^* = \frac{G^*\, A}{H}.$$

containing the dynamic stiffness and damping for the shear spring.

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Case 2

Estimate the dynamic stiffness K_{dyn} at 10Hz for the cylindrical component analyzed in previous cases. The rubber material is assumed to have the dynamic shear modulus $G_{dyn} = 0.9$MPa at low frequencies.

Solution:

The the static spring compression stiffness K_c given by (3.8) as $K_c^{stat} = E_c(S)A/H$ where $E_c(S) = E_0\,(1+2k\,S^2)$ was calculated in case 1 in Chapter 3, with $E_0 = 2.1$MPa, giving K_c=273N/mm.

An estimate of the complex dynamic stiffness is obtained by replacing the static Young's modulus E_0 with the complex modulus E^*. Applying the static relationship between Young's modulus and the shear modulus for an incompressible material to the complex modulus yields $E^* = 3G^*$ and

$$K_c^{dyn} = \frac{3G_{dyn}}{E_0} K_c^{stat} = \frac{3 \cdot 0.9}{2.1} \cdot 273N/mm = 351N/mm$$

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Dynamic stiffness models will be dealt with more thoroghly in Chapter 8.

5.4 Capabilities in FE-codes

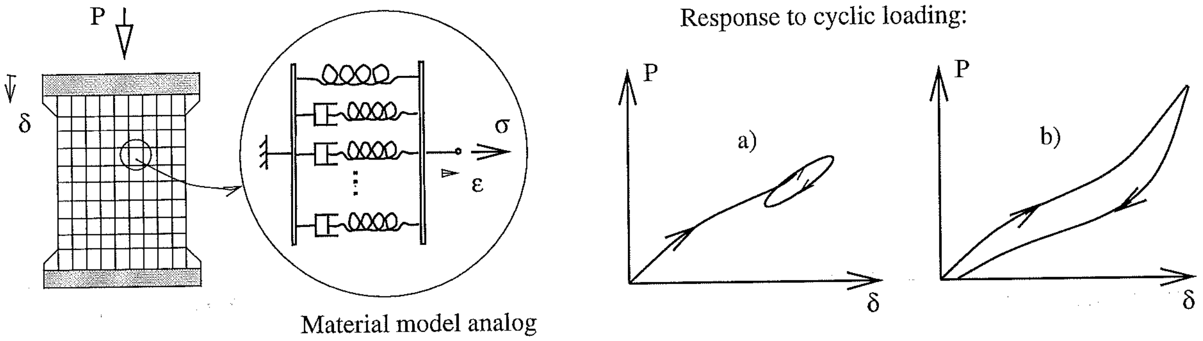

Figure 5.12: *Viscoelastic FEA for large strains: a) Small vibrations on a large static elastic deformation b) Large strain transient viscoelastic analysis.*

Constitutive models in FE-codes should be able to handle multiaxial stress and strain. The established models applicable to the behavior of rubber are the hyperelastic models and the viscoelastic models discussed earlier. Viscoelasticity provides a way of taking into account the time- and frequency-dependent properties of rubber behavior. However, only a few codes have the ability to handle large strain dynamic problems. The constitutive models in these codes combine hyperelasticity and viscoelasticity. These visco-hyperelastic models are modified linear viscoelastic models applied to large strains for which there is a purely rate-dependent damping (cf Figure 5.12). A few commercial codes include options for steady-state dynamic analysis considering small viscoelastic vibrations overlaid on a large static elastic deformation.

A model for viscoelastic analysis of large and arbitrary strains, the elastic behavior being based on hyperelasticity, have been developed by Simo [10]. His model also includes the modeling of damage. Viscoelastic models involving large strains for both nearly incompressible and compressible materials are implemented in the finite element codes ABAQUS and MARC.

A modification of the theory for the important case of small steady-state vibrations superposed on a large static hyperelastic state of strain was proposed by Morman [9]. This model has also been implemented in the finite element codes ABAQUS and MARC.

Experimental investigations suggest that unfilled rubbers can be dealt with by viscoelastic models [7], whereas in terms of the previous discussion the behavior of filled rubbers requires some form of rate independent damping model. The amplitude dependence of the dynamic stiffness is significant in highly filled rubbers and its influence is generally greater than that of frequency and temperature. Amplitude dependence is not included in the viscoelastic three-dimensional large-strain models available in finite element programs, which is a serious limitation in dynamic finite element analysis of filled rubbers. One-dimensional models that incorporate modeling of the amplitude dependence through rate-independent damping is disscussed in Section 5.5.

5.5 Nonlinear inelastic material models

The introductory discussion here concerns damping properties of filled rubbers, which will be discussed in terms of the simple one-dimensional model shown in Figure 5.13. The of this model.

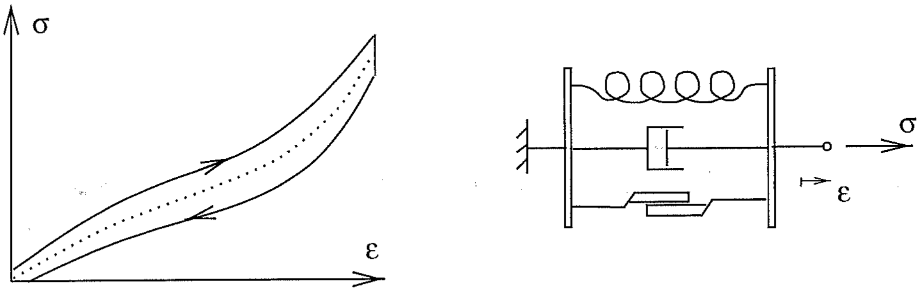

Figure 5.13: *Simple one-dimensional rheological model including nonlinear elastic, viscous, and frictional properties. A mechanical analog of filled-rubber behavior.*

The elastic behavior is provided by the spring element, which is assumed to be nonlinear. The rate-dependent part, represented by the viscous damper and the rate-independent part, is symbolized by a frictional element, consisting of two blocks with sliding contact between the surfaces. The elastic, viscous, and frictional stresses act in parallel, the sum of the stresses yields to the total stress,

$$\sigma = \sigma_e + \sigma_v + \sigma_f,$$

where σ_e is the nonlinear elastic stress, σ_v is the viscous stress, and σ_f the frictional stress. The viscous stress σ_v corresponds to dissipative stresses in the rubber network. Stresses in the filler phase and in the rubber-filler interfaces are responsible for the rate-independent contribution σ_f. If the model is subjected to loading followed by unloading, the response as plotted in a stress-strain diagram will exhibit the behavior as shown in Figure 5.13, yielding a difference between the loading and unloading paths. The elastic response, i.e. the nonlinear spring characteristic, is indicated by the dotted line. The viscous part of the stress will gradually vanish if the strain rate approaches zero. This is a reasonable assumption for rubbers without fillers. However, for filled rubbers there will always be a difference between the loading and unloading curves, even if the strain rate approaches zero. A rate-independent frictional stress component is necessary, therefore, to model the behavior of filled elastomers.

This model is consistent with the microstructure of a filled rubber, as shown in Figure 4.1. The elastic stress and the rate-dependent resistance are due to the rubber network, while the rate-independent stress is due to the filler. The forces that develop in the rubber network and between the filler particles act in parallel. The damping due to the filler adds to the damping in the rubber network.

The model incorporates some important aspects of the mechanical behavior of filled rubbers. It provides a qualitative and conceptual understanding of such properties as frequency dependence, effects of static load on the dynamic modulus, distortion of the hysteresis loop, and amplitude dependence. However, it has some apparent nonphysical properties, and it should not be interpreted as an exact quantitative model of elastomeric materials. For example, the stress response can be discontinuous even if the strain is continuous. Another limitation of the model is the absence of relaxation behavior. However, the sketched model is a starting point for the modeling of damping mechanisms in filled rubbers.

5.5.1 Frictional solid models

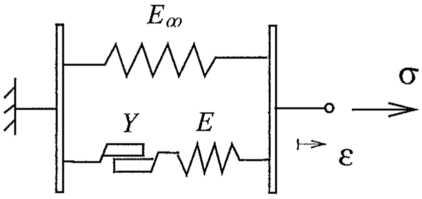

Figure 5.14: *A one-dimensional rheologic model including elastic and frictional properties.*

This section concerns the modeling of rate-independent damping by a model including a frictional element. Frictional one-dimensional models have been shown to provide a good representation of the behavior of filled rubbers under cyclic loading [1] (Chapter 10).

An important need is to model the amplitude dependence, i.e. the decrease in the dynamic modulus E_{dyn} with increasing strain amplitude.

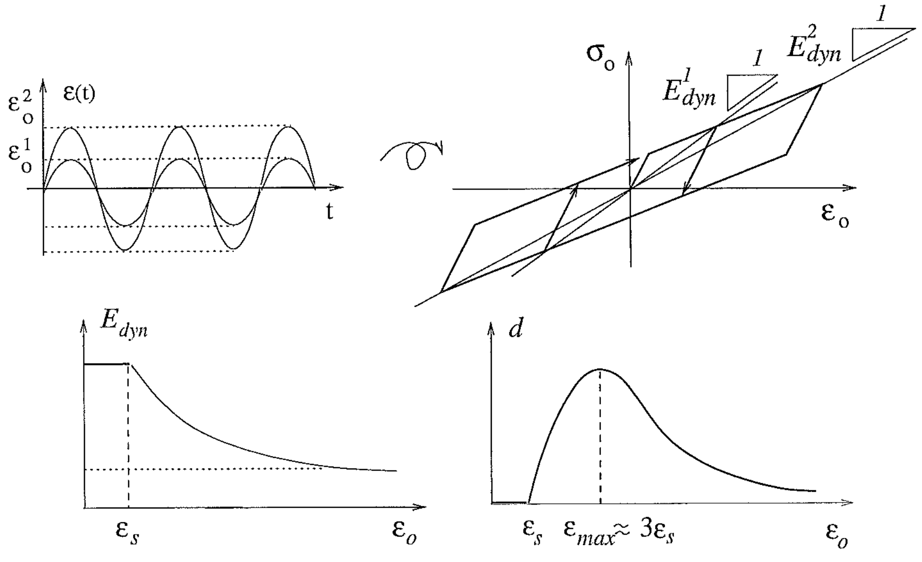

Figure 5.15: *Amplitude dependence of the dynamic modulus and the phase angle for the simple frictional model.* ($\epsilon_s = Y/E$)

The rheologic model shown in Figure 5.14 represents a solid model with rate-independent damping properties. Except for the dissipative element, the dashpot having been replaced by frictional blocks, this model is analogous to the Zener model. For fully developed friction, the rate-independent stress in the frictional element is limited to $\sigma_f = \pm Y$. As was shown in [1] (Chapter 10), a generalized version of this model could fit the amplitude dependence of the dynamic modulus and the phase angle for the steady state dynamic data reported by Harris and Stevenson [7].

The amplitude dependence of the dynamic modulus in the simple frictional model is shown schematically in Figure 5.15. The qualitative behavior is in accordance with experiments on filled rubbers.

The rate-independent frictional element thus introduces a nonlinearity that is seen in the amplitude dependence and the parallelogram-shaped hysteresis loop, contrary to linear dynamics, for which a sinusoidal strain results in a sinusoidal stress and thus in an elliptic hysteresis loop.

The stress response for the frictional solid model can be resolved into Fourier components which are odd multiples of the input frequency, in accordance with the experimental findings of Harris and Stevenson [7].

The simple frictional model has also been discussed qualitatively by Gregory [6]. He concludes that for heavily filled rubbers the hysteresis loop approaches a trapezoidal form, in accordance with the behavior of the simple model. Models with frictional elements coupled in series are discussed by Turner [11] and Coveney et al. [4]. These models are shown to fit the dynamic modulus and the phase angle obtained in shear experiments with highly filled rubbers.

A one-dimensional model of the amplitude dependence in the case of periodic sinusoidal loading was suggested by Kraus [8]. This model explains amplitude dependence here in terms of continuous breaking and reforming of the van der Waals forces between carbon-black aggregates. The Kraus model has been investigated and evaluated by many researchers, see for example Ulmer [11] and Vieweg [12]. The latter found that, within the range investigated, $0.06 - 20\ Hz$, the sigmoidal decline (in logarithmic coordinates) of the dynamic modulus does not depend on the frequency.

The following summarizes various properties of frictional damping:

- The hysteresis loop has the shape of a parallelogram.
- The hysteresis is independent of the rate (frequency) of loading.
- The dynamic stiffness decreases with amplitude.
- Harmonic excitation yields a non-harmonic response which can be resolved into harmonic components that are odd multiples of the excitation frequency.

5.5.2 Combined viscous and frictional models

The previous discussion focused on two sources of damping in filled rubbers, namely viscous damping in the rubber base and frictional damping in the rubber-carbon and carbon-carbon interfaces. A combination of rate-independent and rate-dependent damping is needed to account for the inelastic effects present in carbon-black filled rubbers.

The experimental observations presented in Chapter 4 indicated that the amplitude and frequency dependence of the dynamic shear modulus were nearly independent of each other. Hence, in terms of rheological models, the viscous and frictional elements should be connected in parallel, since the total damping stress is the sum of the viscous and the frictional stresses.

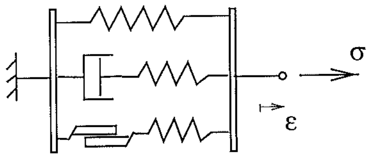

Figure 5.16: *Five-parameter model including elastic, viscous, and frictional properties.*

The one-dimensional model shown in Figure 5.16 is a model of this sort, to which viscoelastic and frictional stress contributions have been added. A step-size dependent relaxation modulus and an amplitude dependent dynamic modulus are properties of the model.

This model was, in [1] Chapter 11, shown to agree well with quasi-static and dynamic simple shear tests obtained for a highly filled rubber vulcanizate. A model with only five parameters that was fitted to stationary dynamic data, was shown to model the frequency and amplitude dependence of the dynamic modulus well.

Similar models have been used for vehicle dynamics and earthquake protection applications, for example. For dynamic analysis of rail vehicles, Berg [2] has proposed a five-parameter rubber spring model with elastic, frictional and viscous forces in parallel.

There appears to be no three-dimensional large-strain counterpart to the one-dimensional models that incorporate the modeling of rate-independent damping (cf. Figure 5.16), which is required for finite element analysis of filled rubbers.

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Chapter 6

General aspects of vibration isolation

U. CARLSSON AND L. KARI

Noise and vibrations have a negative effect on the human environment, as well as on various equipment and materials people make use of. A large part of the vibrations and noise many people are exposed to comes from machines and industrial processes. A useful strategy to prevent this from occurring is to interrupt the pathway between the source and the receiver which noise and vibrations follow. The use of elastic supports is one way of achieving this. In practical terms, this involves placing vibration isolators at particular points along the conduction route. For example, strongly vibrating machines in factories, apartment houses and office buildings are often mounted on elastic supports, just as springs between the chassis and the axles in cars are used to isolate the passenger compartment against vibrations generated by contact between the wheels and the road. The mounting of machines on elastic supports, if done in an appropriate way, is effective in reducing the transmission of noise and vibrations which the machines produce. It also need not be expensive.

Chapters 6, 7, and 8 aim to provide a basic understanding of the design of vibration isolaton and the physical principles involved, as seen in terms of current research in this area.

6.1 Source and receiver isolation

Vibration isolation aims at reducing the level of vibration at one or more locations. The idea is to limit the transmission of vibrations along the transmission route, see Figure 6.1. Vibration isolation can be carried out in many different ways. For one thing, the locations at which isolators are placed along the transmission path can differ. For another, isolators can be of quite differing size. It is important that as satisfactory a combination as possible of the placement and design of isolators be achieved.

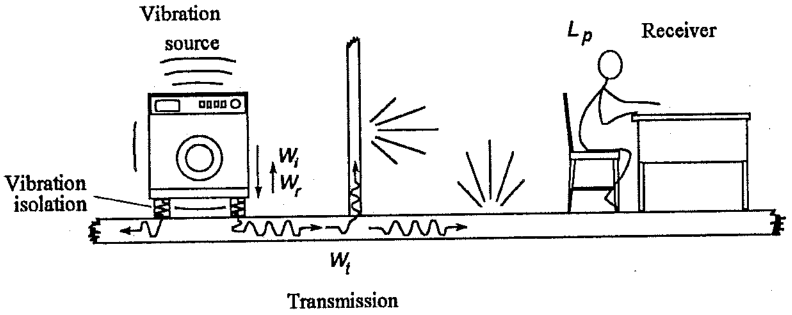

Figure 6.1: *Example of a situation in which a machine is isolated against the spreading of vibrations. Power W_i from the machine reach the vibration isolator, which reflects back the power W_r toward the source, the power W_t being transmitted to the floor.*

Regarding where isolators are placed, two opposite approaches can be distinguished: placing them near the source and near the receiver. The first case, involving isolation of the source from the surroundings, is referred to as source isolation. The second, involving isolation or protection of the receiver from the surroundings, is termed receiver isolation. The two approaches are shown schematically in Figure 6.2. Note that source isolation and receiver isolation can also be combined. If achieving as low a level of vibration as possible is called for, the decision is often made to isolate both the source and the receiver.

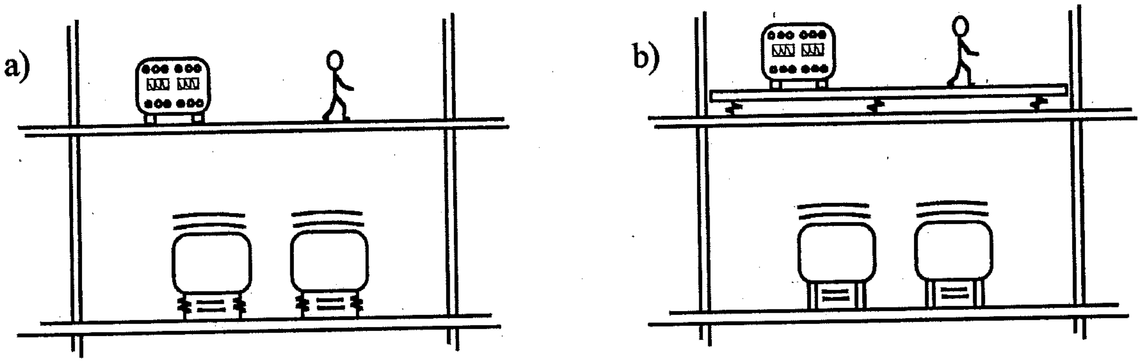

Figure 6.2: *Two different strategies for vibration isolation: a) source isolation of machines and b) receiver isolation of sensitive equipment.*

6.2 General principles for vibration isolation

A problem involving vibration isolation is often described schematically as consisting of a number of different sub structures, including a source structure, a receiver structure and vibration isolators located between the two. Vibration isolation aims at reducing vibration in some particular part of the receiver structure.

It can be carried out in differing ways. The task of the designer is to select some appropriate form of isolation.

Vibration isolation is governed by simple physical principles. When a wave transmitted through an elastic medium encounters a region in which the characteristics of the medium change, only a part of the wave passes through, the remainder of it being reflected back at the approaching wave. The magnitude of the reflected part of the wave is dependent upon the size of the change in the medium. Vibration isolation reduces the transmission of vibrations by effecting changes in the medium along the transmission path.

The most usual way of achieving a change in the medium is to introduce an element of lower stiffness than the parts of the medium adjacent to it, see Figure 6.3 a. Such an element is termed a vibration isolator. Examples of vibration isolators available commercially are rubber isolators of different kinds and coil springs made of steel. Note that the stiffness of the medium can also be changed by introducing an element stiffer than the adjacent parts.

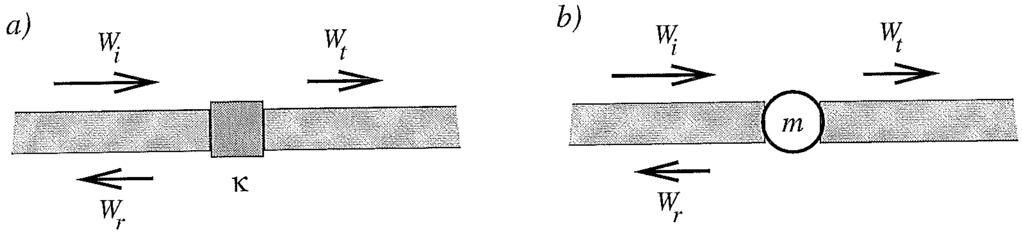

Figure 6.3: *Two different methods of vibration-isolation: a) Reflection by a soft element. b) Reflection by a mass.* W_i *= incident power,* W_r *= reflected power och* W_t *= transmitted power.*

Reflection of vibrations can also be produced by an element that changes the inertia of the system. Such an element is often treated computationally as representing a rigid mass, see Figure 6.3 b. Elements of this sort, are in Figure 6.4 exemplified by auxiliary blocks placed at soft foundation points and seismic blocks.

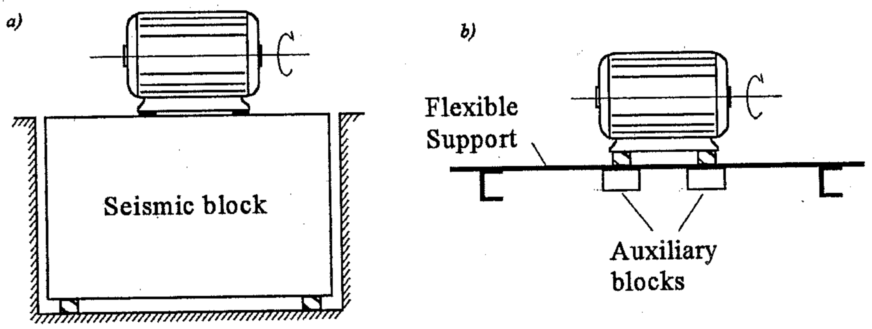

Figure 6.4: *a) Seismic block. b) Auxiliary blocks placed at points of flexible support.*

The most usual construction materials, such as steel and concrete, are relatively stiff. Thus, the simplest and most frequently employed approach to achieving major changes in conductive media is to use soft elements as isolators, although in some cases stiffer elements are needed for purposes of stability and strength.

In designing vibration isolators for machine use have long been made of principles of trial and error combined with the making of rough estimates. Although this can yield good results in the low-frequency range below approximately 100 Hz, it fails to adequately indicate how vibration isolators should be designed in the entire acoustic frequency range. This calls for the use of more advanced theoretical and experimental methods.

6.3 Measures of transmission isolation

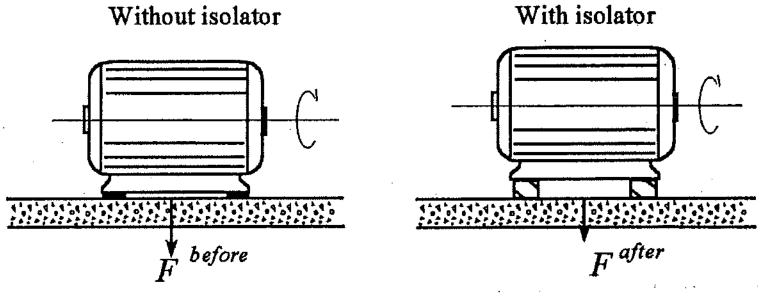

Figure 6.5: *Isertion loss can be defined as the difference in force level before versus after isolation.*

Designing a vibration isolaton in an optimal way requires not only determining what vibrations are produced, but also having some measure of the vibration isolation the isolator can provide. There are various measures of this sort. These vary in the applications to which they apply. *Insertion loss*, D_{IL}, is the measure most frequently employed. This can be defined in any of three different ways:

$$D_{IL}^{v} = L_v^{before} - L_v^{after} \qquad [dB] \tag{6.1}$$

$$D_{IL}^{F} = L_F^{before} - L_F^{after} \qquad [dB] \tag{6.2}$$

$$D_{IL}^{W} = L_W^{before} - L_W^{after} \qquad [dB] \tag{6.3}$$

where L_v, L_F and L_W - or the velocity, force and sound power level, respectively - are defined as

$$L_v = 20log\frac{\hat{v}}{v_{ref}} \qquad [dB] \tag{6.4}$$

$$L_F = 20log\frac{\hat{F}}{F_{ref}} \qquad [dB] \tag{6.5}$$

$$L_W = 10log\frac{\overline{W}}{W_{ref}} \qquad [dB] \tag{6.6}$$

the wavy bar denoting a rms value and the straight bar a mean value, the reference levels being $v_{ref} = 10^{-9}$ m/s, $F_{ref} = 10^{-6}$ N and $W_{ref} = 10^{-12}$ W.

Insertion loss is thus defined as the difference, at some given point, between the velocity, force or sound power levels prior to and after applying the vibration isolator in question, see Figure 6.5. Other, related measures can be defined through different weighting of the various frequency components or frequency bands, such as in the case of A-weighting, the latter being achieved by use of a filter similar in effect to the perception of different vibrations by the human ear. The choice among the types of measures listed in (6.1) - (6.3) depends upon the specific application.

6.4 Computations of vibration isolation

For designing vibration isolaton for frequencies of up to about 1000 Hz, complex computations are required. In principle, computational models for the machine, the isolator and the foundation, correctly describing the relationship between loads and the accompanying deformations within the entire frequency range, are needed. One soon discovers that the computational tasks required are extremely difficult. Even computations using the finite elements method, carried out in detail are so time-consuming and yield such uncertain results as to be of very doubtful value.

In practice, therefore, one is forced to use simplified computational models for the various structural components. Although such models should not be used for obtaining exact values for the degree of vibration isolation achieved within narrow frequency bands, they can be used to advantage, with the help of octave or tertiary band analysis, for comparing alternative solutions. They can also be used to indicate basic tendencies and to suggest how vibration isolation can be improved. Fairly reliable results can be obtained through utilizing so-called frequency-response-function methods, using measured indata. The measurement of frequency response functions, in particular in connection with vibration isolators, is an important research area in itself.

Chapter 7

Different models for the computation of vibration isolation

U. CARLSSON AND L. KARI

In order for comparisons of alternative vibration isolations to be practically achievable on a computational basis, simplified models are needed. The overall problem is that at each point of connection between different parts of the structure the transmission of vibrations involves a large number of degrees of freedom. To simplify the problem, one can

(i) assume that the connecting areas are very small, i.e. of point-wise character.

(ii) assume that the vibration transmission at such points involves only one or two degrees of freedom.

(iii) disregard connecting points that contribute only slightly to the transmission of vibrations.

(iv) combine different transmission routes to form a single route that is equivalent to these.

The last-named approach is applicable if the points connecting the isolator with the machine and with the foundation move in phase with each other, the amplitude of the movement being the same in each case.

7.1 Rigid bodies, ideal springs and foundations

When the excitation frequencies are very low, a simplified model of the component structures can be employed. Assume, for example, that a machine resting on a concrete joist is supported at four points and that the machine has an axle generating sinusoidal forces at the bearings, generating them at the same frequency as the rotation of the axle. At very low excitation frequencies (axle-rotation frequencies), the deformation of the machine is very slight, it behaves as a rigid body. In physical terms, the forces acting upon the machine vary over time so slowly that all parts of the machine react to the forces before the next change in forces occur. Mathematically, the machine's movements can be described in terms of rigid body mechanics. The machine's position is determined completely by six degrees of freedom, whereby three represent translation and three rotation. In practice, the number of degrees of freedom can often be reduced to one or two through eliminating those that are unimportant.

When the rotation frequency of the axle increases, the forces applied finally come to vary so rapidly that not all parts of the machine succeed in reacting before the forces at the point of application change anew. Now, the transmission of waves within the machine begins to play a role.

If the rotation frequency increases still more, an excitation frequency is reached at which the amplitude of the machine's deformation rises quickly to a maximum, due to the deformation waves and the reflections of these supporting each other. This represents the resonance phenomenon. At such frequencies, the machine no longer behaves as a rigid body. A rule of thumb here states that the principles pertaining to a rigid body can be employed for frequencies of up to 1/3 of the first resonance frequency of the machine.

A rigid body model corresponding to that of the machine can also be applied to model the foundation. Consider yet another case involving the machine just referred to. At very low interfering frequencies, the joist on which the machine rests reacts with a quasi-static downward bending due to the slowly varying force acting upon the points on which the machine rests. When the excitation frequencies are so low that the deformation of the joist is negligible compared with the deformation of the isolator, the joist can be described, in terms of vibration isolation, as a stiff foundation. Note that this does not mean that the foundation fails to be set into vibration at all. If such were the case, no vibrations whatever could be transmitted to the foundation.

Let the excitation frequency increase now in the same way as it did with the machine. When the frequency becomes sufficiently large, the deformation it produces can no longer be neglected, the transmission of waves in the foundation increasing. If the boundaries of the foundation are nearby, the first resonance frequency of the foundation soon appears. Thus, it is only at low frequencies, up to approximately 1/3 of the foundation's first resonance frequency, that it can be described as rigid.

Assume now that one wants to reduce the vibrations transmitted from the machine to the supporting joist by inserting soft vibration isolators between the machine and the surface of the joist, doing so at the points on which the machine rests. The force exerted by the machine deforms the spring which each of the isolators represents. At low excitation frequencies, all parts of the isolator are able to react to changes in the load upon it. Thus, the force exerted on the isolator is the same throughout. Accordingly, no appreciable wave transmission occurs. The isolator can therefore be considered at this point to be without mass. Since in contrast to the joist, however, the isolator is soft, the deformation it experiences when subjected to excitation forces cannot be neglected. Under such conditions, the isolator can be described as an ideal spring without mass. As the frequency increases, however, the movements assume increasingly the character of a wave. Again, beyond a particular excitation frequency, resonance appears. A rule of thumb analogous to that described above applies here, stating that up to about 1/3 of the isolator's first internal resonance frequency the the conditions of an ideal spring apply.

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Case 1

Consider the electric motor shown in Figure 7.1 a, mounted on four identical isolators attached to a 2 cm thick steel plate.

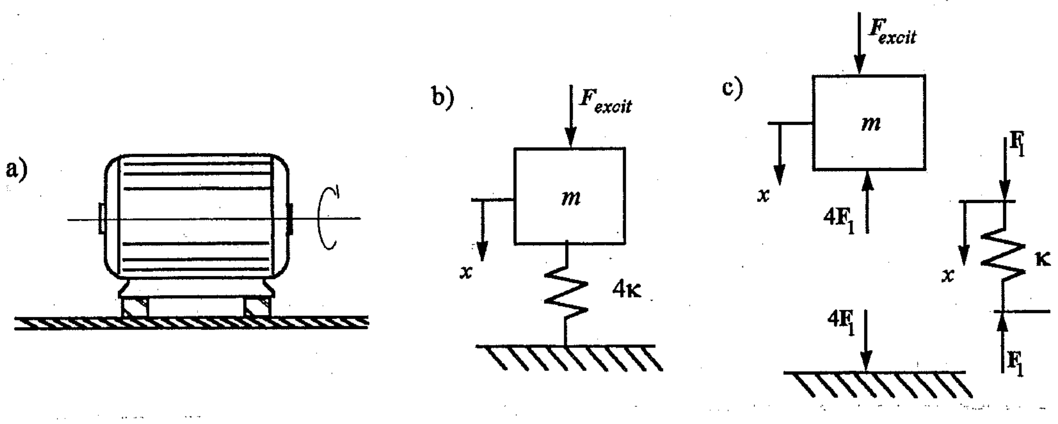

Figure 7.1: *a) An electric motor mounted on a large steel plate. b) A simplified model of the system shown in case a. c) The system in case b represented by subsystems.*

When the motor is in operation, the rotating parts of it generate a vertically-directed sinusoidal excitation force between the machine and the steel plate supporting it. For the total force directed at the steel plate, compute the ratio of the force when the vibration isolators are installed to the force when they are not present. For the computations pertaining to low frequencies, assume that the electric motor, when in operation, generates a vertical harmonic excitation force of $\boldsymbol{F}_{excit} = \hat{F}exp(i\omega t)$. Assume too that the motor has a mass of 100 kg and that the complex stiffness of each isolator is $(1.0 + 0.01i)10^4$ N/m.

Solution:

Assume that the excitation frequency is sufficiently low that

(i) the motor can be considered to behave as a rigid body,

(ii) the foundation can be considered to be rigid and infinitely stiff and

(iii) each isolator can be described as an ideal spring without mass.

Assume, in addition, that the movements of the motor are dominated by vertical translations of low amplitude. Under these conditions, the one-degree-of-freedom model shown in Figure 7.1 b provides a useful description of the problem.

With isolators:

For the exposed system shown in Figure 7.1 c, the following two equations, for movements of the mass m and for the action of the springs, can be expressed

$$m\frac{d^2\boldsymbol{x}}{dt^2} = \boldsymbol{F}_{excit} - 4\boldsymbol{F}_1$$

and

$$\boldsymbol{F}_1 = \boldsymbol{\kappa}(\boldsymbol{x} - 0) \; .$$

The complex sinusoidal displacement $\boldsymbol{x} = \hat{x}exp(i\omega t)$ is eliminated in the two equations just listed. The force exerted on the foundation, normalized to the excitation force, then becomes

$$\frac{4\boldsymbol{F}_1}{\boldsymbol{F}_{excit}} = (1 - \frac{\omega^2}{4\boldsymbol{\kappa}/m})^{-1} = (1 - \frac{\omega^2}{\omega_0^2})^{-1}$$

ω_0 being the rigid body resonance frequency, i.e. the resonance frequency of the vibration of the machine's mass against the vibration isolators.

Without isolators:

As one can readily see, when no isolator is employed the force against the foundation becomes $\boldsymbol{F}_{excit}$. The ratio of the force against the foundation with versus without an isolator is thus given by

$$\frac{\boldsymbol{F}_{without}}{\boldsymbol{F}_{with}} = (1 - \frac{\omega^2}{\omega_0^2}) \; .$$

Defining insertion loss on the basis of this ratio, one gets

$$D_{IL} = 20log\left|1 - \frac{\omega^2}{4\boldsymbol{\kappa}/m}\right| = 20log\left|1 - \frac{\omega^2}{\omega_0^2}\right| = 20log\left|\frac{\boldsymbol{Y}_m + \boldsymbol{Y}_i}{\boldsymbol{Y}_m}\right| \qquad (7.1)$$

where $\boldsymbol{Y}_m$ and $\boldsymbol{Y}_i$ represent the mobility of the machine and of the isolator, respectively, defined as the ratio of the velocity to the force involved. Note that the equation also applies to receiver isolation.

Insertion loss has certain definite characteristics in such a case, see Figure 7.2. For one thing, for excitation frequencies markedly lower than the first resonance frequency f_0, corresponding to ω_0, the insertion loss has a value of null. For another, for excitation

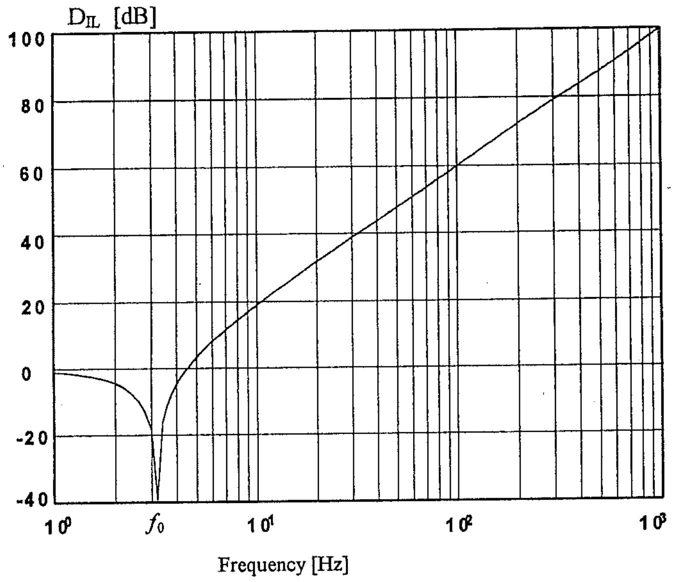

Figure 7.2: *Insertion loss of a rigid body elastically mounted on a rigid surface,* $f_0 = 3.18$ *Hz.*

frequencies that approach the rigid body resonance frequency, the insertion loss assumes large negative values. Thus, in the vicinity of the rigid body resonance frequency, forces directed against the foundation tend more to be enhanced than to be reduced. Finally, as the frequency increases above the level of the first resonance frequency, the insertion loss increases rapidly so as to assume high positive values. The asymptotic increase approaches 40 dB per decade, which means an increase of 40 dB for each tenfold increase in the excitation frequency. For excitation frequencies that exceed the rigid body resonance frequency, isolation thus appears to be highly effective. Unfortunately, this apparent effectiveness is largely the result of using a highly simplified model. In reality, the increase in insertion loss stops when the excitation frequency exceeds a level of $10f_0$.

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An important conclusion to be drawn on the basis of Case 1 is that in designing vibration isolators one should see to it that the rigid body resonance frequency does not coincide with any major excitation frequency. One should thus endeavor to design isolators in such a way that the rigid body resonance frequency is as low as possible. In practice, machine foundations are often so designed that the rigid body resonance frequency is in the range of 2 - 10 Hz.

7.2 A flexible foundation

As the excitation frequency increases, the state is soon reached in which the deformation of the foundation by the interfering force is no longer negligible. This calls for the use of a model concerned with a flexible foundation. Various models, differing in their characteristics, are available. If, for example, the foundation is a joist of large dimensions, a plate infinite in its dimensions can serve as a model for the deformation occurring. If the deformation of the surface involves resonance, a mass-spring system can be used to provide an initial approximation of it.

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Case 2:

Consider the machine and its installation described in Case 1. Assume that the model involving a steel plate of infinite dimensions is applicable to the deformation of the foundation. Compute the ratio with versus without isolators for the total force directed against the foundation.

Solution:

Assume the deformation of the foundation to be the same at each of the four legs on which the machine is standing. Regard the assumptions (i) and (ii) made in connection with Case 1 as also applying here.

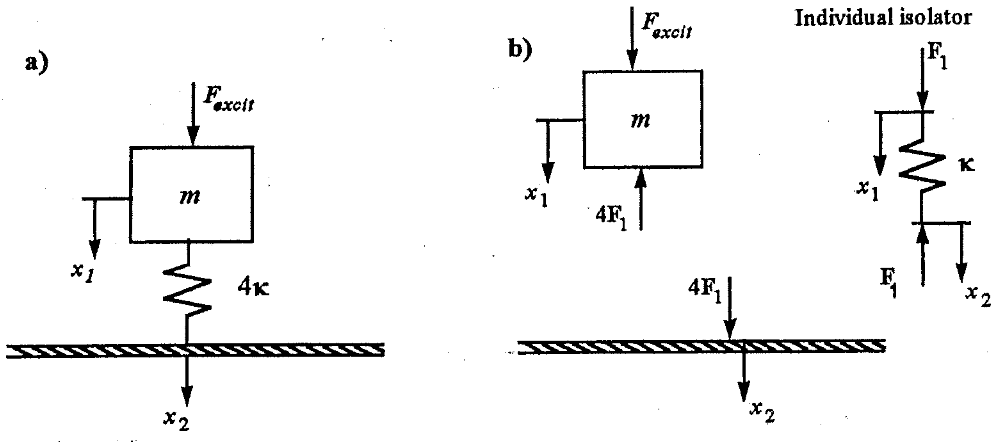

Figure 7.3: *Simple model of a machine mounted on a flexible supporting surface.*

The following system of equations given by the equation of motion,

$$m\frac{d^2\boldsymbol{x}_1}{dt^2} = \boldsymbol{F}_{excit} - 4\boldsymbol{F}_1$$

the spring stiffness expression

$$\boldsymbol{F}_1 = \boldsymbol{\kappa}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$$

and the mobility expression of the plate

$$\boldsymbol{x}_2 = (i\omega)^{-1}\boldsymbol{Y}_{plate}4\boldsymbol{F}_1.$$

applies according to Figure 7.3. Eliminating $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ gives

$$\frac{4\boldsymbol{F}_1^{with}}{\boldsymbol{F}_{excit}} = \frac{-4\kappa/m\omega^2}{1-4\kappa/m\omega^2+4\kappa/(i\omega)\boldsymbol{Y}_{plate}} = \frac{\boldsymbol{Y}_m}{\boldsymbol{Y}_m+\boldsymbol{Y}_i+\boldsymbol{Y}_{plate}},$$

where $\boldsymbol{Y}_m = 1/i\omega m$ and $\boldsymbol{Y}_i = i\omega/4\kappa$. When no isolators are present, the forces on the foundation are determined by eliminating the second equation above concerning the spring stiffness and setting $\boldsymbol{x}_1$ equal to $\boldsymbol{x}_2$. The resulting system of equations has the solution

$$\frac{4\boldsymbol{F}_1^{with}}{\boldsymbol{F}_{excit}} = \frac{1/i\omega m}{1/i\omega m+\boldsymbol{Y}_{plate}} = \frac{\boldsymbol{Y}_m}{\boldsymbol{Y}_m+\boldsymbol{Y}_{plate}}.$$

The insertion loss is then

$$D_{IL} = 20log\left|\frac{\boldsymbol{Y}_m+\boldsymbol{Y}_i+\boldsymbol{Y}_{plate}}{\boldsymbol{Y}_m+\boldsymbol{Y}_{plate}}\right|. \tag{7.2}$$

Equation (7.2) can be shown to also apply to the case of receiver isolation. By use of appropriate formulas, the mobility of a very large plate can be derived;

$$\boldsymbol{Y}_{plate} = \frac{\sqrt{3(1-\nu^2)}}{4h^2\sqrt{\rho E}}.$$

For a 2 cm thick steel plate, with the parameters $\rho = 7800kg/m^3$, $E = 2.0\cdot10^{11}Pa$, $\nu = 0.3$, and $h = 0.02m$, the mobility becomes $\boldsymbol{Y}_{plate} = 1.35\cdot10^{-6}m/Ns$.

Inserting into the equation for insertion loss (7.2) the values obtained yields the curves shown in Figure 7.4, which displays results for both a flexibel and a rigid foundation.

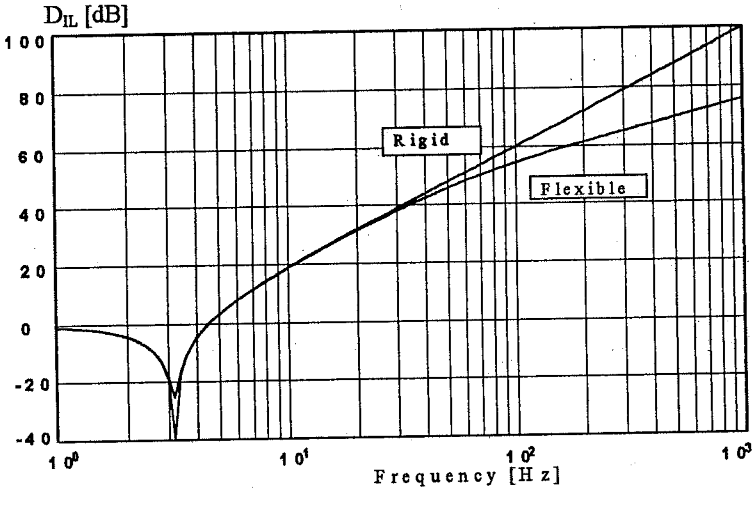

Figure 7.4: *Insertion loss of a rigid body mounted elastically on a steel plate of infinite size.*

A surface of the latter type obviously affects the insertion loss in two different areas: that in which the rigid body resonance frequency is found and that lying above approximately 50 Hz. At the rigid body resonance frequency an increase in insertion loss is observed due to the energy absorbing ability of the infinite plate. In the region above 50 Hz, there is a considerable drop in the isolation efficiency because of the flexible supporting plate. The isolation there being largely determined by the ratio of the mobility of the isolator to that of the plate. At high frequencies the isolation increases asymtotically by 20 dB per decade, as opposed to the 40 dB per decade it increases with the rigid foundation.

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7.3 Wave transmission in the isolator

If the excitation frequency increases so much that a wave movement develops in the isolator, the ideal spring model is no longer adequate for describing the isolator. Different models for wave transmission can then be employed, depending upon the design of the isolator. Case 3 provides an example of a simple wave transmission model of an isolator.

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Case 3:

Consider again the machine and its mounting dealt with in Case 2. Assume the isolator to be a circular cylindric rod that mainly becomes deformed in the axial direction. Since the deformation is axial, movements within the isolator consist primarily of axially directed longitudinal waves. To permit a direct comparison with the previous case, assume each isolator to be 0,05 m in height and to have a cross-sectional area of 0,005 m^2. Assume also the density of the isolation material to be 2500 kg/m^3 and that the complex Young's modulus is $\boldsymbol{E} = 0,1(1 + i0,01)\, MPa$. This agrees with the stiffness of the isolator at low frequencies as found in cases 1 and 2 above.

Solution:

Block one end of the isolator. This allows the frequency-dependent dynamic isolator stiffness κ_{dyn} to be computed on the basis of the ratio of the force at the blocked end to the displacement produced at the free end. If the stiffness κ obtained in Case 2 is replaced by this dynamic stiffness, one obtains an insertion loss that takes account of the longitudinal wave transmission through the isolator. This result can be compared with the results described earlier, shown in figures 7.4 and 7.5.

The longitudinal transmission of waves through the isolator leads at certain frequencies to the isolator's becoming very stiff. This is due to the combined action of the ongoing and the reflected waves. At such frequencies, the isolator no longer functions as a soft element. At these frequencies the insulating effect is very low. In the present case, these higher resonance frequencies are 64 Hz, 130 Hz, etc. As can be seen in Figure 7.5, the second resonance frequency results in the insertion loss failing then to increase. At higher frequencies, the average insertion loss remains approximately constant, remaining in the case above, except at the higher resonance frequencies, at about 40 dB.

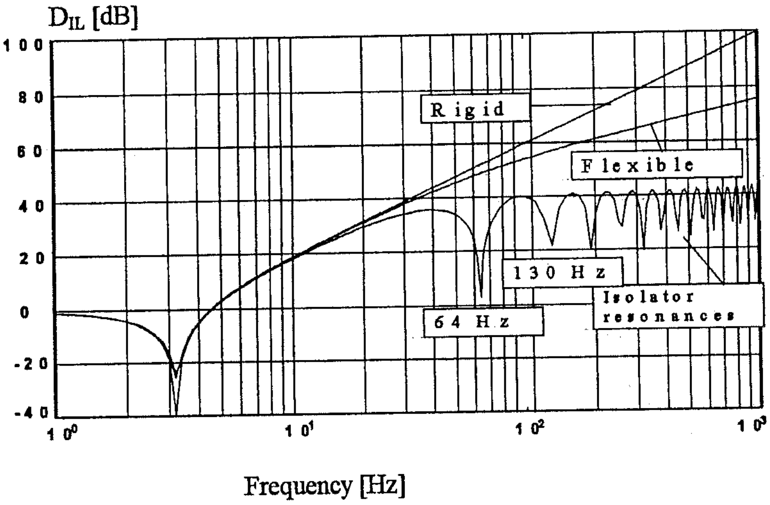

Figure 7.5: *Insertion loss. Summary of the results for cases 1 through 3*

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Case 3 shows clearly how wave transmission through the isolator stops the rise in insertion loss.

7.4 A deformable machine

For cases 1 to 3 it was assumed that the machine moves like a rigid mass in one coordinate direction. At excitation frequencies in the region of the first resonance frequency, the insertion loss thus becomes very low. If several or all of the machine's six degrees of freedom for a rigid body are included in the model, this results in there being several different rigid body resonance frequencies. In the most general case, the insertion loss thus disappears at six different rigid body resonance frequencies. In a real situation, a machine also displays internal resonances at certain frequencies. For a compact machine with a mass of 100 kg, such as a small combustion engine, the first internal resonance frequency is typically found in the region of 100 Hz - 500 Hz. If a machine contains different softly coupled parts, the first resonance can appear at a still lower frequency.

The possibility of wave transmission also affects the insertion loss of an elastically mounted machine. The insertion loss that results depends partly on the comparative stiffness of the machine versus that of the isolator. If the stiffness of the machine varies markedly due to resonance and antiresonance, the insertion will vary markedly as well. Above the first internal resonance frequency, the average degree of isolation declines. Figure 7.6 shows the insertion loss of a simple system consisting of a machine with internal resonances, together with the same rigid foundation and the same type of isolators as described in Case 1.

In this case the machine has resonances at 185, 345 and 535 Hz and antiresonances at 160, 205 and 495 Hz. Figure 7.6 indicates that at resonance frequencies

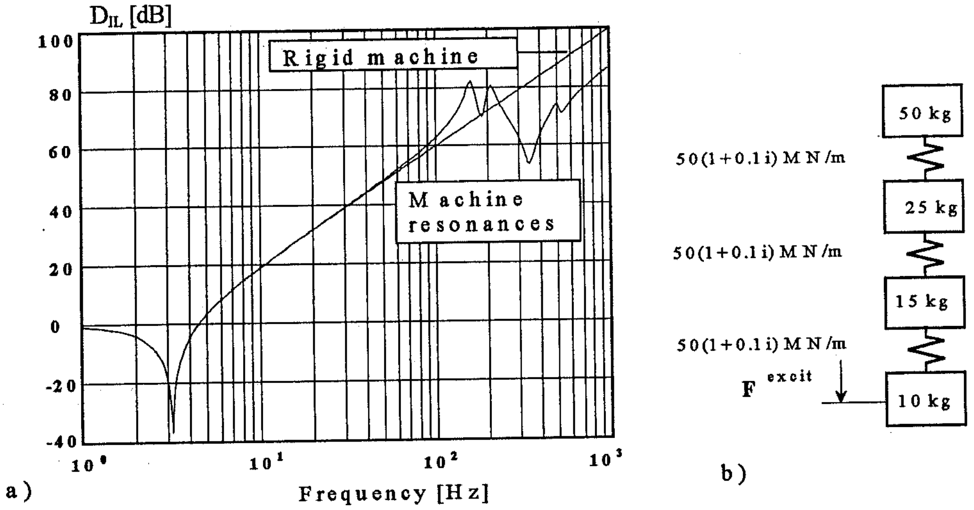

Figure 7.6: *a) Insertion loss for the machine mounting in Case 1, taking account of the internal resonances in the machine as well. b) A mechanical model of the machine.*

where the machine becomes soft there is a decrease in insertion loss, whereas at antiresonance frequencies the insertion loss increases, the machine becoming very stiff. As the excitation frequency increases within the frequency region above the first resonance of the machine, the average insertion loss successively declines. The decline is caused by the effective mass of the rigid fondation that moves with the isolator successively decreases as the number of resonance frequencies that have been passed increases.

7.5 General formula for computing insertion loss

In solving cases 1 to 3, a general approach to computing insertion loss was developed, though not commented on there. This is an approach applying to a structure of the type shown in Figure 7.7. The approach is based on its being possible to replace the mobility of each of the three initial structures $\boldsymbol{Y}_m$, $\boldsymbol{Y}_i$ and $\boldsymbol{Y}_{plate}$ by the true, computed or measured mobilities $\boldsymbol{Y}_M$, $\boldsymbol{Y}_I$ and $\boldsymbol{Y}_S$.

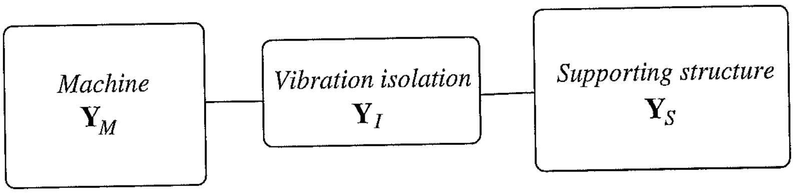

Figure 7.7: *The general vibration isolation problem.*

This results in a formula for insertion loss involving two different characteristic structures and the presence of one or more isolators such that

$$D_{IL} = 20log\left|\frac{\boldsymbol{Y}_M + \boldsymbol{Y}_I + \boldsymbol{Y}_S}{\boldsymbol{Y}_M + \boldsymbol{Y}_S}\right|. \tag{7.3}$$

The system must also satisfy the condition of its being possible to regard the transmission routes between the machine and the foundation as a single route. If one needs to consider different routes of transmission, a generalized form of (7.3) is required. Note that (7.3) also applies to the case of receiver isolation.

Chapter 8

Practical aspects of vibration isolation

U. Carlsson and L. Kari

It was shown in a series of cases above that at high frequencies it is not possible to obtain the high insertion loss which the simplified computational models predict. This is due to the assumptions of the simplified models not applying to conditions of high frequency. Under such conditions, the machine and the foundation on which it stands are no longer rigid and the isolators are no longer soft.

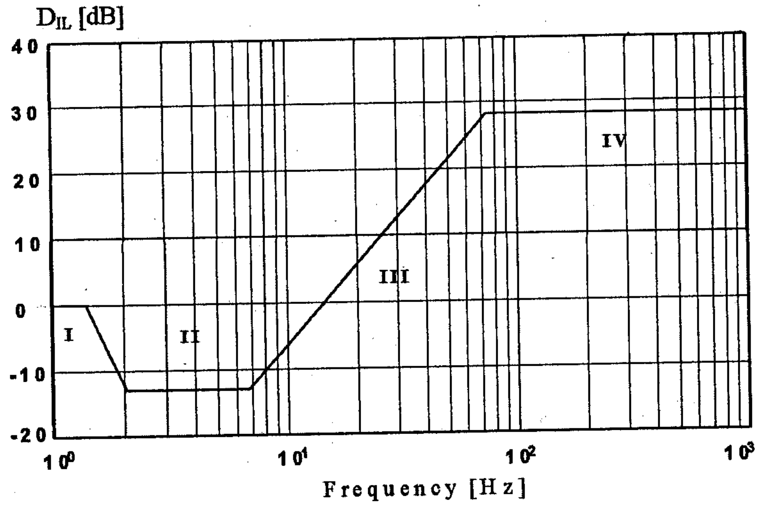

Figure 8.1: *Typical curve for insertion loss. I.) Frequencies below the rigid body resonance frequencies. II.) Rigid body resonance frequencies. III.) Stiff machine - soft isolator - rigid foundation. IV.) Internal resonances in the machine, the foundation and the isolator.*

In practice, the simple models are often found to give acceptable results up to about 100 Hz. Above that point, insertion loss tends to remain at a relatively constant level, between about 20 and 30 dB, see Figure 8.1. Under some conditions, not even that level of insertion loss is achieved at high frequencies. If the point of support is fairly soft, the isolation obtained at high frequencies is often no more than one of 15 dB. Figure 8.2 shows, for a machine mounted elastically on an aluminum section of a vehicle, how the the choice of support affects insertion loss. The machine can be described as a rigid mass and the isolator as an ideal spring. Curve a) shows the insertion loss when the points of support can be considered rigid. The other two curves show the insertion loss for each of two alternative types of mountings of the machine, the one type being stiffer than the other, being supported by two crossed ribs, the other being softer and being supported by a single rib. If use of the softest points of support is selected, the insertion loss at high frequencies becomes no more than 7 - 8 dB.

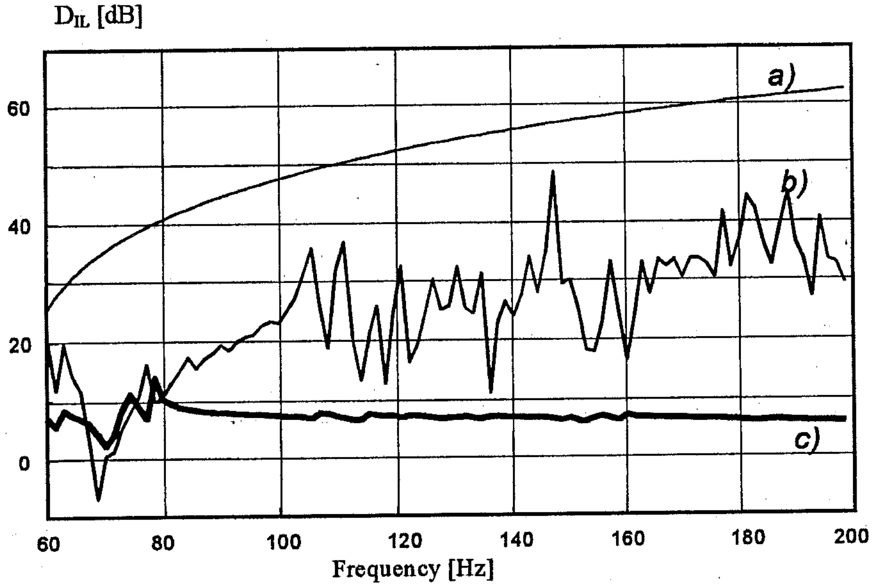

Figure 8.2: *Vibration insertion loss. a) Rigid support. b) Flexibel support, mounted on the intersection of two ribs. c) Flexibel support, mounted on a single rib.*

8.1 Designing of vibration isolators

There are a number of rules of thumb to follow in designing vibration isolatons. If these are followed, the results obtained should be acceptable.

(i) A (static) stiffness of the isolator should be chosen that is low enough so that the highest rigid body resonance frequency is considerably lower than the lowest excitation frequency.

(ii) The points of support should be as stiff as possible.

(iii) The points at which the machine is joined to the isolators should be as stiff as possible.

At low frequencies, one usually has no difficulties in applying rules (ii) and (iii). At higher frequencies, however, problems appear due to internal resonances.

(iv) Insofar as possible, an isolator should be so designed that its first internal anti-resonance frequency is higher than the highest excitation frequency of interest.

This last rule is very difficult to adhere to in practice. If it cannot be followed, one should carry out appropriate measurements or computations and see to it that the following rules are adhered to:

(v) The isolator should be so designed that its internal anti-resonances do not coincide with any of the strong components of the excitatory spectrum.

(vi) Insofar as possible the isolator should be so designed that its anti-resonance frequencies do not coincide with the resonance frequencies of the support.

In addition to these rules, one have to follow a number of other design rules, concerning for example the geometry, the durability and the stability of the system.

8.2 Various methods of improving insertion loss

Under some conditions, one can achieve a considerable improvement in insertion loss by use of rather simple methods. If particularly effective isolation is called for, so-called double vibration isolation can be employed. This represents a combination of elastic elements and a mass, see Figure 8.3.

In practice, double vibration isolation can be achieved by introducing between the machine and the supporting surface a structure which possesses considerable mass and still acts as a rigid body at as high a frequency as possible.

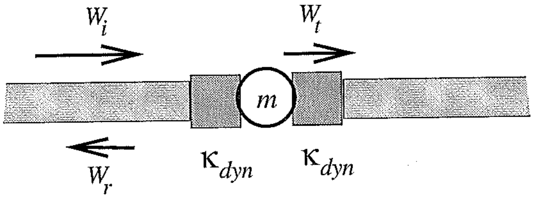

Figure 8.3: *Schematic representation of double vibration isolation.*

Machinery contained within a vehicle, for example, can be mounted elastically on a stiff and heavy supporting structure which, in turn, is mounted elastically on a supporting structure, see Figure 8.4.

If the double-elastic mounting is carried out properly, it can lead to an improvement in insertion loss. Due to a rigid body's being added, the isolation system possesses then six separate internal rigid-body resonance frequencies i.e. six separate secondary resonance frequencies above the basic rigid body resonance

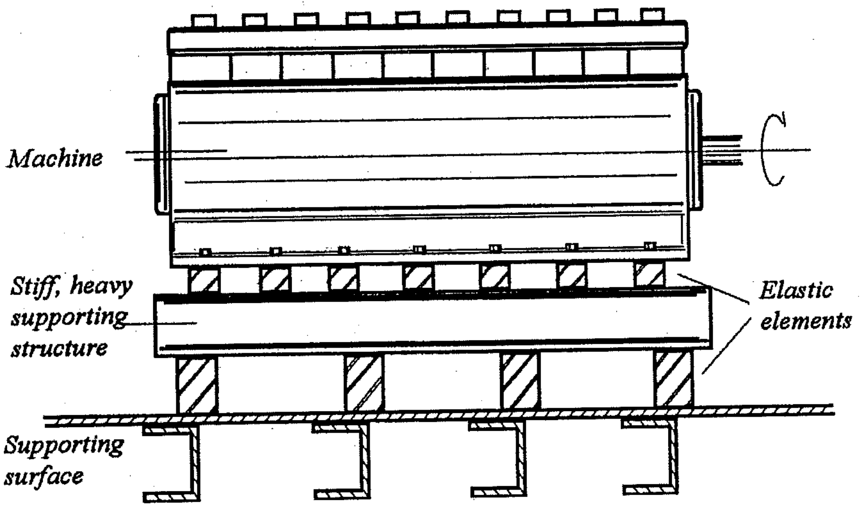

Figure 8.4: *Double-elastically mounted diesel engine in a vehicle.*

frequencies, the insulation efficiency thus being very low. For this reason, the isolators should be designed so that the secondary resonance frequencies are lower than the first excitation frequency of interest. If the additional structures are designed so that their mass and moments of inertia are of the same magnitude as those of the machine, the internal resonance frequencies of the isolation system lie fairly near to the rigid body resonance frequencies. This can be shown in the lower limit being rather low for area III in Figure 8.1, where the insertion loss increases rapidly. Vibration isolation there has a positive effect at low frequencies. If the design of the double-elasic system has been successful, the insertion loss within area III increases then by 80 dB per decade, twice as much as in the case of conventional isolation.

Some types of mechanical constructions, such as water or land vehicles of different types, require that machines be mounted at relatively soft points. Motors of small boats such as those used for recreation purposes are often mounted, by way of vibration isolators, directly on the thin, soft body of the boat. Vibration isolation has no effect then, due to the difference in impedance between the points of contact and the isolators being too small. One method of increasing the difference in impedance is to use inserts that add additional mass at the points of contact, see Figure 6.4 b. If the mass of these inserts is sufficient, the insertion loss can be increased considerably.

8.3 Vibration isolators available commercially

There is a large market for vibration isolators. Among the types available commercially are spiral springs made of steel, isolators made of rubber and gas springs. The two fundamental characteristics of an isolator are dynamic stiffness and a loss factor. The stiffness is the characteristic that largely determines whether an isolator can be used at all, as was shown above. The loss factor is of importance as an amplitude-limiting factor in the case of resonance. Both of these parameters are dependent upon the frequency and are usually determined experimentally.

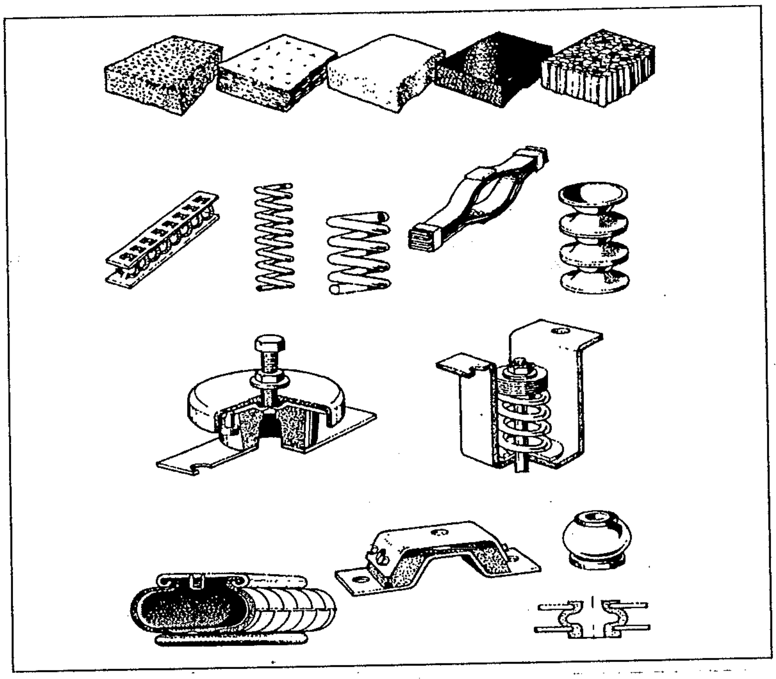

Figure 8.5: *Examples of commercially available vibration isolators (Diagram: Brüel & Kjær.)*

Spiral springs of steel can be designed so as to be of very low stiffness. If the lower limit for isolation needs to be very low, say 2-3 Hz, spiral springs are appropriate. A disadvantage with them, however, is that the loss factor for them is very low.

Rubber isolators are the type of isolators used most. They can be constructed so as to be appropriate for deformation both in terms of shear and of compression, being useful for shear down to about 3 Hz and for compression down to about 5 Hz. A special problem with rubber isolators is that their characteristics can vary markedly from one example to another of a given type. The variation in static stiffness for a particular type of rubber isolator can vary, for example, by 30 - 40 %. In critical cases it can thus be necessary to determine the characteristics of different isolators individually. Gas springs can be more appropriate to use in situations calling for particularly low resonance frequencies. In train cars and buses, gas springs are sometimes used to isolate the body of the car or bus from the chassis or bogie.

8.4 The dynamic stiffness of an isolator

An isolators stiffness characteristic can be described in many different ways. The ability of an isolator to stop transmission of vibrations is strongly dependent upon the so-called transfer stiffness. This can be expressed as the inverted ratio of the displacement occurring on the incoming side to the force on the outgoing side which results when the latter is blocked.

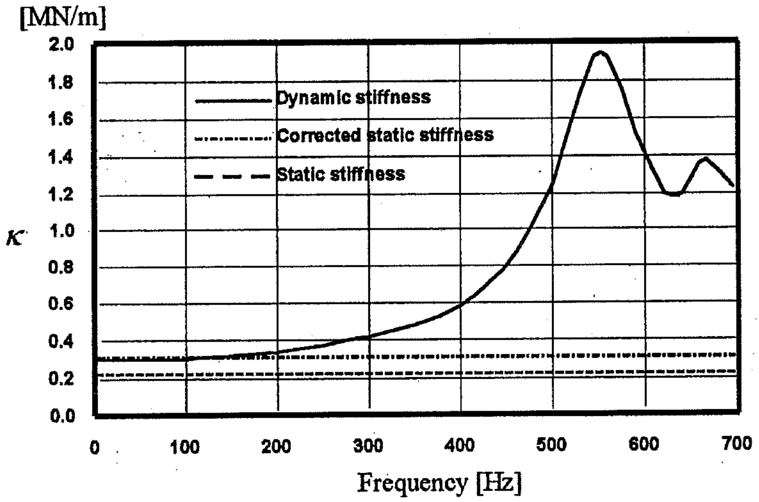

Figure 8.6: *The measured dynamic transfer stiffness for a circular cylindric rubber isolator of 50mm length and 75mm diameter.*

Obtaining reliable data on the dynamic stiffness of isolators requires that one measure the dynamic stiffness of the individual isolator. The data on dynamic stiffness provided by manufacters consists simply of only partially corrected figures on static stiffness. At high frequencies the discrepancies between the true dynamic stiffness and the corrected static stiffness is often very large. This is illustrated in Figure 8.6, showing the dynamic stiffness of an ordinary circular cylindric rubber isolator. The corrected static stiffness and the true measured stiffness can differ by as much as several thousand percent!

8.5 More refined dynamic stiffness models

The simplest dynamic stiffness model is that of the frequency- and static load-independent complex stiffness corresponding to the hysteretic damping model mentioned in Chapter 5, as given by

$$k = k_0(1 + \eta i), \tag{8.1}$$

where η is the loss factor, i the imaginary unit and k_0 the static stiffness. For an long body, the static stiffness can be computed as $k_0 = E_0 A/L$, where E_0 is the static Young's modulus of the material in the spring, and A and L are its area and length. Correcting for the vibration isolator's form is done by setting

$$k_0 = E_0A/L(1+2k_1S^2), \tag{8.2}$$

where k_1 is a correction constant obtained partly on an empirical basis. S is the form factor

$$S = \frac{loaded\ area}{load\ free\ area}. \tag{8.3}$$

Equation (8.2) apply to compression loading as discussed in Chapter 3. For loads involving shear the stiffness k_0 is replaced by

$$k_0 = G_0A/L, \tag{8.4}$$

where G_0 is the static shear modulus of the spring material. In order for the computed stiffness to apply to the dynamic case, the frequency dependency of the module must normally be corrected for. This can be done most simply by using the correction constant k_2, which denotes the relationship between the dynamic and the static values, which typically varies between 1.1 and 1.9, depending on the rubber material involved. The dynamic stiffness then is for compression

$$\boldsymbol{k} = k_2E_0A(1+2k_1S^2)(1+\eta i)/L \tag{8.5}$$

and for shear

$$\boldsymbol{k} = k_2G_0A(1+\eta i)/L. \tag{8.6}$$

These models are satisfactory enough in the low-frequency area. When the excitation frequency increases, however, a wave transmission in the the isolator must be accounted for. The simplest *longitudinal-wave model* [3], that was used in Case 3 assumes the displacement field to be planar and uniform and that movement occur in one direction only. The dynamic transfer stiffness is then

$$\boldsymbol{k} = \frac{\boldsymbol{E}A2\pi f/\sqrt{\boldsymbol{E}/\rho}}{sin(2\pi fL/\sqrt{\boldsymbol{E}/\rho})} \tag{8.7}$$

and

$$\boldsymbol{k} = \frac{\boldsymbol{G}A2\pi f/\sqrt{\boldsymbol{G}/\rho}}{sin(2\pi fL/\sqrt{\boldsymbol{G}/\rho})} \tag{8.8}$$

where ρ is the density and $\boldsymbol{E}$ is the dynamic Young's modulus and $\boldsymbol{G}$ is the dynamic shear modulus. The dynamic stiffness in shear (8.8), in particular, is highly predictable, no corrections on the theoretical values normally being called for. The compression stiffness (8.7), on the other hand, tends to be underestimated in the low-frequency area. The underestimation can be reduced by correcting (8.7) for the vibration isolator's form, giving

$$\boldsymbol{k} = (1+2k_1S^2)\frac{\boldsymbol{E}A2\pi f/\sqrt{\boldsymbol{E}/\rho}}{sin(2\pi fL/\sqrt{\boldsymbol{E}/\rho})}\ . \tag{8.9}$$

This compression stiffness model is rather satisfactory for estimating the low-frequency characteristics and the first internal resonance frequency. However, the stiffness value for this frequency tends to be overestimated, which holds also for (8.7).

A more refined model, taking some account of movements in the radial direction is the so-called *Love model* [3]. This model assumes radial movements to be directly proportional to axial movements in accordance with Poisson's ratio. The dynamic transfer stiffness then becomes

$$\boldsymbol{k} = \frac{\boldsymbol{E}A2\pi f\sqrt{\rho(1-(2\pi f\nu R)^2\rho/\boldsymbol{E})/\boldsymbol{E}}}{sin(2\pi fL\sqrt{\rho/(\boldsymbol{E}(1-(2\pi f\nu R)^2\rho/\boldsymbol{E}))}\,)} \tag{8.10}$$

where R is the polar radius of gyration of the cross-sectional area of the isolator and ν is Poisson's ratio, for rubber normally 0.5.

Particularly when vibration isolators are large in width and are made of soft rubber material, the Love model can overestimate the effect of radial movements and do so to such an extent that the stiffness computed may be less accurate than with use of (8.7). To obtain better estimates, account must be taken of the shear stresses stemming from radial movements. A model that corrects for such stresses is the so called *Bishop model* [4].

During the last few decades, the development of analytical models for predicting the dynamic stiffness of vibration isolators has more or less stood still. Rather, emphasis has been placed instead on finite element analysis. An exception to this is the newly presented *wave guide model* [1], providing an exact solution to the basic (linear) equations without any assumptions being made regarding movement patterns and the like.

Figure 8.7 shows the dynamic transfer stiffness computed for an ordinary circular cylindric rubber isolator of 50mm length and 100mm diameter, together with the measured results. The limitations of the approximative methods are obvious, whereas the wave guide solution largely agrees with the measurements.

8.6 Models of the rubber material

Within a large range of frequencies, the rubber material damping is clearly the most important characteristic for its serving as a vibration isolator. Modelling this characteristic is far from simple, however. A large number of different research projects within this area are being conducted. It is often assumed that rubber exercises its damping effect only in terms of shear. The simplest model for this assumes the presence of a frequency- and static load independent complex shear modulus

$$\boldsymbol{G} = G_0(1+\eta i), \tag{8.11}$$

where η is the loss factor equivalent to (8.1) and G_0 is the static shear modulus. The damping can be expressed either as the loss factor or as the loss modulus,

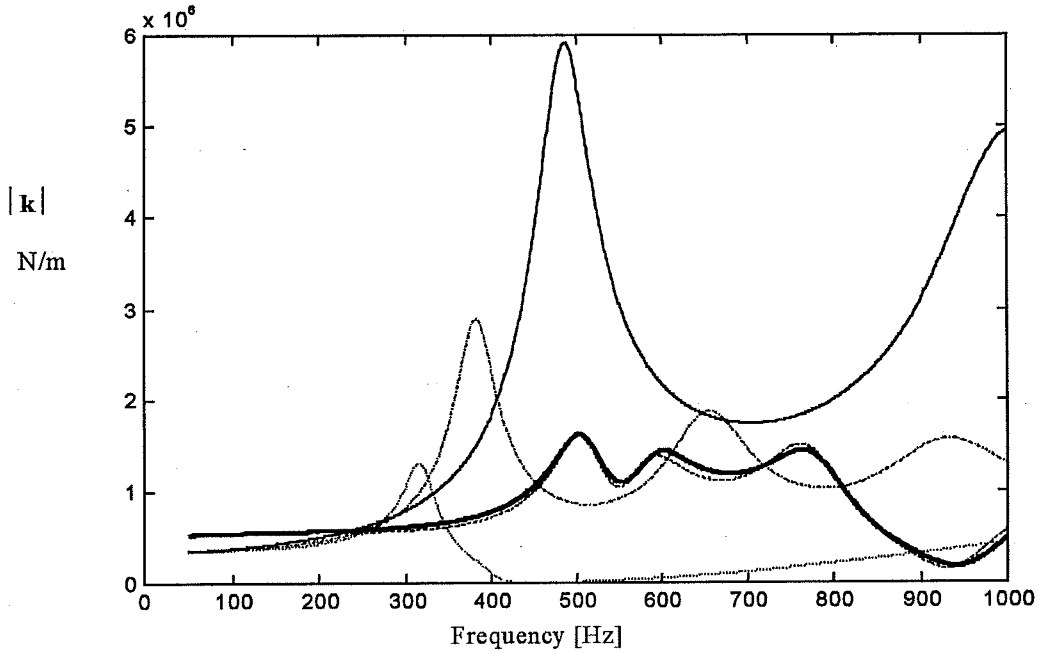

Figure 8.7: *Computed dynamic transfer stiffness for a circular cylindric rubber isolator compared with measurements (dashed line). Wave guide model (thick solid line), longitudinal wave model (solid line), Love model (dotted line), and Bishop model (dash dotted line).*

which is the imaginary part of the complex shear modulus. A correction constant for the dynamic processes involved is usually required such that

$$\boldsymbol{G} = k_2 G_0 (1 + \eta i), \tag{8.12}$$

where the constant k_2 is the same as in (8.5) and (8.6).

Equations (8.11) and (8.12) approximate a slightly damping material such as unfilled natural rubber. For strongly damping rubber, a viscous model is required,

$$\boldsymbol{G} = G_0 (1 + ci\omega), \tag{8.13}$$

where the constant c is the viscosity and ω is the angular frequency.

Most rubber, however, displays neither frequency independency nor viscous damping, but rather

$$\boldsymbol{G} = \frac{\sum_{n=0}^{N_p} p_n (i\omega)^n}{\sum_{m=0}^{N_q} q_m (i\omega)^m} \tag{8.14}$$

where p_n and q_m are material constants. In principle, all rubbers can be characterized in terms of this relationship if the conditions are such that the nonlinear dynamic effects are negligible.

Intensive research is being carried on to reduce the number of material constants needed, thus simplifying computations and measurements. An example of how this has succeeded is development of the so-called *fractional derivative model*

[1], in which the integer exponents in (8.14) are replaced by non-integer numbers. This yields a significant reduction in the number of material constants needed, without loss of agreement to experiments in a broad frequency range.

Research aimed at arriving at more non-phenomenological models related to the structure and movements of the rubber molecules is also being carried on.

8.7 Measurement methods

One distinguishes between measuring the dynamic characteristics of a vibration isolator and the material characteristics of rubber. The former concerns the dynamic stiffness of the isolator as a whole, based both on its geometry and the material of which it is made, whereas the latter concerns only the characteristics of the rubber itself, without account of the geometry.

Research aimed at using the first-named approach also as a means of determining the purely material characteristics of rubber is underway, involving partly the use of so-called computer-based generic algorithms.

8.7.1 Dynamic stiffness

Dynamic stiffness can be measured either directly or indirectly. Direct methods measure both the dynamic force and the motion of the isolator, with the ratio of the two being the dynamic stiffness. Direct methods often cannot be used at high frequencies, however, due to resonances in the measuring equipment. One is forced then to use indirect methods.

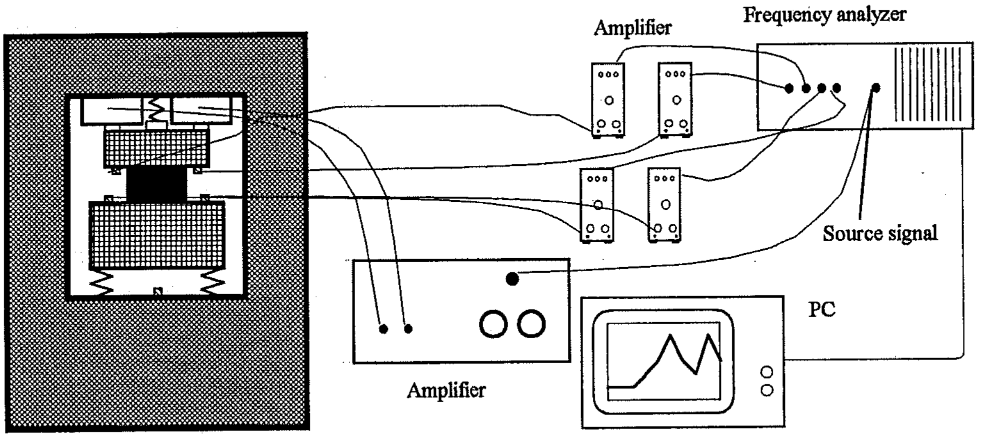

Figure 8.8: *Measuring-setup and measuring equipment.*

Figure 8.8 illustrates such a method, in which a static deformation is imposed through pressing two masses together by means of screws or hydraulic pistons. Vibration isolators are placed between the two masses. The upper block is then set into dynamic motion by means of electrodynamic excitators. Through measuring the movements of the blocks by accelerometers, one is able to compute the dynamic transfer stiffness indirectly. This can be done in terms of each of the six degrees of freedom. Figure 8.9 shows the measured dynamic stiffness [1] obtained for a large circular cylindrical rubber isolator of 170mm length and 165mm diameter, containing several metal plates vulcanized into it.

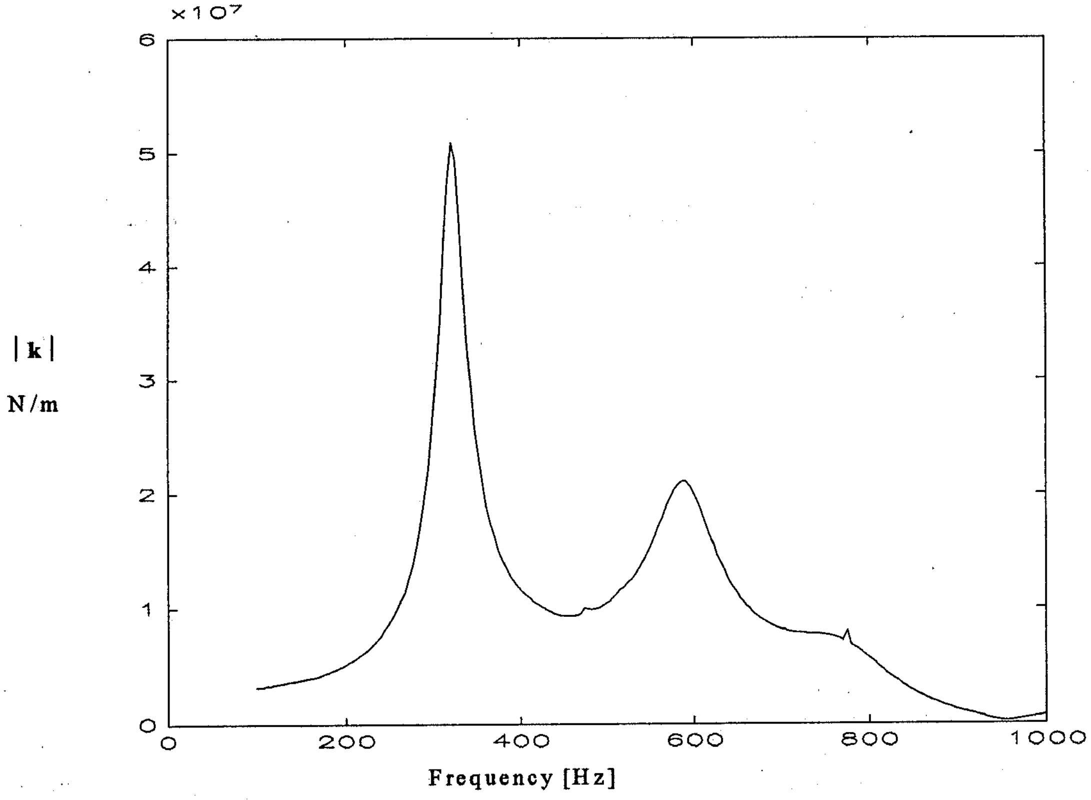

Figure 8.9: *Dynamic transfer stiffness of a circular cylindric rubber isolator laminated with metal plates, measured for a pre-load of 10 kN.*

Work on developing international standards and research on developing improved methods of measurement is underway [1].

8.7.2 Material characteristics

Since within a large range of frequencies the damping in shear is the most important material characteristic for a vibration isolator, measurement work tends to be focused upon this mode of deformation. Here too, both direct and indirect methods are employed, together with certain standards. Due to the need of minimizing the effects of the geometry of the object, and to the fact that deformations in the objects studied should occur primarily as shear, the objects measured need to be very small, objects about the size of a sugar lump being rather usual.

8.8 Finite Element methods

When the geometries and the material characteristics are complex and heavy static loads are employed, finite-element computations are needed for obtaining accurate results [1]. If one has access to a powerful computer, the computations themselves require comparatively little research effort. On the other hand, the damping effect of the rubber must be modelled correctly if close agreement with measurements is to be obtained over a wide range of frequencies. Figure 8.10 shows the dynamic transfer stiffness [1] computed for the same circular cylindric rubber isolator as in Figure 8.7, but with the use of static pre-loads, ranging from zero up to the maximum load employed.

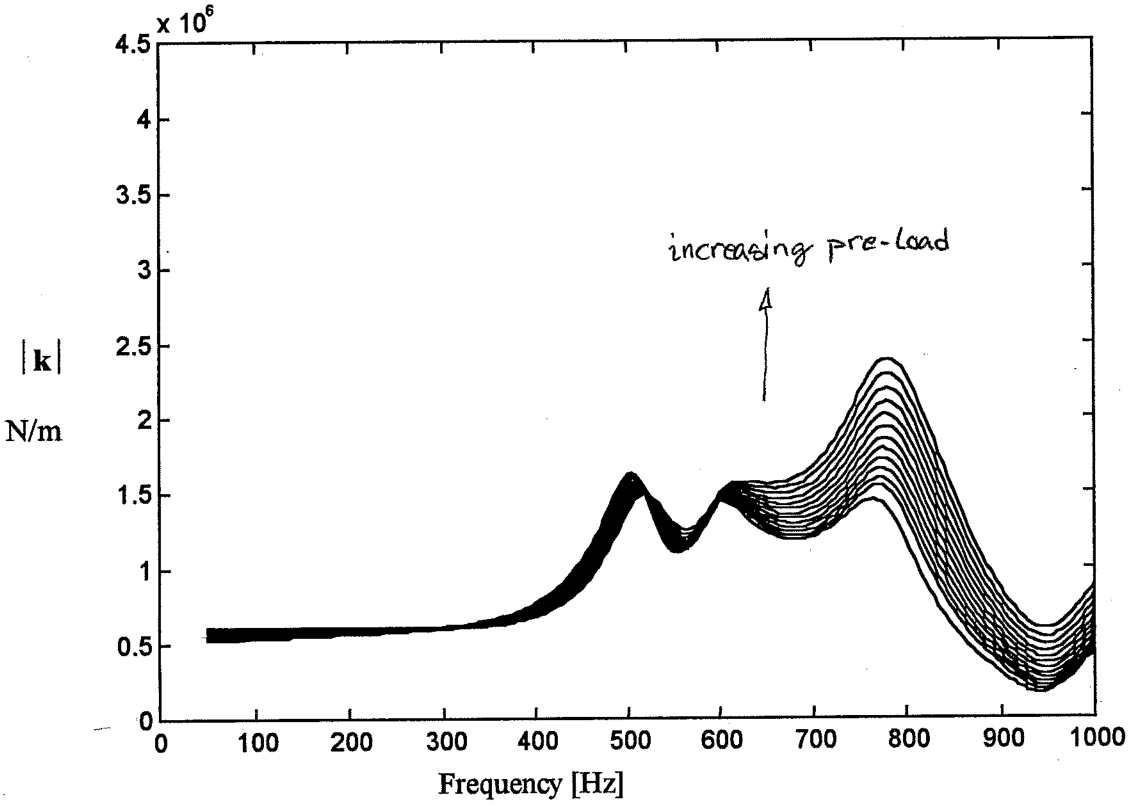

Figure 8.10: *Dynamic transfer stiffness for a circular cylindric rubber isolator as obtained by finite element analysis for different pre-loads starting from zero.*

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Chapter 9

Fracture mechanics and fatigue

P.-E. AUSTRELL

Considerations of fracture mechanics and fatigue are essential in the designing of rubber components. Knowledge of these matters makes it possible for engineers to more fully exploit the behavior of elastomers and to optimize the performance of these in applications.

The analysis and prediction of crack initiation and crack propagation has become increasingly precise due to the development of fracture mechanics, originating from the work of Griffith [3], who in the early 1920s formulated the first criterion for crack growth. The fracture criterion Griffith proposed, as well as the related tearing-energy concept and considerations of fatigue are dealt with in the present chapter.

9.1 Crack initiation and growth

The initiation of a crack that occurs in a rubber material is influenced by flaws, holes and irregularities. A rubber compound contains various additives of differing particle sizes that form inhomogeneities in the material. Large-scale geometrical features in loaded rubber units, such as roughness of cut and molded edges, can also lead to the initiation of cracks. The feature common to all such inhomogeneities is that they all lead to localized concentrations of stress that greatly exceed the nominal stress in the material, thus initiating crack propagation.

In addition, environmental effects such as exposition to ozone or to chemicals can increase the crack growth and reduce service life of a rubber unit. Matters relating to the effect of the environment are discussed further in Chapter 10.

9.1.1 Crystallizing and noncrystallizing elastomers

Under static conditions, crystallizing and noncrystallizing elastomers differ in crack growth. The formation of crystallites at the tip of the crack where high local strains occur inhibits crack growth. Hence, crystallizing rubber such as NR and CR show no crack growth under static loading conditions. In contrast, noncrystallizing rubber materials such as NBR and SBR are subject to crack growth even under static conditions.

9.2 Fracture criterion

Griffith [3] proposed a fracture criterion based on the balance of energy, involving the change in mechanical strain energy due to the formation of new crack surfaces. According to this criterion, the energy required to propagate a crack and create new surfaces in the material is equal to the elastic energy released in the material surrounding the crack. The criterion states that a crack can grow if the strain energy applied to the material is just sufficient to cause a small increase dA in the crack surface area.

The growth of a crack depends on the tearing energy T, also referred to as the fracture energy. This quantity is related to the rate of release of elastic energy in terms of

$$-(\frac{\partial U}{\partial A})_\ell = T \tag{9.1}$$

where U is the total elastic energy stored in body studied, A is the area of new crack growth, and ℓ indicates that the partial differentiation is with respect to constant deformation. Under fixed displacement conditions, the external force applied can do no work, and crack growth being driven by the release of elastic energy.

Rivlin and Thomas [7] applied the criterion to the tearing of rubber vulcanizates. They showed that the tearing energy required to propagate a crack was independent of the geometry of the test-piece. This property is desirable, simplifying the relationships and the understanding of the mechanical factors involved in determining crack gowth. The concept used for tearing of rubber is thus the same as that originaly proposed by Griffith for fracture of metals. Although rubbers are more complicated, due to elastic nonlinearties, there is no development of a plastic zone at the crack tip in these materials. A fracture mechanics approach thus being simpler in this sense than it is for metals.

The criterion (9.1) can be used for the estimation of catastrophic crack growth, which occurs if the elastic energy release rate, given by the left-hand side in (9.1), exceeds a certain maximum tearing energy T_c which the material can withstand. The criterion can also be used for calculating the fatigue life of rubber. The tearing energy T is considered then as a measure of the mechanical straining at the crack tip, the development of the crack being a function of T. This will be discussed further in Section 9.3.

9.2.1 Tear energies for different test-pieces

Test pieces of different geometries for evaluation of the tearing energy will be considered next. The difficulties in applying criterion (9.1) in practice arise from the need of calculating the tearing energy for different geometries of the engineering components. The initial work by Rivlin and Thomas [7] and subsequent work in the area provide solutions for various cases involving thin rubber strips. The studies following provide solutions for bonded rubber layers in cases of simple shear and of uniaxial compression. The solutions are based on theoretical investigations of the energy balance between the stored elastic energy and the fracture energy.

The trousers test piece

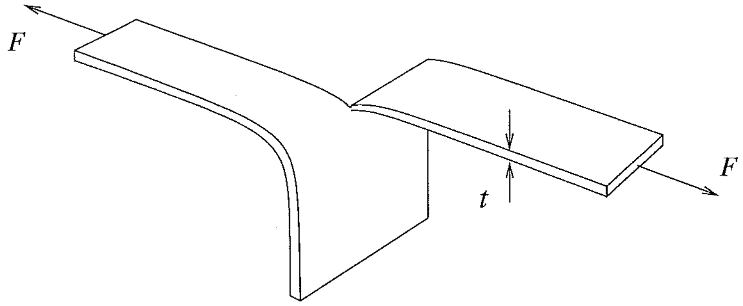

Figure 9.1: *The trousers test piece.*

The trousers test piece shown in Figure 9.1 is a widely used test piece and one of the first for which a solution was obtained. Regarding the energy balance of the test piece, the work done by the force that is applied is approximately equal to the sum of the energy required for tearing and the increase in strain energy in the legs. This can be shown to yield

$$T = \frac{2F}{t} \tag{9.2}$$

the fracture energy of the trousers test piece depending only on the force F applied and the thickness t of the test specimen, the latter being constant.

The pure shear test piece

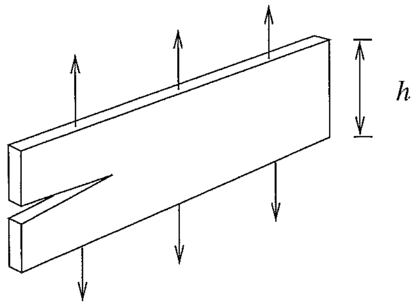

Figure 9.2: *The pure shear test piece.*

Another widely used test piece is the pure shear test specimen shown in Figure 9.2, of which the fracture energy is

$$T = Wh, \tag{9.3}$$

where h is the height of the specimen and W is the strain energy density discussed in Chapter 3.

The tensile strip

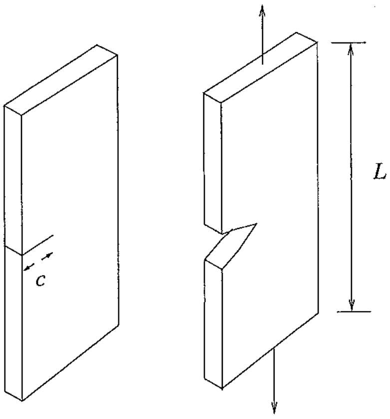

Figure 9.3: *A tensile strip-test piece with a crack of length c.*

Yet another case is that of the tensile strip shown in Figure 9.3. The length L of the test piece is large compared with the length c of the cut. If the cut length was zero, the test piece would be in simple extension. For a small edge crack it is assumed that the tensile strip is approximately in simple extension. The fracture criterion then can be written as

$$T = 2kWc\,, \tag{9.4}$$

where k is a constant. The value of this constant have been investigated by Lindley [5] useing finite element analysis and by Greensmith [2] in experiments showing k to be approximatly $3/\sqrt{\lambda}$, with λ being the longitudinal stretch.

Simple shear

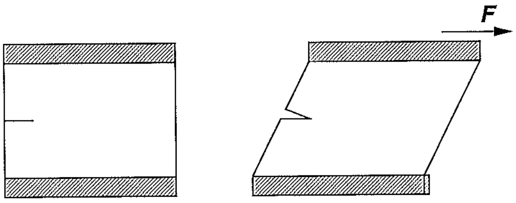

Figure 9.4: *A simple shear test-piece with a crack.*

The following expression applying to the tearing energy T in the case of simple shear was derived by Lindley and Teo [6]

$$T = kWh \tag{9.5}$$

where the constant k is approximately 0.4 but can vary, depending on the size and configuration of the crack. The expression (9.5) is an approximation and it holds only if the crack length is smaller than the height h of the specimen.

Constrained compression

A solid rubber cylinder, bonded to rigid metal discs of the same diameter is employed, yielding constrained compression, which is an inhomogeneous deformation, in contrast to the other cases previously discussed. For this case, the strain energy W will depend upon the shape factor in terms of the compression modulus E_c.

An approach for this specimen was suggested by Stevenson [9].

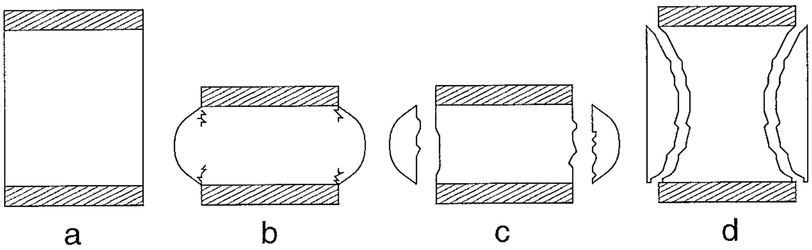

Figure 9.5: *Typical stages of crack growth in compression. (a) unstrained; (b) compressed - crack initiation at bond edges; (c) compressed - bulge separates from core; (d) unstrained - showing parabolic crack locus.*

On the basis of experimental observations, Lindley and Stevenson [5] suggested that in bonded rubber units under cyclic compression the locus of the crack growth defines a surface which is approximately parabolic in section, as shown in Figure 9.5 (from Stevenson [9]).

An approximate expression for the tearing energy T is

$$T = \frac{1}{2}Wh = \frac{1}{4}E_c e_c^2 h\ , \tag{9.6}$$

which is valid for shape factors above 0.5 and compression strains e_c below 50%, where e_c is the compression displacement divided by the original height.

The tearing energy expressions presented for different test specimens here are approximations. More accurate expressions can be obtained by use of finite element analysis. One way of doing this is to use the finite element model to calculate the elastic energy for the body at different crack lengths.

9.3 Fatigue of rubber components

Since the service life of rubber components subjected to cyclic loading is limited, due to fatigue, efforts to estimate service life are important. The major causes of fatigue cracking in rubber material are mechanical, thermal, environmental (oxygen, ozone and ultraviolet radiation) and chemical (e.g. oil, gasoline) in character. Rubber materials are thus provided with protective agents to prevent these problems.

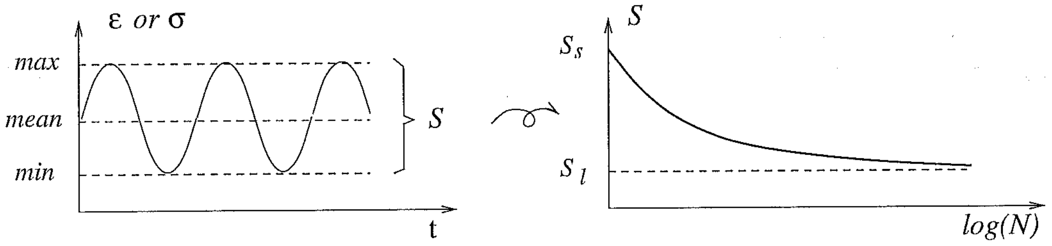

Figure 9.6: *Wöhler curve. The dynamic stress or strain range S is plotted against the number N of cycles to failure.*

The original manner of presenting the fatigue characteristics of a material has been to determine a so-called Wöhler curve, in which the dynamic stress or strain range is plotted against the number of cycles to failure. Such a curve, which also includes a fatigue limit representing infinite fatigue life is shown schematically in Figure 9.6. The limiting value for fatigue life is denoted in the figure as S_l the static limit load S_s also being shown. One should note that it is only crystallizing rubbers that display a fatigue limit S_l that differ from zero.

Obtaining Wöhler curves for rubber useing specimens without a pre-defined crack, requires a considerable amount of testing due to the large scatter in the experimental data this approach yields. A different approach is usually taken in the fatigue analysis of rubber. Pre-defined cracks are applied and the tearing energy concept is inroduced through use of test specimens of the type discussed earlier. A review of the fracture mechanics approach to fatigue of rubbers is given by Seldén [8].

The fatigue properties are usually shown in a diagram in which the crack growth rate dc/dn is plotted against the tearing energy T as examplified by Figure 9.7. Since the tearing energy is independent of the geometry of the test-piece it is useful in characterizing crack growth. The curve shown in Figure 9.7, which is characteristic for NR cycled under relaxing conditions, where the the specimen is unloaded completely in the cyclic loading, can be divided into four regions.

Since in region I the tearing energy T, as shown in Figure 9.7 is less than the threshold value T_0, no mechanical fatigue will occur. In the region below this mechanical fatigue limit T_0, crack growth is caused by ozone only. Crack growth in region II is dependent both on ozone and mechanical factors, so-called mechanico-oxidative crack growth occurs. In region III, a power law has been

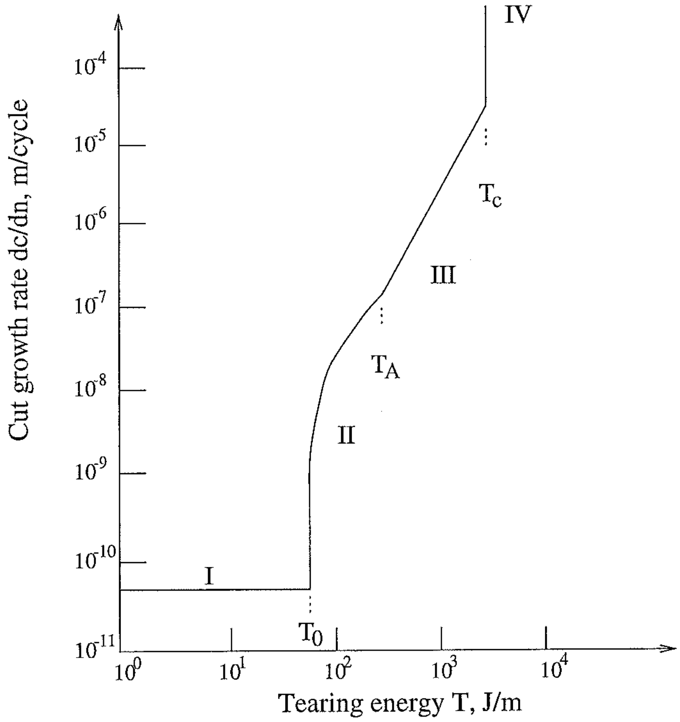

Figure 9.7: *An example of the crack growth characteristics of an NR vulcanizate in a laboratory atmosphere at 100 cycles/min under non-relaxing conditions.*

found to work well for both crystallizing and noncrystallizing elastomers. The relationship is

$$\frac{dc}{dn} = BT^{\beta} , \tag{9.7}$$

which is valid for $T_A < T < T_c$, where T_c is the critical tearing energy, T_A being located between regions II and III as shown in Figure 9.7. B and β are constants characteristic of region III. For natural rubber, β is often about 2, and for noncrystallizing rubbers such as SBR it may be about 4. In region IV the tear energy is limited by the critical value T_c when catastrophic failure is approached asymptotically.

9.3.1 Calculating fatigue life

Predicting values for the theoretical number of cycles to failure can be done on the basis of fatigue crack growth characteristics. A methodology for predicting the number of cycles is illustrated in Lake [4].

Crack growth can be expressed as a function of the tearing energy T,

$$\frac{dc}{dn} = f(T) \tag{9.8}$$

as shown in Figure 9.7. For moderate to high strains, as was indicated earlier, it can be approximated, by the power law relationship. The number of cycles required for a crack to grow from length c_0 to the final failure size c_c is given by

$$n = \int_{c_0}^{c_c} \frac{dc}{f(T)} \tag{9.9}$$

which can be evaluated, provided the function (9.8) of the tearing energy T and the crack length c_0 and c_c are known. An example of the evaluation of the expression for the tensile strip (9.4) and for the power law (9.7) is given by Ellul [1].

9.3.2 Factors influencing fatigue life

Several factors influence the mechanical fatigue properties of rubber. Ellul [1] (section 6.4) discusses such factors in detail. The following is a brief summary of matters taken up:

- **Static strain/stress:** Nonrelaxing conditions are present if the minimum strain or stress in a fatigue test is non-zero. A parameter R, which is the ratio of the minimum to the maximum load (cf. Figure 9.6), is normally used to characterize nonrelaxing conditions. Under nonrelaxing conditions ($R > 0$), fatigue life of rubbers is prolonged. This effect is particularly significant for strain-crystallizing rubbers.

- **Frequency:** Whereas for crystalizing rubbers the influence of frequency is small, for rubbers of the noncrystallizing type the effect is significant due to the time dependent behavior of these rubbers. Changing the frequency also changes the duration of the load.

  A complicating factor in the use of high frequencies and maintained amplitudes is the heat generation which can cause thermal degradation.

- **Temperature:** The effect of temperature is also more significant for noncrystallizing rubbers.

9.3.3 Fatigue life for different rubbers

Fatigue crack growth rate as a function of tearing energy at 22°C for different rubbers are shown in Figure 9.8 (from reference [1]). The minimum tearing energy in each cycle is equal to zero.

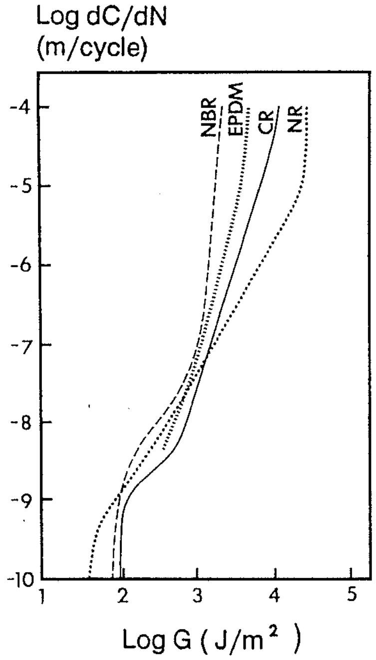

Figure 9.8: *Fatigue crack growth rate as a function of tearing energy at 22°C for various elastomers under relaxing conditions.*

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Chapter 10

Longevity characteristics

M. BELLANDER AND B. STENBERG

When the long-term properties of a product are to be predicted, several details not found in the common descriptions of different rubber types must be taken account of. These include the degree of stress relaxation of a material at different temperatures, the creep and compression set of the material, and the material's dynamic-mechanical characteristics as described by frequency and temperature spectra, the latter being especially important when a product has requirements to be met at widely different temperatures. Often, all that is available are single-point values, such as the $tan\delta$ for a certain temperature. This is far from satisfactory when the characteristics and behaviour of a product under varying conditions are to be predicted. On the other hand, it is difficult to develop a data bank providing all sufficient information on such characteristics, as every rubber compound has its own characteristics depending on e.g the type and amount of additives and fillers.

10.1 Methods for estimating the lifetime of a material

The properties of a rubber material change over time. Both mechanical strain and conditions in the environment, such as oxygen and ozone, and the exposure to sunlight and to various chemicals, can produce gradual changes in a material. Changes of this sort will sooner or later lead to deterioration of the material, and they may also lead to failure of the function of the product considered. How long this takes depends on many factors.

When the lifetime of a material is to be determined, the function of the product in question first needs to be clarified, and what factors that may have a decisive effect on its lifetime. For example, an O-ring can lose its sealing properties due to stress relaxation of the material, leading to declined contact pressure since the rubber is no longer elastic enough. An O-ring can also break down due to exposure to such reactive media as acids, bases or ozone, or it can harden during use, losing its elasticity and ability to seal. Many rubber materials harden when exposed to the oxygen in the air, but the opposite may also occur: softening of the material. It is not uncommon that many different factors influences the lifetime of a rubber product, e.g. oxygen and ozone in the atmosphere in combination with UV-light (from the sun). In addition the product may be mechanically loaded, thereby complicating the situation yet more. All of these factors need to be taken into account in the prediction of the lifetime of the material.

When the decisive factors leading to failure of the rubber product in charge are known, a failure criterion can be established. The failure criterion do not necessary need to be identical with the break of the product, it can as well be when a certain property has reached, say, 60% of its original value. The following are factors that can be of help when establishing a failure criterion of a rubber product:

- Development of cracks, perhaps in the presence of ozone. Static or dynamic load.
- Hardening due to oxidative aging. Cracks may be formed at the surface, leading to further crack propagation to the interiour of the product.
- Swelling in the presence of solvents.
- Chemical deterioration in the presence of aggressive chemicals.
- Stress relaxation, with loss of resilience and loss of sealing capability, due to a decrease in elasticity.
- Deformation changes due to creep phenomena.
- Loss of mechanical properties, e.g. measured by tensile testing of the material. This can occur either in the presence of oxygen or at elevated temperatures without oxygen.
- Thermal deterioration at elevated temperatures.

Figure 10.1 is a schematic presentation of the approach used in determining the lifetime of a rubber product.

How much data on lifetime that is needed for designing a rubber product properly, depends on the demands placed on the product. There is less need for a careful analysis of the lifetime of a doorstop, for example, than of an O-ring in a nuclear plant. How carefully the lifetime of a rubber product needs to be assessed depends partly, therefore, on what the consequences of a breakdown would be. If it is perfectly acceptable for a product to simply break down and be replaced, a rough estimate of its lifetime can suffice. On the other hand, if it is essential that the product last, far more thorough assessment and careful examination of the lifetime curves of the product or material in question are needed (see the following sections). Between these two extremes, there are no set of principles on how to proceed. Often, it is a matter of "intuitive feel", such that one invests the time and effort that seems reasonable for obtaining an acceptable result.

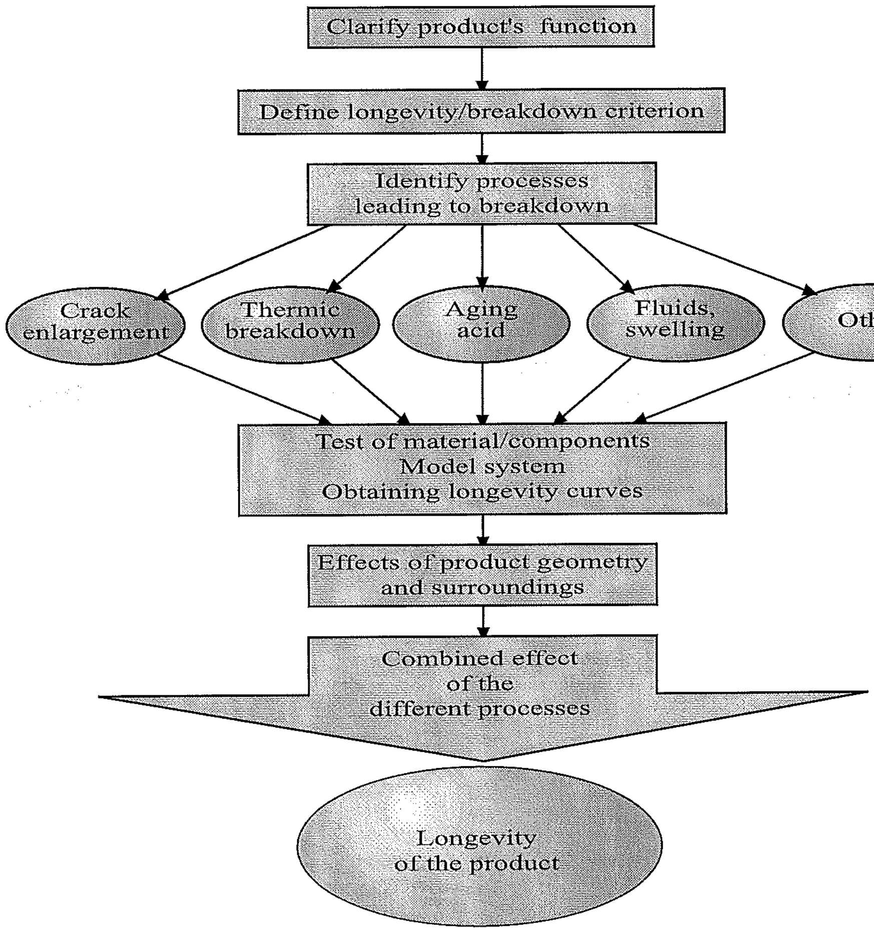

Figure 10.1: *Approach used in determining the longevity of a rubber product, adapted from A. Stevenson and R. P. Campion in "Engineering with Rubber", edited by A. Gent, Oxford University Press, 1992.*

10.2 Type of polymer

Selecting an appropriate type of rubber is of critical importance. Most rubber types are hydrocarbon-based, and the chemical bonds are susceptible to deterioration in various ways. A proper selection of a rubber material which fulfills all criterion can always be done. However, as in so many contexts, matters of cost can play an important role. The types of rubber able to withstand extreme conditions are often quite expensive, while the cheaper types perform less satisfactorily. A rough overview of the characteristics of different rubber materials is provided in Chapter 1. By carefully specifying the demands placed on the finished product, and analyse it thoroughly, it is possible to avoid purchasing unnecessarily expensive material or to obtain a product that breaks down before it is expected to.

10.3 Environment

The environment will obviously affect the lifetime of the rubber product. Aggressive chemicals such as oil, fuels, acids, and bases are examples of media that rubber material may be exposed to. Applications in such environments may require the use of special types of rubber for the product to last. Although many rubber products are only exposed to the surrounding air, this may suffice to deteriorate the material, resulting in failure of the product. The oxygen in the air plays a central role here. Most rubber polymers react chemically with oxygen, and create weak sites in the polymer chain. These weak sites (carbonyls, peroxides) can react further and lead to breakdown of the material. In the absence of oxygen, no weak sites are created, and the material will last much longer.

When a rubber product is exposed to air, the surface will be attacked by the oxygen and form a kind of oxidation skin. If the product does not deform, this oxidation layer will prevent the oxygen to penetrate the material, thereby leaving the interiour of the material fresh. In this way rubber products can look like they are deteriorated on the surface, but behave perfectly as a virgin material due to the unaffected core of the material. A good example of this phenomenon is bridge bearings of natural rubber which have been used for more than 100 years! The same is true for products that have been into the water for long time. Parts from crashed planes have been picked up and found to be almost as new, even though they have been laying at the ocean bed for 30-40 years.

This shows the importance of the geometry of a rubber product. A thick piece of rubber displays a completely different type of aging behaviour than a thin piece. The oxide layer formed on the surface of a thick piece prevents the major part of the product from being broken down. A thin piece, in contrast, is easily oxidized all the way through, becoming disfunctional much earlier. In some cases, the presence of chemical agents that protect the rubber from aging can prevent the formation of an oxidized layer, and thereby reduce the product's lifetime.

On the other hand, the oxidized layer may have negative influence on the performance if the product is loaded mechanically. Very often small cracks are formed at the surface (look on the tires of your car or bicycle!). These cracks can initiate catastrophic crack propagation through the whole product and thereby failure. During accelerated aging tests (aging at elevated temperature to accelerate the deterioration) this may be seen as a drastic decrease in elongation at break. It is not necessary that the same behaviour is seen if the stress at break is measured. The reason is that the stress at break is governed by the whole cross-section of the material, and not only the surface. In this way, an aged rubber material can still show a high stress at break, but very low values for elongation at break.

10.4 Loading

The lifetime of a product exposed to mechanical loading is often limited by creep or relaxation. Creep is defined as deformation under constant load, while relaxation, in turn, is defined as a reduction in the mechanical stress under constant deformation. The speed at which creep or relaxation occurs depends on the temperature. Two major types of processes steer relaxation and creep phenomena. The distinction is made between *physical* and *chemical* relaxation. The first stems from the polymer chains' gliding either across or away from each other, making the material unable to hold the load it is subjected to. Chemical relaxation, in contrast, occurs as a result of chemical reactions that breaks either the polymer chain or the crosslinks in the material. Both leads to the material losing its load-bearing capacity. A further factor that can be important for how rapidly the breakdown of a rubber product occurs is the type of loading to which it is subjected. Products subjected to static loading can behave quite differently than those subjected to dynamic loading. This too is due to the possibility of oxygen's gaining access to the rubber polymers. A dynamically-loaded rubber test object cannot form a protective oxide coating in the same way that a statically loaded object can. Dynamic loading leads to cracks developing in the oxide layer, exposing new and fresh material to oxidation and causing further disintegration. Although most rubber mixtures contain substances that protect them against aging, generally in the form of waxes that migrate to the surface and form a thin protective film, this film can crack and lose its effect if the object is subjected to dynamic loading. It is important under such conditions that an antioxidant be present not only on the surface as a protective layer, but also in the bulk of the material being active inside.

10.5 Geometry

Few rubber products are directly comparable with the standardized rubber test specimen used to investigate the characteristics of the rubber material itself. Thus, it can be difficult to translate the material characteristics determined by means of such tests to product characteristics. It would be the best, of course, to test a real product, but this is seldom economically feasible, if indeed possible at all. In utilizing the lifetime data from standard test specimen, therefore, one must often make a subjective evaluation of the applicability of the data to the product at hand.

10.6 Lifetime estimations

It is important to understand what is meant by lifetime. There are often definite requirements to be met regarding various characteristics, in the form of either a minimum or a maximum value, that an aging material shows. This may be the stipulation, for example, that the stress at break may not be less than 60% of the original value.

Many norms for rubber involve a simple aging test of some sort, such as hanging a test object in a heated air-circulated oven for a particular period of time, say for 3 days, and using as norm the maximum decline the specimen is permitted to show on some measured characteristic. This is a simple approach, but it is not always particularly informative. A more enlightening approach that provides a better prediction of the lifetime of a material is to determine a so-called lifetime curve. However, it is often completely impractical, or nearly so, to perform aging tests at room temperature, due to the time this would require. One way to try to overcome this is to accomplish accelerated aging. Using elevated temperatures when carrying out aging tests accelerates deteriorating processes, allowing the results of an experiment performed within a relatively short period of time to serve as a basis for predicting lifetimes involving considerably greater periods of time. Testing at different temperatures and measuring the time it takes for a material to fail (or to pass the limit for some particular characteristic) allows lifetime curves to be constructed. In these, lifetime is plotted against temperature (cf. Figure 10.2).

Within an IFP (Swedish Institute for Fiber and Polymer Research) project lifetime curves for rubber and for other elastic materials under conditions of exposure to air, to water and to oil have been obtained. The results of this work [2] are available upon request.

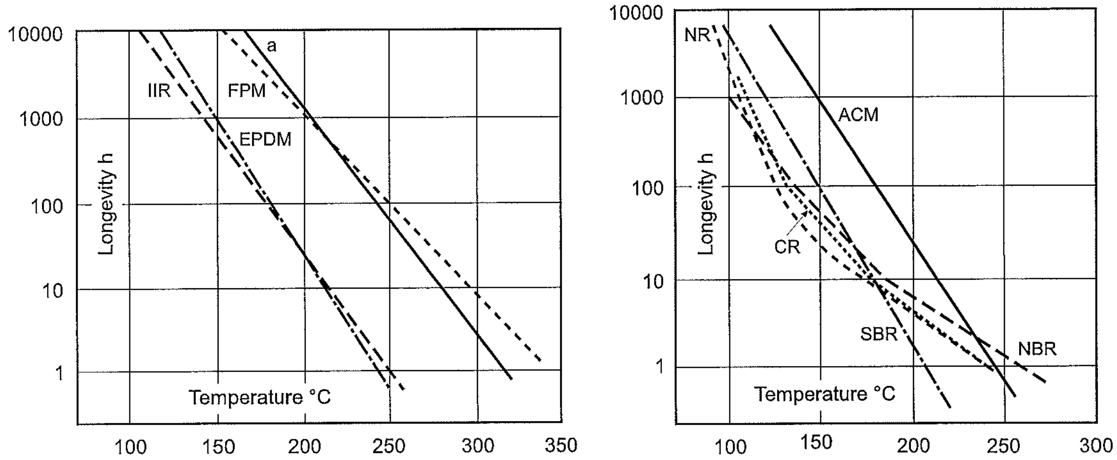

Figure 10.2: *Lifetime curves.*

Since measurements were only made at high temperatures (= shorter lifetime), it was necessary to extrapolate to lower temperatures (= longer lifetime). This saves the time that would otherwise be required for aging the material to the state of breakdown at lower temperatures. Such an approach assumes that the same basic processes steer aging under both low and high temperature conditions. However, at extremely high temperatures, the processes that steer aging differ, producing a different slope of the curve (see Figure 10.2, right-hand diagram).

Bearing this in mind, it is important not to perform the accelerated aging at a too high temperature, as this will yield a misleading result.

Lifetime curves is usually obtained for test specimen of a certain, standardized type. Thus, they provide a measure of the material's lifetime in only a general sense. As already indicated, the lifetime of individual products made of rubber of a particular type can vary to a great extent, depending upon the geometry of the product and the type of loading to which the product is subjected.

The product may also be subjected to other harmful factors than exposure to air. Exposure to ozone is one such factor. Examples of other factors are exposure to oil, to solutions of chemicals and to the UV radiation in sunlight. Strictly speaking, the lifetime prediction obtained, applies only to the conditions under which the measures were obtained. These conditions seldom coincide with those a particular product made of this material is exposed to. Thus, it is important to handle data on lifetime very cautiously and to not regard them as providing any absolute measure of the lifetime of a particular product.

10.7 Ozone

Ozone is present not only in the upper atmosphere, but also at ground level where its concentration is about 0,1-1 ppb (parts per billion. It can also be expressed as pphm, parts per hundred million. 1 ppbm = 10 pphm). Ozone reacts with the double bonds in polymer chains, causing them to break. This results in "ozone cracks", which can be observed on tires for example. A condition that must be fulfilled for such cracks to develop is that the rubber product in question is stretched somewhat, in the order 5-10%. Typical for ozone cracks is that they develop perpendicular to the direction of stretch. On tires, they are readily visible to the naked eye. Such cracks may well have only a cosmetic effect, and no influence at all on the lifetime of the product. Alternatively, they may initiate the formation of larger cracks, leading to breakdown of the product.

Crack formation in rubber samples exposed to ozone has been studied thoroughly, models having been developed to predict how long it should take for breakdown to occur. Some types of rubber are fairly insensitive to ozone and can be a good choice if the product in question will be exposed to significant amounts of ozone.

10.8 Examples

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Case 1

A sulfur vulcanized carbon black-filled natural rubber is selected for gaskets to be used for sealing pipe joints in a drain. What processes contribute to determining the gaskets' lifetime?

Solution:

The gaskets are usually at ground temperature, which in the summer is about 10^oC and in the winter about 4^oC. At such temperatures only physical relaxation processes occur. When physical relaxation or creep processes are dominant, it is usually quite simple to predict how rubber will behave over long periods of time. If relaxation or creep behavior on a linear scale is plotted against logarithmic time, a linear relationship is obtained. Hence, the speed with which relaxation or creep occurs can be expressed as percentage of change per time-decade (cf. Figure 10.3). Relationships of this sort hold as long as physical processes alone determine the behavior of the rubber material.

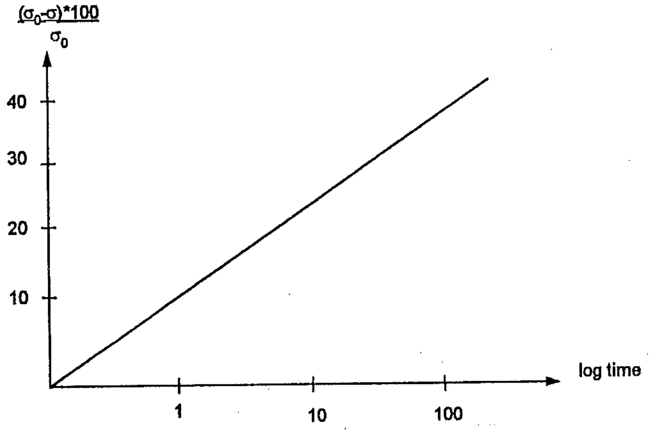

Figure 10.3: *Percentage of relaxation or tension/compression-set plotted against logarithmic time.*

In some parts of the drain system, the gaskets are exposed for brief periods of time to warm water from washing machines. The water can be as warm as 90^oC. Depending upon the material, this may suffice for thermooxidative processes to begin.

Case 2

Figure 10.4 shows relaxation curves for fluoro rubber FKM, perfluoro rubber PFE, silicone rubber, fluorsilicone rubber, polyacrylate and nitrile rubber. At 150^oC only fluoro rubbers and perfluoro rubbers possess acceptable characteristics.

Figure 10.4 illustrates the difficulties in interpreting such curves. The curves for PFE and FKM probably reflect only the the physical component of relaxation, whereas

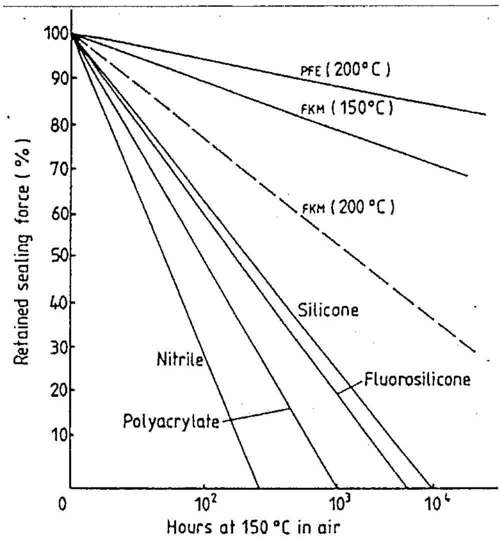

Figure 10.4: *Relaxation of different types of rubber according to R. J. Weston in A. Stevenson, "Rubber in Offshore Engineering", Adam Hilger Ltd 1984.*

the curve for nitrile rubber reflects primarily the thermooxidative component. Silicon rubber and fluorsilicon rubber are shown to be relatively stable at 150 ^oC, the curves obtained showing primarily thermooxidative breakdown.

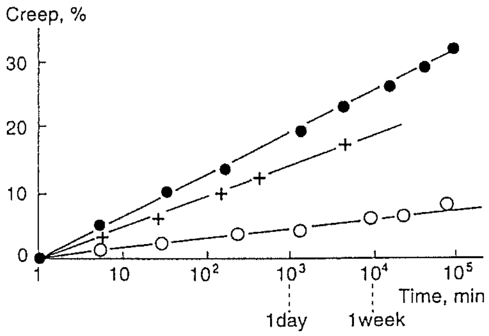

Figure 10.5: *Creep of a sulfur vulcanized natural rubber either containing or not containing filler. P. B. Lindley, Engineering Design with Natural Rubber, MRPRA 1992 p. 8.*

Figure 10.5 shows creep developing in a sulfur vulcanized natural rubber. (o) represents NR without filler, (+) non-reinforcing carbon-black (50 phr), and (•) reinforcing carbon-black. Note that when creep is displayed as a function of logarithmic time, almost a linear relationship is obtained. The rate with which creep develops can be defined in terms of the slope shown in the diagram, the speed of creep being expressed as deformation/time decade. Relaxation speed, in turn, is expressed as percentage of decrease in stress per time decade.

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Bibliography

[1] A. Stevenson and R. P. Campion in "Engineering with Rubber", edited by A. Gent, Oxford University Press, 1992

[2] PGI report 1986.

[3] R. J. Weston in A. Stevenson, "Rubber in Offshore Engineering", Adam Hilger Ltd 1984.

[4] P. B. Lindley, Engineering Design with Natural Rubber, MRPRA 1992

Chapter 11

Environmental effects, recycling and processing effects

M. BELLANDER AND B. STENBERG

Many different chemicals are used in rubber as additives for obtaining the product characteristics desired. The rubber material is chosen for its elasticity. The additional characteristics called for can be achieved by the selection of appropriate elastomeres, fillers, vulcanizing agents, preservatives, and the like.

11.1 Rubber manufacture

Particular attention has been directed at rubber manufacturing for the following reasons:

- *vulcanizing agents* contain accelerators for regulating the speed of vulcanization. These usually consist of organonitrogenous sulfurous compounds that, through producing nitrosamines, can be injurious to health. Nowadays, accelators are available that are safer to use than those employed earlier.

- *chloroprene rubber* may contain traces of chloroprene monomer that can produce cancer; it is also vulcanized by thiourea, which is suspected of likewise producing cancer.

- *vulcanization fumes* which develop during the vulcanization of rubber to form the finished product are generally removed by ventilation, preventing contamination of the air in factories. However, opening the moulds can free such fumes nevertheless.

- *the aromatic oils* employed as softeners are effective in technical terms. Their use has been questioned, however, from an environmental standpoint. Particularly the highly aromatic oils should be avoided.

11.2 Recycling

All over the world, large numbers of worn-out tires are collected. It is estimated that in Sweden some six million tires are discarded annually. A certain portion of these (about 20%) are recapped.

What is problematical about the recycling of rubber products that are worn out? *Answer:* Rubber material that has been vulcanized cannot be melted as a thermoplastic substance can. When vulcanized rubber is heated to a sufficiently high temperature, it either ignites (in air) or is broken down (in an inert atmosphere).

A wide variety of techniques are available for getting rid of worn-out tires. A frequently used technique is to incinerate them in concrete ovens in the manufacture of cement (thermal recycling). The combustion energy of rubber is nearly as high as that of oil. Tires have a sulfur content of several percent, which is desirable in manufacturing cement. Another technique for getting rid of tires is to incinerate them in heating plants and in garbage disposal plants.

11.2.1 Grinding

Grinding and pyrolysis are two alternatives to the recapping and thermic recycling of tires. The following are three different approaches employed:

- *cryo-grinding*, which involves cooling the cut-up tires to a low temperature by means of liquid nitrogen, in which the rubber material becomes brittle. The cold and brittle rubber material (at -100 ^oC) is ground to a powder (several tenths of a mm in size) in a mill.
- *warm grinding*, which involves the risk of the material's igniting.
- *wet griding*, a technique that increases fire-safety but may require a drying step afterwards.

Powdered rubber can be put to the following uses:

- Blending it with newly-formed rubber mass. Up to 10% can be added in this way without the mechanical characteristics of the resulting mass being negatively affected to any appreciable extent. To reduce the impairment of the mechanical properties, the surface characteristics of the rubber particles are often modified chemically so that adhesion to the newly formed rubber mass is improved.
- Blending it with asphalt. This improves the surface of roads, in the winter in particular.

11.2.2 Pyrolysis

Worn-out tires and other worn-out rubber products are warmed up in an inert atmoshere and broken down into gases, oil and solid residue. The gases consist of hydrocarbons that can be burned for heating purposes. The oil produced is often contaminated and needs to be purified, but it can also be used for heating. The solid residue consists in part of soot that can be employed as filler. Several plants for the pyrolysis of worn-out rubber products have come into operation.

11.2.3 Unvulcanizing of rubber products

The basic idea here is to use thermal treatment to split the crosslinks that vulcanization produced. The most frequently employed techniques involve the use of microwaves or of ultrasound. For removing the rubber from combinations of rubber and metal, use has been made in Japan of fluid masses of abrasive particles, although difficulties may come about in separating the rubber material from the fluidized particles.

11.3 Processing effects

The processing of the rubber material affects the characteristics of the final product. Vulcanization results not only in crosslinks, with an accompanying increase in elasticity, but also in a certain shrinkage, which can generally be noted in the extrusion of tubing.

Another important phenomenon that occurs is anisotropy, or differences in the characteristics of the material depending upon the direction through the material considered. Anisotropy can be desirable in different products that are subjected to mechanical strain. One sometimes endeavors to orient a fibrous filler (either of glass or of polyamide, polyester or kevlar fiber) in the direction of the strain exerted on the product. In order for the characteristics that are desired to be achieved, close adhesion between the fibers and the rubber material is necessary.