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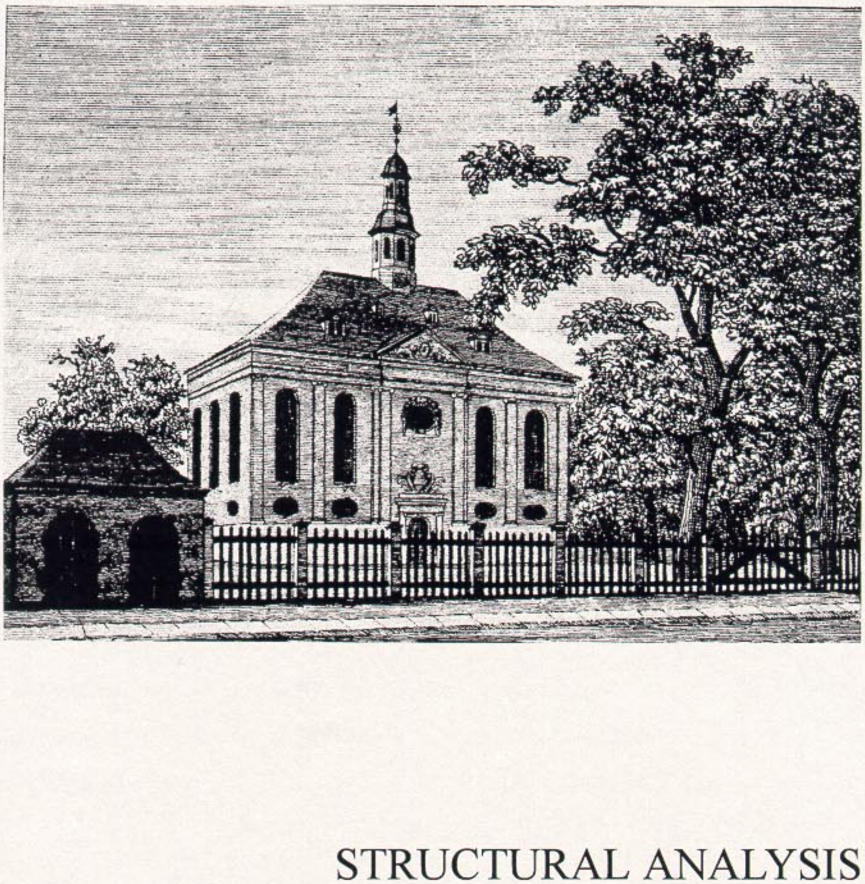

STRUCTURAL ANALYSIS OF THE ROOF STRUCTURE IN THE REFORMED CHURCH

Master's thesis by PETRA DIKE and SARA MACDONALD MALMBERG

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Carried out at the Div. of Structural Mechanics, Lund, Sweden.

Preface

This Master's thesis is the result of our work at the Division of Structural Mechanics at Lund Institute of Technology. The work has been carried out during the summer and the autumn of 1998.

We take this opportunity to thank our examiner Karl-Gunnar Olsson at the Division of Structural Mechanics and our supervisor Svend Jakobsen at Eduard Troelsgårds rådgivende ingeniører in Copenhagen, Denmark, for all their support and encouragement. We would also like to show our gratitude to all the staff at the division for their assistance. We especially thank Jonas Lindemann for the assistance with VRML, Erik Serrano and Mats Gustavsson for helping us with the computer programs, and thank you Bo Zadig for helping us with the figures. A thank is also sent to Varmings Tegnestue in Denmark for the opportunity to use their drawings of the church.

Lund in November 1998

Petra Dike and Sara MacDonald Malmberg

Abstract

The purpose of this Master's thesis is to find and illustrate the behaviour of the roof structure of the Reformed Church in Copenhagen, Denmark. Explanatory visualisation results in an enhancement of the understanding of the statical behaviour.

The introduction of the report contains the building history of the Reformed Church. A restoration took place in 1987 and our Danish supervisor was part of the team who performed this restoration. The team used a method of structural analysis, different from the one performed in this Master's thesis. From this point of view a discussion of the different methods is presented. Questions concerning the statical behaviour of specific parts of the structure which came up during the restoration in 1987 are addressed. Different philosophies used when restoring old historical buildings are discussed.

To be able to conduct the computer analysis a thorough knowledge of the structure is required. The structural, material and the geometric model, as well as the boundary conditions, are investigated to get the correct input data for the analysis.

With help from the visualisation, different load paths and their load carrying meaning are shown and discussed. The deformations, the normal forces and the stresses can become clear through the visualisation even to the less experienced viewer.

Keywords: ABAQUS, church, Copenhagen, FEM, finite element method, history, maintenance, PATRAN, reformation, Reformed Church, restoration, roof, structural analysis, timber, timber structures, wood

Sammanfattning

Syftet med detta examensarbetet är att finna och tydligt illustrera beteendet hos takkonstruktionen i den Reformerta kyrkan i Köpenhamn, Danmark. En tydlig visualisering av det statiska beteendet resulterar i en ökad förståelse för hela konstruktionen.

Inledningen av rapporten innehåller den Reformerta kyrkans byggnadshistoria, från år 1688 då krykan restes fram till idag. Vår danska handledare, Svend Jakobsen, deltog 1987 i en restaurering av kyrkan, vid vilken en annan metod för strukturmekanisk analys användes jämfört med den metod som används i det här examensarbetet. Ur denna synvinkeln hålls en diskussion angående de olika metoderna. Under restaureringen 1987 uppkom frågor kring beteendet hos en del specifika delar av takkonstruktionen, försök att besvara dessa är gjorda i rapporten. Olika filosofier som åberopas vid restaureringar av gamla historiska byggnader diskuteras.

För att kunna utföra en datoranalys av det strukturmekaniska beteendet krävs en grundlig kännedom om konstruktionen. Struktur-, material- och geometrimodell så väl som randvillkor studeras för att korrekta ingångsdata på så vis ska kunna ges till datorprogrammet.

Med hjälp av visualiseringen kan olika lastvägar och deras lastbärande betydelse tydliggöras och diskuteras. Deformationer, normalkrafter och spänningar blir genom visualiseringen tydliga även för den mindre erfarne betraktaren.

1 Introduction

1.1 History

In the 17th century the Reformed Church of Zwingli and Calvin, the Calvinistic Church, had become very independent, with its own distinctly formulated program. It was in its nature radical and democratic, and disassociated itself from the original Christian church, the Catholic. Traditions and customs from the Catholic Church not derived directly from the Bible were purged. Nothing but Jesus Christ was recognised and he was the only and eternal bishop. The essential part of the divine service was the word of God, and the pulpit thus had a prominent role in the church room, which is also the case today. There are no altar, altarpiece or altar candles in the Calvinistic Church; together with the prohibition of pictures according to the Bible (2 Mos. 20:4) this results in a very severe atmosphere.

The Calvinistic Church is sometimes considered to be the source of our modern democracy. The clergymen were all on equal footing and their main mission was to pass on the word of God to the congregation.

The Calvinistic Church started developing in Switzerland and in the Netherlands, probably because of an early democratic structure in the ruling of these countries. After a while the doctrine spread to the surrounding principalities, where belief was secured only if the prince himself was a Calvinist. This was the case in Brandenburg and Hessen. If the prince was a Catholic or a Lutheran this could pose a risk of conflict. The religious wars in the Netherlands and in France at the time indicate these problems.

A quite unexpected situation occurred in Denmark in 1667, when the Danish Crown Prince Christian, later Christian V, married the Hessian Princess Charlotte Amalie. She had inherited a strict reformist belief. Even as the Queen of Denmark-Norway she had no intentions of giving up her faith. When she married she had permission from the King, Frederik III, to continue to practice her religion under the new circumstances.

In 1685 the then King of Denmark, Christian V, gave the reformers in Copenhagen permission to build a church of their own. According to the King this should be a voluntary church, so the Reformed congregation established a collection book. It was not until 1688, when Queen Charlotte Amalie donated a sum of 10,000 rdl. to the construction of a church, that the construction of the building was secured. It was to be built on a piece of land, opposite to the castle Rosenborg, donated by the King.

The Queen's wish was that the two brothers Frederik and Nicolai Müller should be chosen to lead the building of the Reformed Church. The architect was Henrich Brockam.

The foundation rested on piles and on top of that was the visible socle of carved blocks of granite. The masonry was of small bricks and the cornice was of wood with a covering of copper. Copper facing was also used on the pediment and on the skylight windows. The roof was covered with glazed tiles.

In 1728, the year of the big fire in Copenhagen, the Reformed Church was burned out. The roof structure with the spire and all the equipment were totally ruined. The condition of the walls after the devastation is not known. There are no accounts saved from the time right after the fire to help ascertain, whether the church had to be rebuilt from the ground. Everything seems nevertheless to indicate that the walls were saved. The façade as we know it today corresponds to the description in the building contract of 1688. The measurements and the decorations are the same. The destroyed parts of the church were rebuilt in 1730.

The building is about 80 Danish feet (25 m) long and 50 Danish feet (15.7 m) wide. The height, from the ground to the cornice, measures 40 Danish feet (12.5 m) and from the ground to the top of the spire the height is three times this length which means 120 Danish feet (37.6 m); see Figures 1.1, 1.2 and 1.3. This simple relationship between the measurements is probably proof that these were also the measurements before the fire in 1728.

The decorations of the church are almost exclusively placed on the front façade. The architect Henrich Brockam composed the column façade with the help of simple units of measurements, see Figure 1.3. He had been inspired by the Dutch baroque with the columns in the centre. The long sides have five parts, and the short sides have three parts with arched high windows. Under these windows there were oval smaller windows, but these have to some extent been bricked up. Since 1730 the cornice has been of brick instead of the original one made of wood covered with copper.

The roof is today hipped as it very likely was even before the fire. Some ideas of the exterior appearance of the church before the big fire in 1728 can be gathered from the French clergyman Gaspard-Antoine de Bois Clair, who painted a gouache representing the church in 1690. The façade seems to have looked guite as it does today. [1]

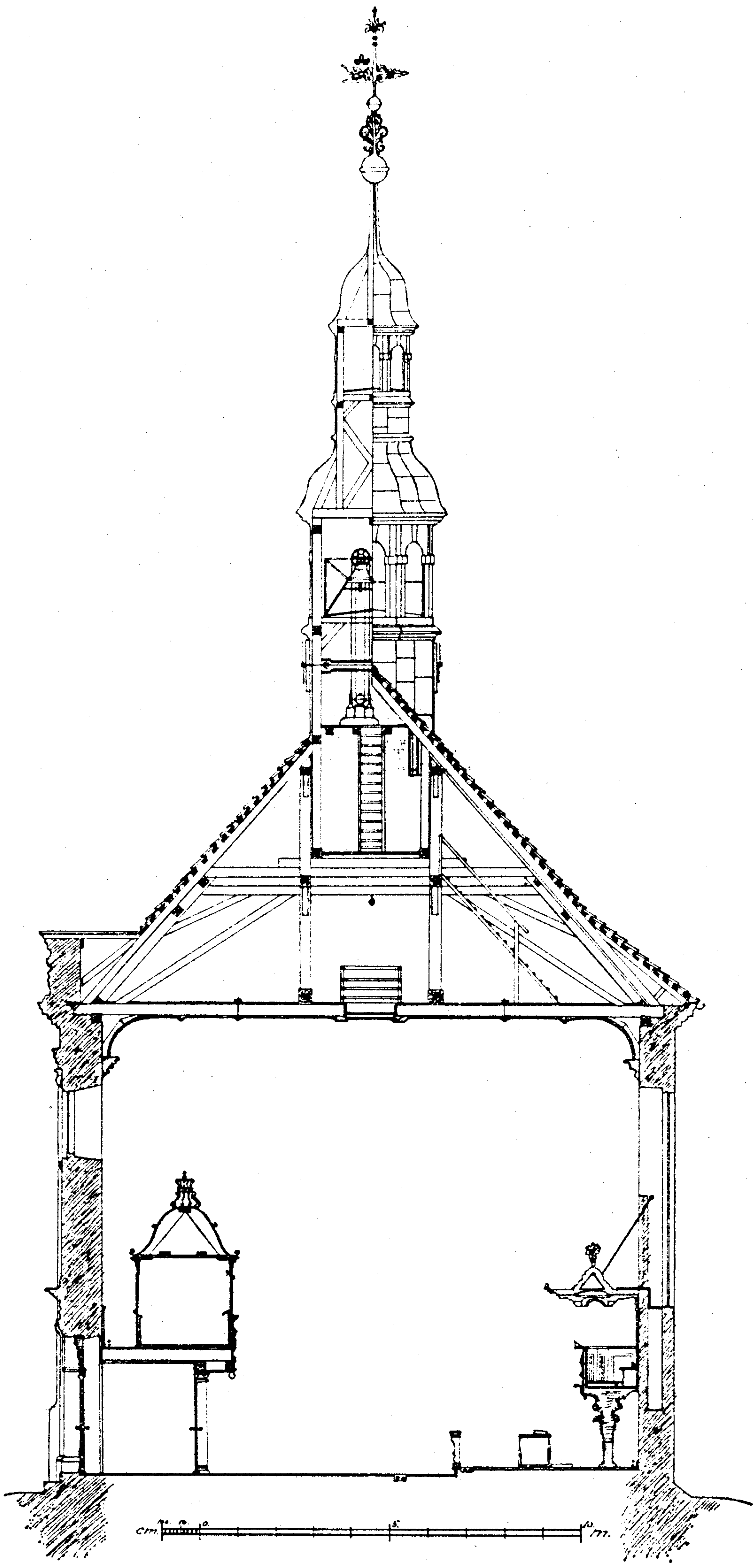

Figure 1.1 Transversal section of the church [1].

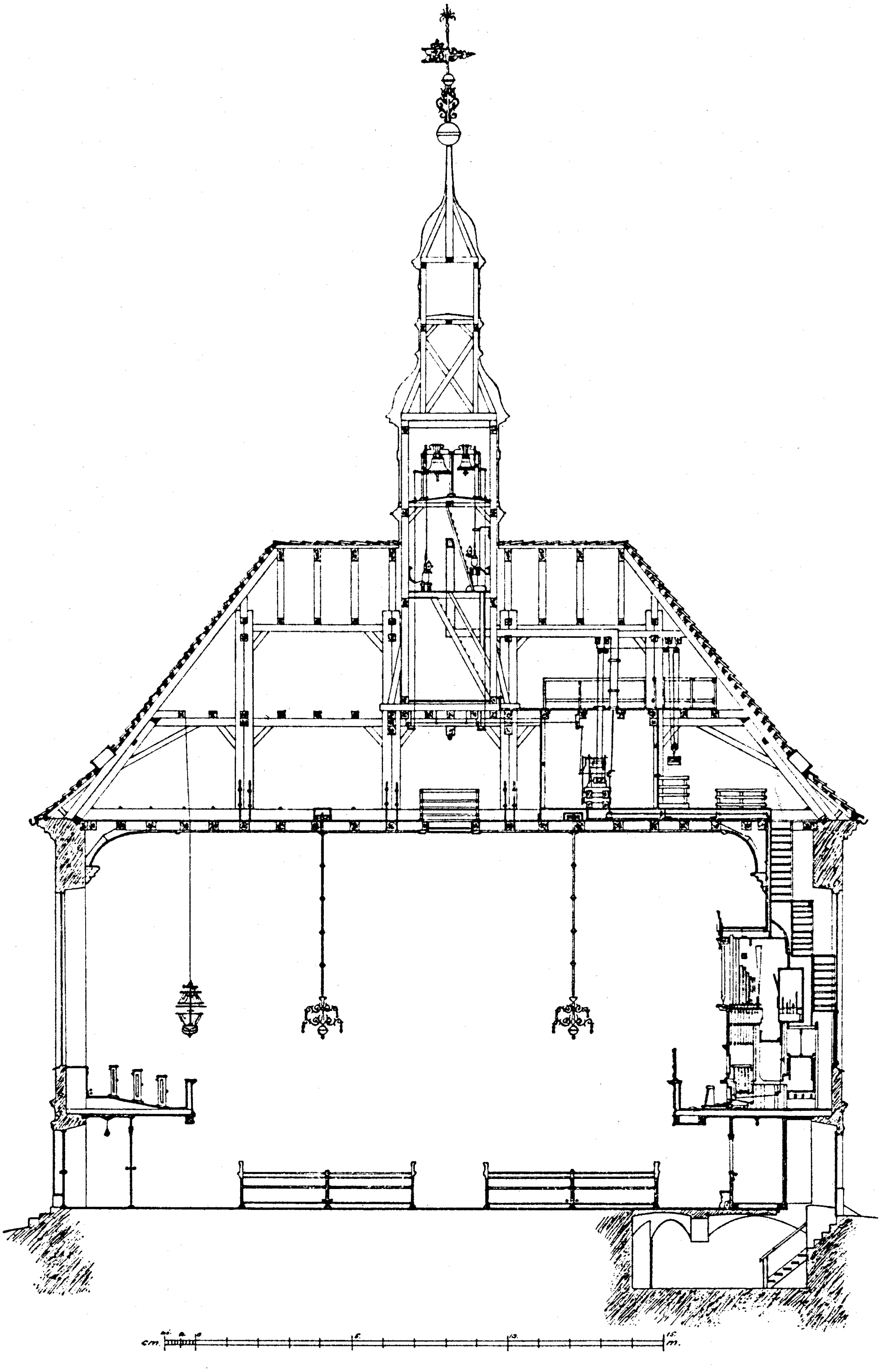

Figure 1.2 Longitudinal section of the church [1].

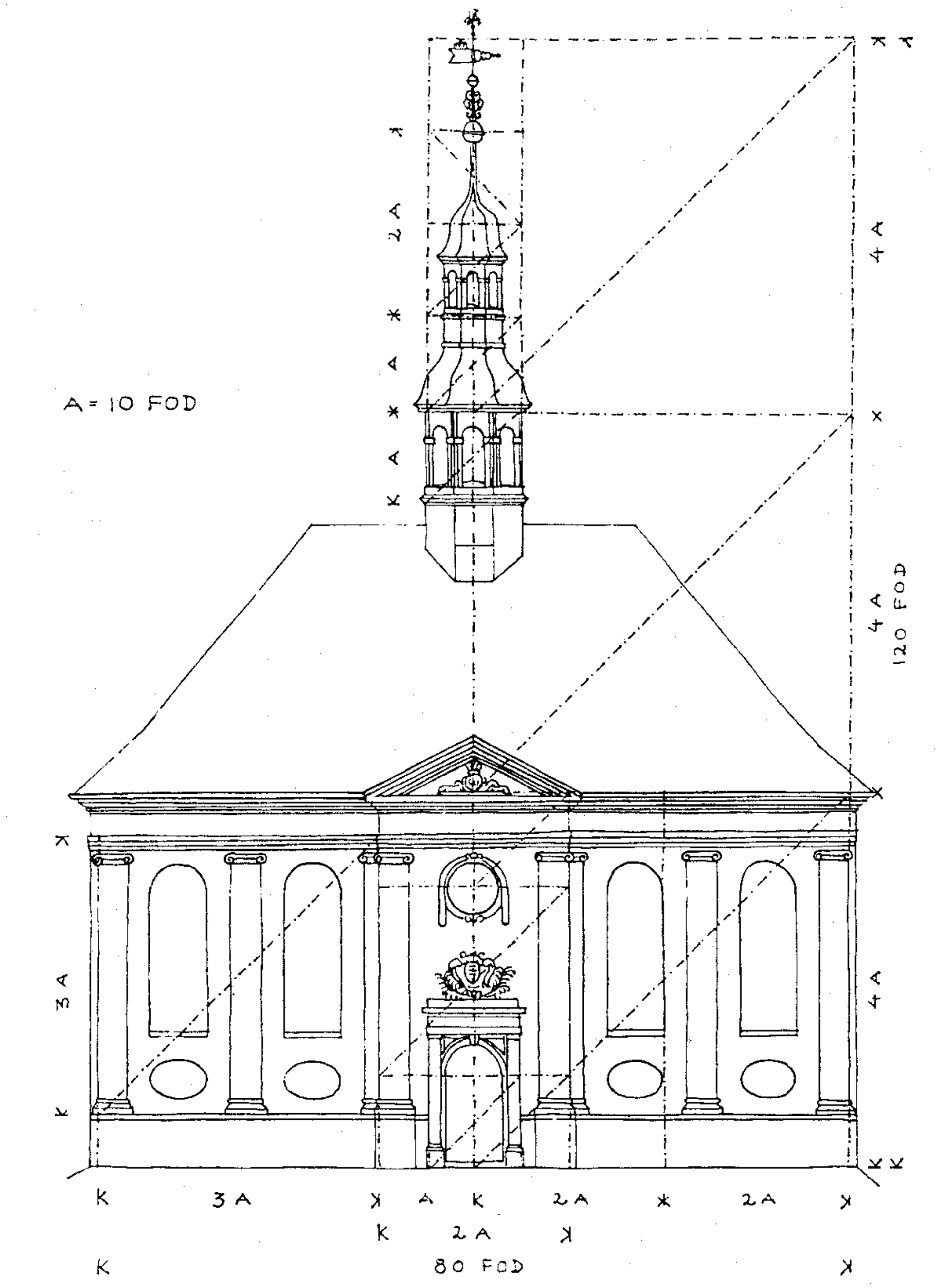

Figure 1.3   The façade including the measurements. (1 fod = 0.313 m) [1].

1.2 The restoration of 1987 and 1989

In 1987 there was a decision made that the mortar joints of two of the walls should be restored. The construction workers started from the bottom and worked their way up. In 1989 the time had come to restore the remaining walls. When the workers reached the top of the wall at the back of the building and the cornice, they found that a restoration of the mortar joins would not be enough. The whole roof of the building was about to fall in because of dry rot. [3]

2 Purpose

The main purpose of our work is to show the potentials of a three-dimensional computer modelling and simulation of a building structure. We would like to show how the science of engineering in particular, and advanced computer modelling including finite element methods, FEM, can be used to increase the understanding of a structure. We intend to show more than simple strength calculations, which is the most common range of application of the finite element method.

We will also demonstrate how to apply this method to a historical building structure. This includes a comprehensive study of the structure in mind, which is done not only to be able to reveal the geometry, but even more to learn to understand the structure and its behaviour. This knowledge is necessary to be able to judge the truth of the results.

When the static behaviour of the structure is found we will illustrate this in a way that is simple to understand even for the less experienced viewer. A historical wooden framework is very likely to be highly complex. That is why it is very important to be able to study the results of a calculation when dealing with the framework as an entirety. This includes plots of deformations and reaction forces on the complete structure, as well as special examination of smaller sections of the structure.

With the computer model needed for the above analysis it is easy to go further and simulate damage to the structure or exposure to special load cases. This can be done either to include realistic or probable damage in the dimensioning when restoring the building or to verify observed damage. This will not be demonstrated explicitly in this work, but the possibility will be obvious.

Svend Jakobsen, our Danish supervisor, was part of the team when the building in question was restored in 1987 and 1989. At that time a different method was used to analyse the structure. The results from our analysis will be used by Svend Jakobsen to compare with that analysis. There are also some parts of the structure in particular where he is uncertain of the behaviour. These parts will be specially treated by us.

As an additional purpose we will also discuss different methods of analysing building structures, especially old historical wooden structures. Originating in the different tools, computer calculations and hand calculations respectively, used in Sweden and in Denmark we will try to investigate how the norms and regulations are used. We are interested in whether the rules must be completely obeyed or whether they may in some cases take advantage of the calculation method in question.

We are also analysing and discussing different philosophies when restoring old historical buildings.

3 Methods

As mentioned in Chapter 2 our main purpose in this Master's thesis is to visualise the behaviour of the rather complicated roof structure of the Reformed Church by means of advanced computer simulation. Thus we need to choose computer programs with a high capability. We need programs that are able to handle the three-dimensional frame model of the roof, with its actual connections and piles, and to calculate deformations, forces and stresses by the finite element method. Our choice is a combination of three different computer programs.

To start with we have to generate the geometry of the structure. To do this we use a program called Microstran, which is a three-dimensional CAD program. All elements and their nodes are drawn in this program. The next step is to export the elements and the nodes from Microstran to PATRAN. With PATRAN as a pre-processor we use a computer program called ABAQUS Standard for the calculations. ABAQUS is a commercial and general finite element program which contains calculation-, pre- and postmodulus. In the program there is an opportunity to choose between many different material models and element types. Linear as well as nonlinear, static and dynamic analyses can be performed. PATRAN was made to facilitate the use of ABAQUS and gives a graphic feedback to the incoming information.

The ABAQUS program applies the finite element method as an approximate and numerical way of solving differential equations. These equations are a mathematical interpretation of the actual structural mechanical problem.

ABAQUS supports different levels of beam modelling. We choose to use a beam element type devised from the Timoshenko beam theory. With the slender design of the beams considered, the Euler-Bernoulli beam theory would have been sufficient. However, due to the lack of constraints in torsional movement in our model, less input data is needed if the Timoshenko beam element is used. As can be seen in Figure 3.1, the Timoshenko theory does however coincide with the classical Euler-Bernoulli theory when the beams are shaped like the ones in the Reformed Church. Both theories assume equilibrium in the undeformed state, but the Euler-Bernoulli beam theory requires undeformable sections of the members. In the Timoshenko beam theory the cross section need not necessarily remain normal to the beam axis.

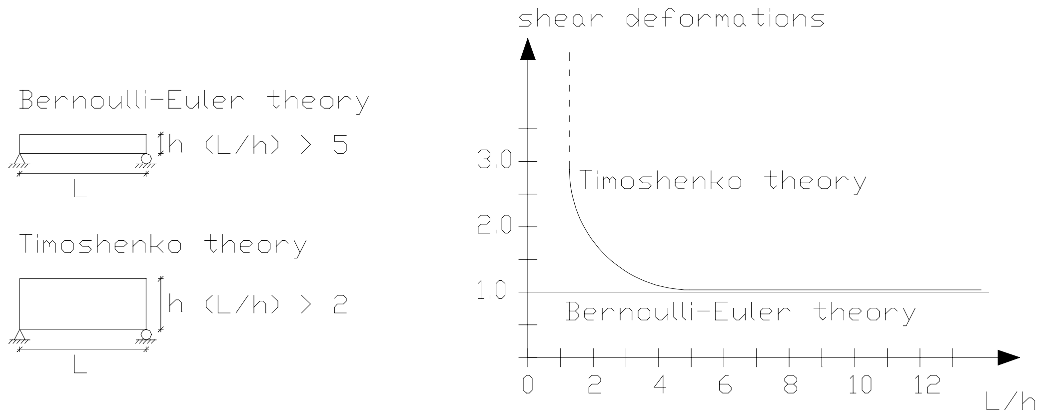

Figure 3.1 Comparison between the Euler-Bernoulli and the Timoshenko theory [2].

The element type is also based on the assumption of small deformations and the calculations are made according to the elastic theory and within the theory of the first degree. For further discussion see Chapter 6.

The frame model, containing beams, is, in comparison with the truss model, made up by bars, the one best corresponding with reality in the Reformed Church. In a truss the assumption of non-friction hinges in all joints is used, Figure 3.2.

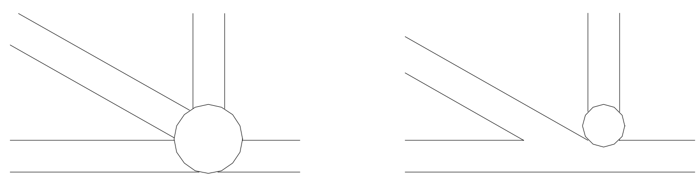

Figure 3.2 Left: frame model – right: truss model.

The dominating forces, which are the normal forces, can often be found by the use of a simple truss model. With PATRAN as a design tool it is, however, quite easy to start with the full framed model, which has been done in this Master's thesis.

When it is decided to use Microstran for the modelling of the structure, knowledge of the geometry is required. The aim is to make the geometric model as similar to reality as possible. The drawings on which we base our dimensioning were made for the restoration in 1987. We use these drawings together with our own measurements. When divergences between these occur we postulate from photos, which we think represent reality more accurately. This means that the photos together with our own measurements are the main source of input to the geometric model. All the parts of the roof structure are co-ordinate referenced, and the cross sections of the members are measured.

The structure is in reality mainly double symmetric, that is symmetric along two axes, but in the geometric model we would like it to be exactly double symmetric. Existing differences are small, which is why we choose to level them out. In this way the structure is forced into a double symmetric state.

Since the model is double symmetric it is very easy to project one quarter into a complete model of the structure. The whole roof model is used in PATRAN, in order to be able to load to the model correctly. This makes it possible to take the single symmetric loads into consideration. Furthermore, this simplifies the interpretation of the results and the need for boundary conditions at the symmetry axis disappears.

We draw the conclusion that the material of the original beams is old pine of high quality [3]. The material parameters are found in a wooden handbook [4]. These values, however, are determined from tests on pine of a younger age than the one in the Reformed Church, so some divergences may occur, see further discussion in Chapter 6.4. This is the platform for the material model.

To make a realistic model of the joints, we model the behaviour of the connections between the members in the roof by means of studies of literature about historical buildings from the same period [5] [6].

The calculations of the loads snow, wind and dead load, are based on Swedish building standards [7].

The visualisation is partly made with PATRAN, but we also use the Internet standard of visualisation VRML. [8]

The above decisions and interpretations are in part derived from discussions with our supervisor, Svend Jakobsen, and our examiner, Karl-Gunnar Olsson. We also had the opportunity to attend a seminar at the castle of Glimmingehus concerning the restoration of its roof in the summer of 1998. This was profitable and gave us some ideas for the explanation of the results.

4 Restoration of historical timber structures

4.1 Different methods of diagnosing

When damage has occurred in a building there are different ways of locating the reasons. One possibility is to do a thorough inventory of the structure. There are certain guidelines for conducting such an inventory, including some especially critical areas to consider.

- It is always important to check areas where different types of material meet. Badly shaped connections of this type are often the reasons for damage to timber caused by dampness.

- Any visible leak must be further investigated. Most of the time the reason for the leakage is quite simple. Sometimes the original reason may, however, be derived backwards to another part of the building. That is, a leak can be caused by a simple hole in the roof, but the hole itself can be caused by something far more complicated. If this is the case it is important to find and eliminate the actual reason for the damage. Simply patching up the hole means that the damage will very likely emerge again.

- Earlier repairs must be investigated very carefully. A previous repair can sometimes indicate problems in other parts of the structure. Perhaps these problems have not been treated accurately and therefore they might have caused new problems.

The kind and the range of every damage must be investigated. Furthermore, the feasible reasons for the damage, the effects that it might have had on other parts and the development of the damage must be investigated.

One part of the inventory can be to carry out a statical analysis by means of a computer program. If the problem is obvious, for example decay damage in a load-bearing part of a structure, there is a great possibility to simulate the actual damage in the model. Then its effects on the rest of the structure can be found. Otherwise, if the cause of the damage is unknown, one way of finding the reason can be to try and evaluate different feasible occurrences in the model.

4.2 Philosophies of restoration

There are many different philosophies when it comes to restoration of historical buildings, see for example Smeallie, Smith [9] and Jakobsen [10].

A general guideline when dealing with a restoration of an old historical building should be to make as few changes as possible; originality should in some way be sought. Originality means different things to different persons, which is why there are several approaches when restoring this type of building.

- Necessary changes can be made to integrate the repairs with the original building so that the entire building is considered as one entity. The same material is used and the methods are the same as when the building was first erected. An imitative philosophy when a restoration or addition is conducted can be used with respect to the architect in question.

  If the building has a historic or symbolic value it is often of great importance that the method of restoration or addition is imitative. The advantages of this method are that the appearance is genuine and there is a possibility to study and learn different techniques even though the details are new.

  One disadvantage is that after awhile it might be impossible to distinguish old from new unless accurate marking and documentation have been made.

- Another way to perform the changes is to make very obvious what is new and repaired and what is old. The use of different materials and modern methods are tools for the adherents of this philosophy. As an example, a restoration of an old timber structure can be made with slim beams of steel. This can be tolerated if the new members can be placed into the old structure without the need to remove or destroy any original members. These new members can be removed in a later stage without damage to the original parts.

  An addition or repair that contrasts to or is abstract from the original becomes a very visible part of the building. The contrasting repair distinctly defines what part of the building is not original. The intention is to draw attention to the new addition, and to tell the building's history and development. In some ways this can be considered as more honest to the viewer both today and in the future.

Regardless of the philosophy used in a restoration, computer analyses can be helpful. If the first imitative philosophy is used, the statical behaviour of the structure can be seen. This provides knowledge of the building's weaknesses, which can help to prevent similar damages from occurring after the restoration. If the other philosophy is used, there is a chance to make the additions as slim and effective as possible. A slim addition is less disturbing than a coarse one.

When the restoration of the Reformed Church was performed, the philosophy was to restore the church to its former condition. They used pinewood, but the timber is not hewn squared. If the documentation is well done, this will not be a problem; otherwise it might be impossible to distinguish old from new in the future.

4.3 The Swedish and the Danish criteria

In this chapter a description is provided of the differences and the resemblances between building and restoration in Sweden and in Denmark.

There are some differences when it comes to the responsibility of the structure during the renovation. Denmark has a system that is very established. This system is built upon authorized structural engineers. The analysing is founded on the tradition in hand calculation which in its turn is based on a statically determined structure.

To become an authorized structural engineer there is a procedure to undergo. It includes a close investigation of the person in question. At least two-thirds of the jury, which contains nine members chosen for a period of four years, has to be in favour of the applicant.

The bases of authorization are:

- A project investigated by the applicant, which is supposed to give an impression of his or her engineering qualifications.

- Two authorized structural engineers and colleagues of the applicant, who have had a close interest in his or her work, should express their opinions.

- A list from the applicant concerning his or her projects which have been treated by the building authorities over the last three years.

- Documentation that the applicant has been involved in statical calculations for three years, of which at least one year was devoted to independent work.

- A written declaration of the applicant's submission to the present rules and regulations.

If deficiency in the work of the authorized structural engineer is found, the board has an opportunity to withdraw the authorisation. This prevents a new application for three years.

If a project containing calculations and drawings is signed by an authorized structural engineer there is no need for a higher authority to further supervise. The authorized structural engineer follows a certain set procedure and adjusts the contents accordingly. [11]

In Sweden there is no such distinct tradition. One system is left, and another is about to be entered. Previously the building authorities examined the strength calculations whenever a building permission was applied for. However, the structural engineer continued to be the responsible party. The difference today is that the authorities do not check all calculations. Random tests of the calculations can still be performed, but the responsibility is more clearly defined.

The responsible party in the structure work is called the quality control official but the responsibility of this person is nowhere near as wide as the responsibility of the authorized structural engineer in Denmark. The Danish system can be compared to the Swedish system when building bridges. Then a person is given a wider responsibility, like the authorized structural engineer in Denmark. [12]

When a new building is constructed there are rules to follow in Sweden as well as in Denmark. However, when a restoration is to take place it is not always possible to refer to these rules. There are, in Sweden, some guidelines which can be used while restoring an old, historical building. Depending on the philosophy used, see Chapter 4.2, the strategies are different.

In Sweden it is always permissible to restore a building to its former condition, even if this means that it is impossible to follow the present building regulations. If a building has been standing since the 17th century, it is most likely to stand even if the today's rules are not obeyed.

If there is no need to reestablish the structure, it is of course allowed to use the current rules and regulations, when restoring the building.

5 Restoration of the Reformed Church

5.1 Terminology

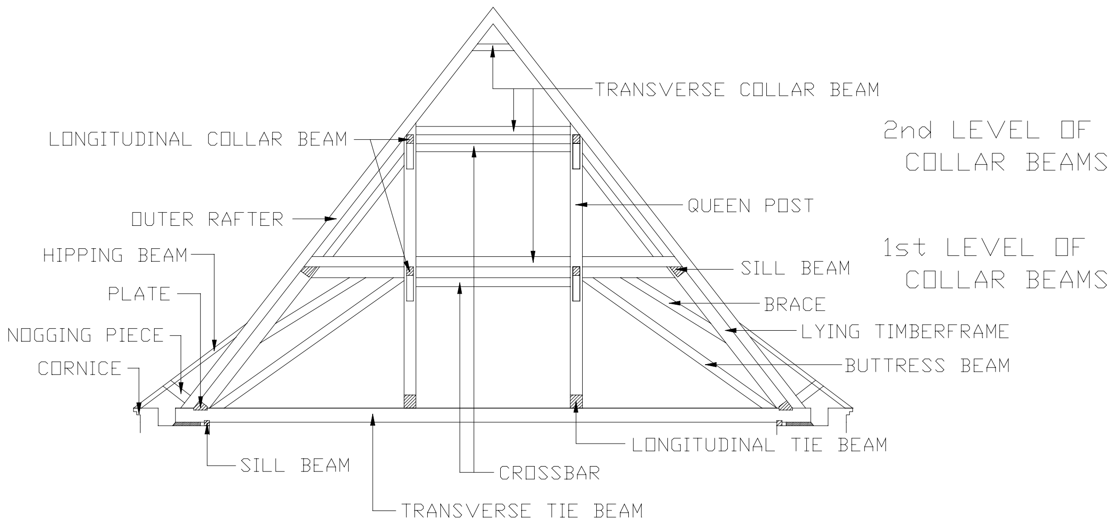

Figure 5.1   A section of the roof structure of the Reformed Church.

Figure 5.1 represents a section of the Reformed Church, and the names of the members are present to facilitate the understanding of this Master's thesis.

The most common connection between members in the Reformed Church is the tenon joint; compare Figure 5.2. The thickness of the tenon is about one-fourth of the dimension of the beam. To facilitate the fitting of the tenon into the mortice, the tenon is chipped off at the end.

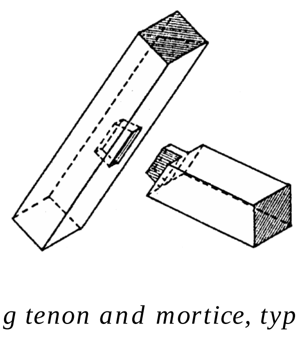

Figure 5.2   Tenon joint including tenon and mortice, typical of the Reformed Church [6].

The length of the tenon is a bit shorter than the depth of the mortice, see Figure 5.3, but there is still enough room to drill the hole for the dowel.

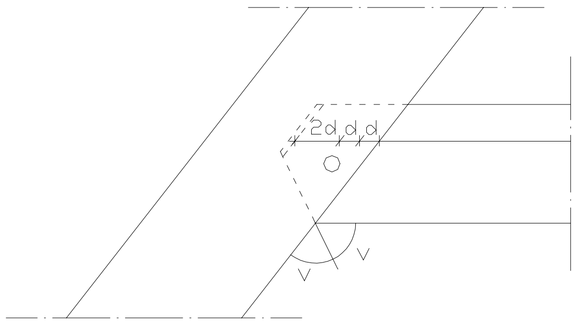

Figure 5.3 Section of a tenon joint, seen from the side.

According to Figure 5.3 the length of the tenon is about four times the diameter of the dowel. The tenon is shaped, compare Figure 5.3, by dividing the angle in the figure into halves, v. This is done to increase the load carrying function of the joint.

To force the tenon and the mortice together the following method is used: The centre of the hole for the dowel is marked in both the tenon and the mortice, but the hole in the tenon is to be drilled two to three millimetres closer to the breast, Figure 5.4. When the dowel is driven in to secure the joint the members are forced together as desired.

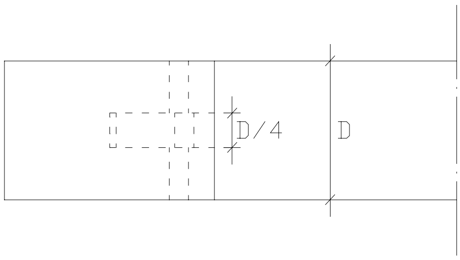

Figure 5.4 Section of a tenon joint, seen from above.

Other often-used connections between two members are the cross halving, Figure 5.5, and the double tenon joint, Figure 5.6.

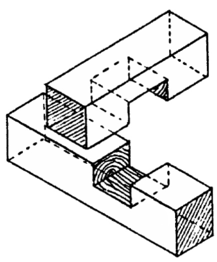

Figure 5.5 Cross halving [6].

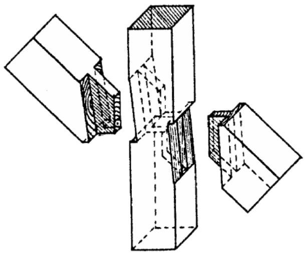

Figure 5.6 Double tenon joint [6].

At the top of the roof, the two rafters are connected according to Figure 5.7. This is called a halving joint.

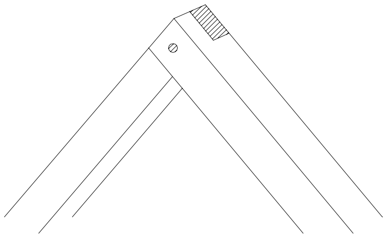

Figure 5.7 Halving joint, connecting the rafters.

5.2 The Restoration of 1989

In 1987 the front façade and one of the gable façades of the building were cleaned and the mortar joints were redone, see Chapter 1.2. In 1989 the restoration was meant to continue with the remaining two sides of the building. It was now realised that the cornice stood out a bit too much from the wall. When it was further investigated, fungi-mycelia was found in the mortar joints. A specimen was sent to a Danish institute for building analyses and it was determined to be dry rot fungi (Serpula Lacrymans).

If the specimen had been active, the traditional treatment would have been to separate all wood in a safety zone of about one meter from the damaged timber. Then the brickwork would have been brought down, the mortar joints scraped and the brick burned. A disinfectant treatment would also have been used.

In this case the fungi were inactive and therefore an alternative method could be used. Parts of the structure that had suffered reduced strength due to the fungi attack were reinforced with new timber, and a chemical protection with Boracol was applied.

The reason for the projecting cornice was also found. In the late 1930s, a steel tie-rod was fixed at the beam ends as a sort of reinforcement. In addition the cornice was bricked up around the tie rods and attached to the rafters with cramps, which means that parts of the roof were enclosed in masonry. This measure resulted in an increased load on the church wall. The church wall, however, was not constructed to carry these horizontal forces, which resulted in the projection of the cornice.

When the beam-ends then became fungi attacked, an attack that was seen to be more widespread than the first inspections of the building had indicated, the cornice of course could not be kept in place.

The restoration ended up with a replacement of all the segmental tiers of beams and joists, the rafter ends and the sill beams. The plate, over the beams, which carries the pressure from the lying timberframe was replaced as well. [13]

6 Framework model of the roof structure

6.1 The Finite Element Method

There are a lot of different physical phenomena in engineering mechanics as well as in other areas of engineering science where differential equations are used as models of reality. The finite element method, FEM, is an approximate and numerical way of solving these differential equations. The process of solving can be seen in Figure 6.1.

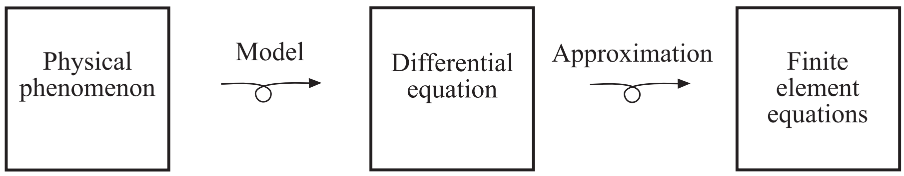

Figure 6.1 Steps in analysis of engineering mechanics [14].

The model of the structure is divided into smaller areas, finite elements, over which the differential equation is valid. The element mesh can be made more or less dense depending on the accuracy, calculation time and cost expected.

The nodal points are points often located at the boundary of each element. The approximation is an interpolation between the known quantities in these points.

To solve the problem, boundary conditions are needed. Boundary conditions describe how the modelled area behaves at the border to the world outside. In structural mechanics, the boundary conditions for a beam supported on walls, which in this case represents the outside world, can be restricted in movements in different directions. These directions are called degrees of freedom.

There is more than one possible load path in structures that are statically indeterminate. This makes it difficult to see all the different paths that the load can take through the structure. It is simpler to solve this type of problem with computer programs based on the finite element method than with hand calculations. The roof structure of this Master's thesis is an illuminating example of a statically indeterminate problem. [14]

6.2 Structural model

In this Master's thesis the model of the roof is built up by elements and nodes. The elements are connected to each other as described later in this chapter, and the support from the walls is taken into consideration. A global co-ordinate system is defined where the x- and y-axes are placed in the horizontal plane and the z-axis represents the height. Compare Figure 6.2.

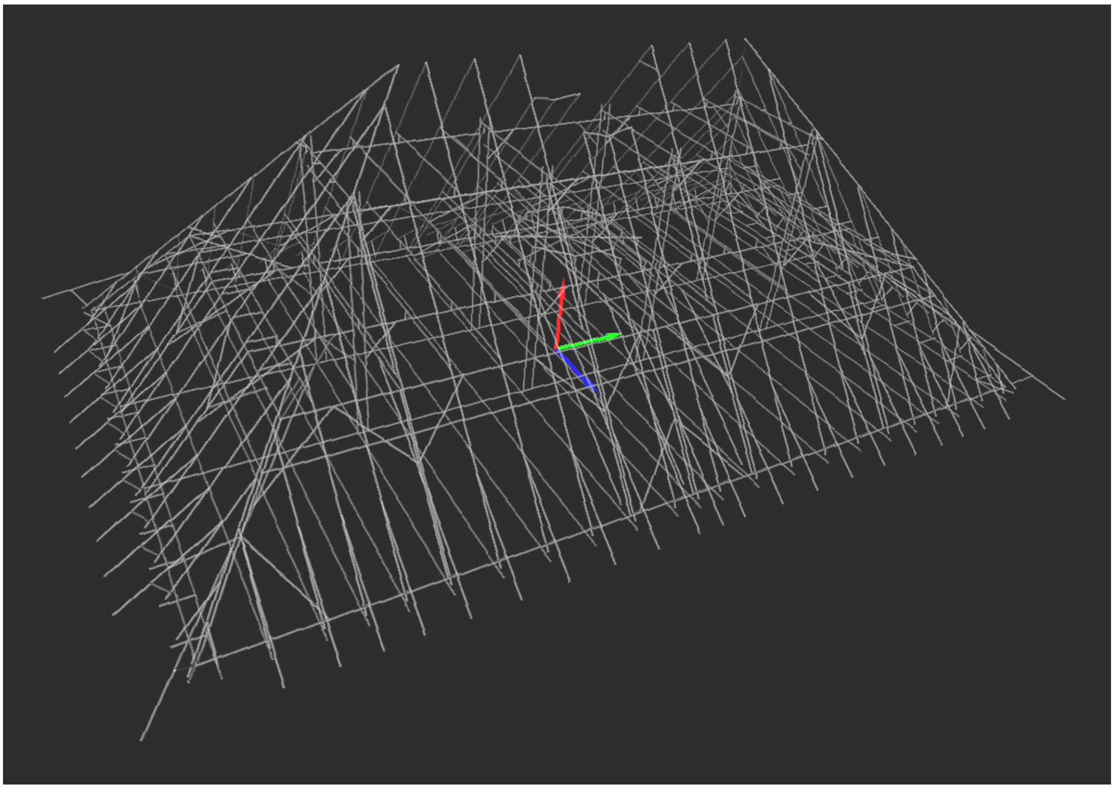

Figure 6.2 Definition of the global co-ordinate system. X-axis green, y-axis blue, z-axis red.

The element type used in this model is a three-dimensional beam element. It contains three nodes, one node in each end of the element and one in the middle. Each node has six degrees of freedom. Displacements are represented by the first three, and rotations by the last three, illustrated by single and double arrows respectively, compare Figure 6.3.

For numerical reasons, prevention of torsional movement, that is, free rotation of a beam around its own axis, is required. This is why the midnode is introduced for every beam. A local boundary condition exists at every midnode. This local boundary condition prescribes the rotational degree of freedom in the local z-direction, that is coincident with the beam axis of the element, to zero. The ends of the beams are still considered to be free to rotate.

The three-dimensional beam elements we use will allow bending, stretching, torsion and warping of the beam. Only axial and bending strains can be considered.

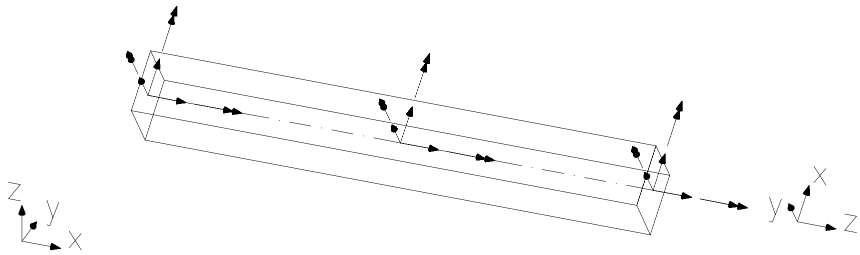

Figure 6.3 Three-dimensional element used in the framework model. Definition of global co-ordinate system (left) and local co-ordinate system (right).

There can be more than one element along a timber beam. If the beam is continuous the connections between these elements are rigid, that is, all the degrees of freedom are coupled.

Each beam is defined in a local co-ordinate system where the z-axis coincides with the beam axis. The local xy-plane coincides with the symmetry lines of the beam section. This means that if the beam is rotated the local xy-plane is not the same as any global plane, compare Figure 6.3. The local co-ordinate system gives the beams the correct stiffness in the different directions. If we had chosen not to rotate the elements around their axes the result would probably have been almost the same.

We assume a beam behaviour according to the Bernoulli beam theory. This presupposes small deformations, and equilibrium in the undeformed state which makes the calculations linear. When using the first order theory, however, it is not possible to estimate security against buckling.

Each beam in the model is represented by its centreline. This gives a distance between centrelines of beams in contact which cross each other at different levels. To solve this problem we use two different procedures.

- In a connection between two elements there are two nodes, one from each element. The degrees of freedom in these nodes can be coupled with any different possible constraint. For example the translations of the two nodes can be connected while the rotations of the nodes are independent of each other. This connection is possible even if the elements do not cross at the same level.

- At some contact points we use fictive elements between the two crossing elements, compare Figure 6.4.

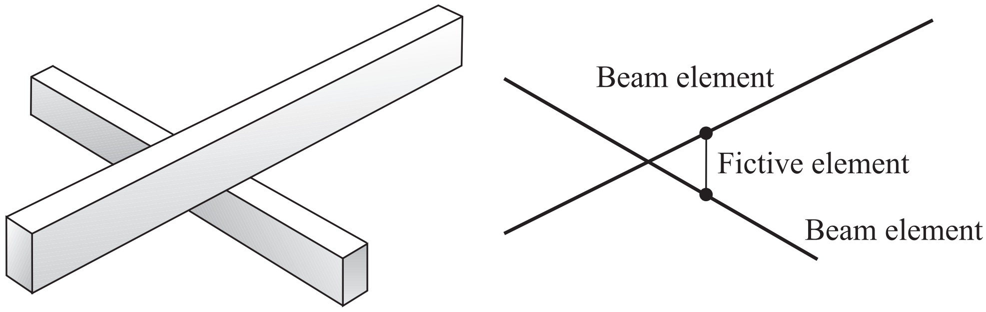

Figure 6.4 Fictive elements used to model elements that cross at different levels.

  These fictive elements are given the Young's modulus of wood perpendicular to the fibres. This connection models the contact as well as the ability of compression.

The first approach was to use the latter procedure, with fictive elements. These were connected to the real elements as regards translations. The rotations were independent. However, this approach resulted in large rotations in some of the fictive elements. These elements were removed and replaced with a simple connection; see the first of the procedures above. Another solution could have been to use fictive elements with a connection where even the rotations were dependent. The results and the effort involved do not differ greatly between the two procedures of modelling.

A fictive element is also needed when an inclined beam meets horizontal or vertical beams. Their centrelines do not meet where the beams connect. In the model you can either extend the inclined beam and let the two beams meet at the same node or use a fictive element perpendicular to the horizontal or vertical beam, compare Figure 6.5.

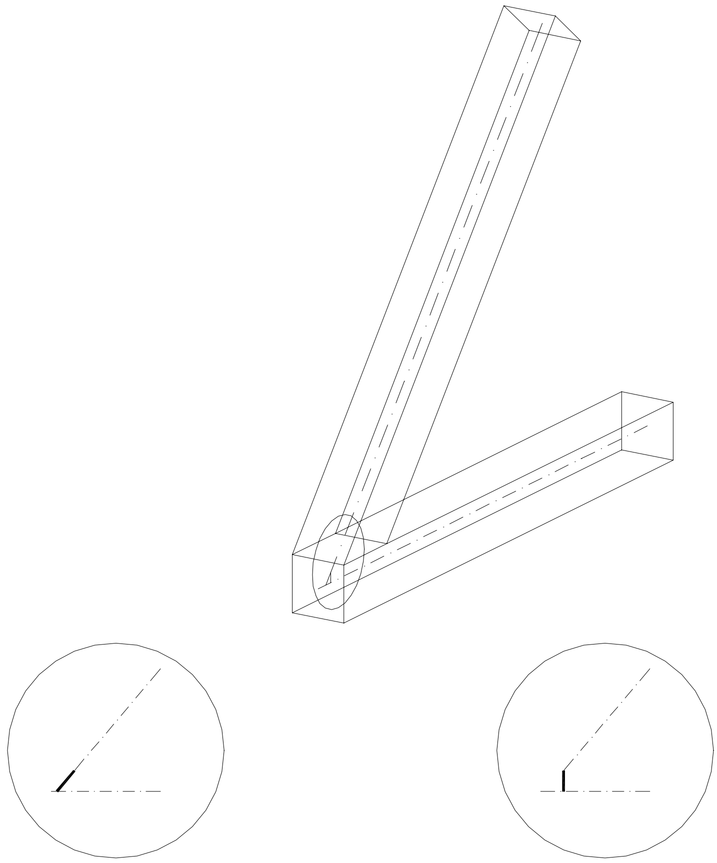

Figure 6.5 Possible connections between one inclined and one horizontal beam: extension of the centreline and a fictive element perpendicular to the horizontal beam respectively.

Our choice is to extend the centreline of the inclined beam into a connection with the horizontal beam. This method requires less input data but will cause slight incorrectness in moments. The distances are however small, see Figure 6.6, which probably makes the fault insignificant.

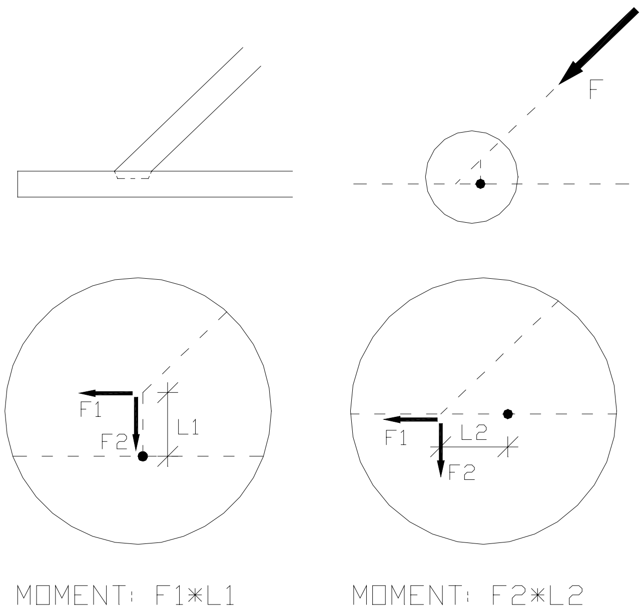

Figure 6.6 Differences in moments. The force, F, is divided into F1 and F2.

Most of the beams in the structure are connected with wooden tenons, and sometimes the connections are also reinforced with wooden dowels. Potential pressure is taken care of by the tenon, but if any tension occurs the dowel is needed.

Wood is a quite soft material, and making rigid connections is almost impossible. That is why we choose to model almost every junction between elements with translations connected, but rotations disconnected; every element is free to rotate.

At the hipping beam there is, for this building, an unusual connection. Two beams are connected and the junction is reinforced with dowel and screws as shown in Figure 6.7. This connection probably has a stiffness somewhere between a rigid connection and a frictionless joint. In the model we choose to connect the beams only regarding to translations. If we had instead made it rigid, the result would have been a heavy load on the hipping beam. The hipping beam is, however, not piled on the cornice and therefore not supported from underneath. This means that large vertical loads cannot be supported. The purpose of the hipping beam is probably just to give the roof its hipped shape. Of course the connection could also be represented by a spring, but then the problem would be to estimate the spring stiffness.

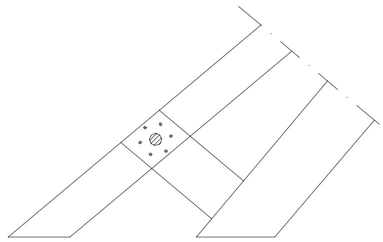

Figure 6.7 Connection in the hipping beam (the location of the detail is shown in Figure 6.9).

We consider the church spire as a rigid body and model it with rigid-body elements. We do not analyse the spire since we are not interested in its behaviour; the reason for modelling it is to get the values of wind and dead loads applied to the structure accurately.

The star structure in the middle of the whole structure on which the spire rests is in reality like the left part of Figure 6.8 but we model it as the right part.

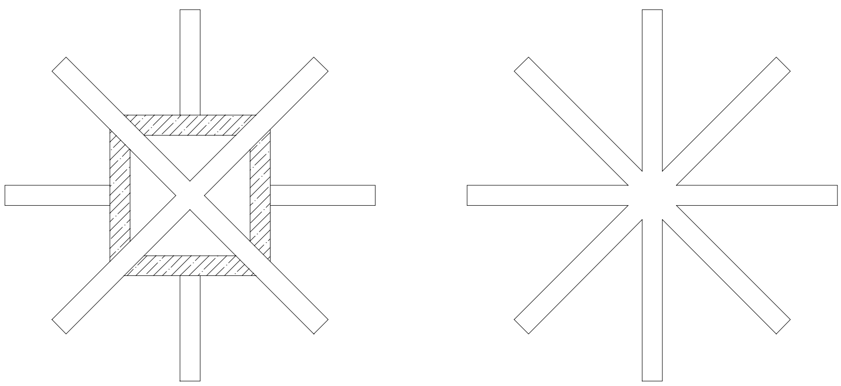

Figure 6.8 The star structure in reality (left) and in the model (right).

This will not make a big difference because the behaviour of the structure will be almost the same. This is explained by the quadratic beam structure (striped) and the other beams' attachment to this. The connected beams are in reality almost unable to rotate in relation to each other. In the model we choose to disregard the quadratic beam structure. Instead we simulate the star structure with crossing beams prevented from rotating in relation to each other. The main purpose of the star structure is to convey the *vertical* loads from the church spire to the roof structure. This means that it does not much matter which model we choose.

6.3 Boundary conditions

We choose to model two different sets of boundary conditions, which represent two extreme support behaviours, that is whether the wall is carrying the horizontal reaction forces or if the wooden structure of the roof is able to carry these forces itself.

Boundary condition case A:
The wall supports the horizontal reaction forces.

Boundary condition case B:
The wall carries no horizontal reaction forces, which means that the roof is capable of handling these.

On the sides of the hipping beams there are flat bars of iron screwed into it, see Picture 6.1 and Figure 6.9. These bars are present to carry the drainpipe and have no other load carrying function.

Picture 6.1 The hipping beam furthest out in the structure.

Before the restoration of 1989 the hipping beam was in contact with the cornice. In the 1930s the hipping beams and the tie beams were surrounded by a clump of masonry, as well, compare Chapter 5.2. This resulted in a co-operation, regarding horizontal reaction forces, between the roof structure and the wall. Since the church wall was not built to carry these forces, the cornice became embossed. This was noticed and taken care of during the restoration in 1989. After the restoration the roof structure once again carries its own horizontal reaction forces, as it was constructed to do.

These are the reasons for the different boundary conditions used in this Master's thesis. Boundary condition case A, where the cornice is charged with loads from the beams as it was before 1987, and boundary condition case B, where the wooden structure carries its own horizontal reaction forces.

6.3.1 Boundary condition case A

The inclined beams in the section are connected to the transverse tie beam, compare Figures 5.1 and 6.9, by a tenon joint. The only exceptions are the inner rafter, resting on the plate, and the hipping beam, see Figure 6.9.

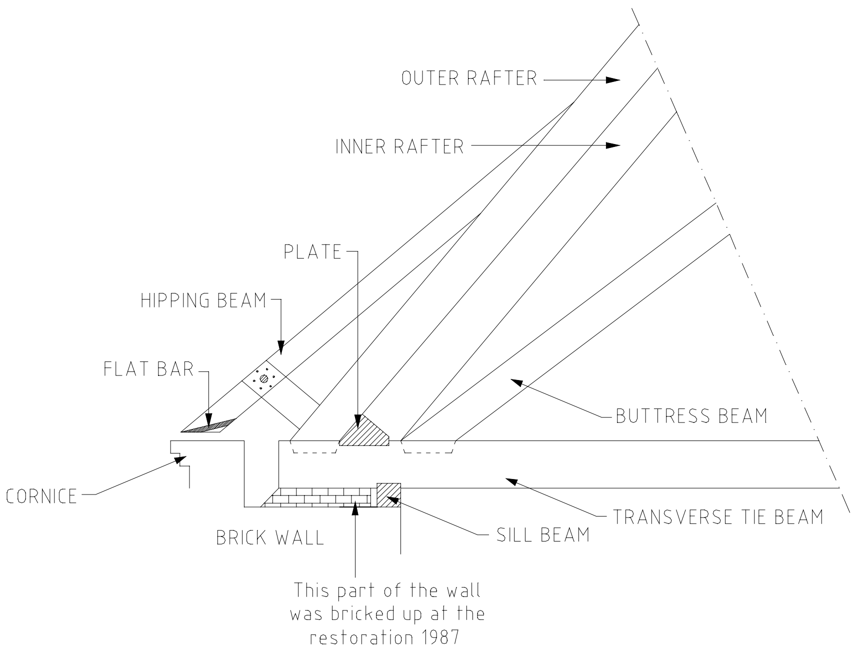

Figure 6.9 Section of the roof structure, showing the inclined with corresponding boundary conditions.

The transverse tie beams are in their turn simply supported on a sill beam which is resting on the wall. This naturally means that the degrees of freedom in the global z-direction are prescribed to zero. In this boundary condition case, however, the degrees of freedom in the global x- and y-directions are prescribed to zero as well. This means that the wall supports the horizontal forces from the roof structure.

6.3.2 Boundary condition case B

In this case we want the wooden structure to carry its own horizontal loads. This is what happens if the degrees of freedom in the global x- and y-directions are not prescribed. To prevent the structure from moving like a rigid body, one degree of freedom in the global x- and y-directions have been prescribed in the symmetry line of the roof. Prescribing one of the degrees of freedom in either x- or y-direction in one of the corners prevents rigid body rotation.

6.4 Material model

Wood can be regarded as a linear elastic material, that is we only use the elastic part of the stress-strain curves in Figure 6.10 where the relationship is close to linear. If the theory of elasticity is used the behaviour of the loaded structure can only be simulated up to the proportional limit. The theory is almost always used while dimensioning in the serviceability limit state, and very often also used while dimensioning in the ultimate limit state.

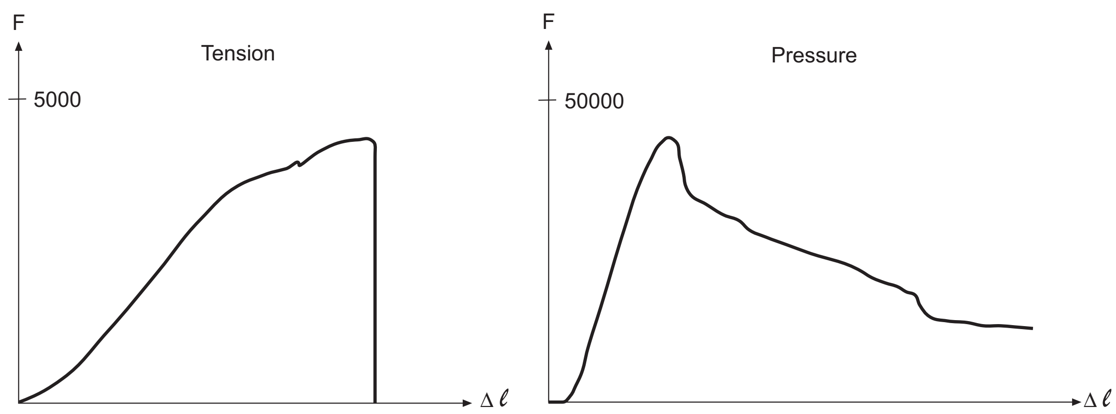

Figure 6.10 The relationship between forces (stresses) and strains.

In the Bernoulli beam theory the shear deformations and the deformations caused by warping are neglected. Because of this we do not have to consider the orthotropy of the material. The parameters needed as input to the computer program are the Young modulus, E, and the Poisson ratio, ν.

The Young modulus parallel to the fibres for the pine in the building is 11.5 GPa [4]. However, this value is not the characteristic value, but the Young modulus can be regarded as a scale factor in the equation system.

The mechanical behaviour of the structure will still be the same but the values of the results might be affected.

The value of the Poisson ratio is not really important to us, since we do not consider shear deformations, so it will be given the value 0 merely to suit the input properties for isotropic material. The density is needed, to calculate the dead loads for the wood as well as for the tiles [4].

The material data used for the pine have maybe been underestimated. The tabulated data available concerns pine that is felled today. Timber from the time of the construction of the Reformed Church is often of a better quality, since the trees grew in a less stressful environment. This entailed wood with closer annual rings and consequently higher strength. The wood in the Reformed Church is on the other hand very old, which might reduce its strength. This means that the results from our modelling will probably be accurate.

6.5 Geometric model

The drawings that we have at our disposal are based on the measurements made in 1987. If we compare the line measurements with the sum of the part measurements, there is no correlation. We use our measurements together with the drawings, the procedure is described in Chapter 3. These measures, together with photos, are the input to the geometric model.

The roof structure and the loads are double symmetric, except for the point loads from the structure connected with the church bell.

6.6 Load models

The building rules of Boverket, BKR 94 [7], are the basis of all load calculations in this work. The calculations are presented in Appendix.

Calculations are made for two load cases:

load case A: Dead load + Snow load
load case B: Dead load + Wind load

6.6.1 Dead load

All the beams are made of Pomeranian pine and the density we are using is 690 kg/m^3 [3]. There will also be dead load caused by the roofing tiles, which is estimated to be 795 N/m^2 [15]. The church spire will cause a point load of approximately 83 kN [3].

6.6.2 Snow load

Copenhagen will approximately belong to the same snow zone as Malmoe, that is snow zone 1 according to BKR 94. This will result in a snow load, s = 0.24 kN/m^2, which acts on the roof structure, except for the roof of the spire because of its steep angle.

6.6.3 Wind load

We choose to use the 50-year wind load, which is an extreme load that will statistically occur during one short period in 50 years. The roof structure is mainly built up by tenon joints, see Chapter 5.1. Hence the friction of the joints can absorb the energy caused by an extreme wind. This friction subdues the tops of the loads. Wood is an elastic material, which further facilitates the handling of these loads. When using such an extreme wind load it would be wise not to underestimate the bearing capacity of the wood. We have used the best values for the material parameters available, but since these data are for pine timber of today there is still a slight risk of underestimation of the bearing capacity as well as the stiffness, see Chapter 6.4.

Topography II is used and the pitch of the roof is about 50°. The characteristic wind power g_k = 1.12 kN/m^2 is given by BKR 94.

Four different wind cases can occur depending on which direction the wind is blowing from and depending on whether the inner suction is regarded or not, see Figure 6.11.

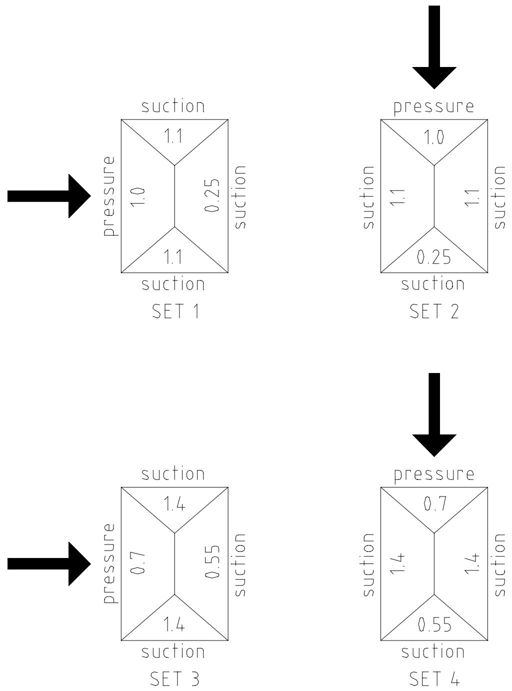

Figure 6.11 Possible wind cases. The arrows show the wind direction.
Case 1 and 2: Inner shape factor 0.
Case 3 and 4: Inner shape factor 0.3 (all areas are assumed to be equally untight).

Case three is probably the worst, and therefore the one we use in the calculations.

Sometimes when calculations are made on a very slim structure, for example towers and chimneys, the consideration of wind-induced vortex is required. The whirls on each side of the tower are superseding each other. When a vortex is generated on one side of the structure the velocity of the wind on the other side increases, which means a decrease in the pressure. A fluctuating load perpendicular to the wind direction acts on the tower because of the force induced on the side of the tower where the vortex occurs.

A slim structure is recognised by a relationship between the height and the diameter, where the height exceeds the diameter by at least five times. This is, however, not the case for the spire of the Reformed Church. Furthermore there are several openings to simplify the spreading of the sound of the church bells see Figure 1.3, which further decreases the risk of vortex. [7]

7 Results

The deformations are visualised to get an overall picture of the behaviour of the loaded structure. The deformation scale specifies the value of displacement in every point in comparison with the undeformed structure. The directions of the deformations are shown in the figures, in which the displacements generally are enlarged 200 times to make the behaviour clearer.

To be able to show the main load path, the normal forces are visualised. The elements are given different colours depending on the value of the occurring force.

The bending moments are not explicitly visualised, but they are implicitly included in the visualisation of the stresses. The maximum normal stress of a cross section is derived in ABAQUS according to the three dimensional equation of Navier:

$$\sigma_z = N/A \pm M_x/W_x \pm M_y/W_y$$

σ_z : normal stress
N : normal force
M_i: bending moment about the i-axis
W_i: bending resistance about the i-axis

This maximum value of the stress specifies the colour of the elements. Shear stresses are visualised as the normal stresses, but according to the equation:

$$\tau = 3{*}V/(2{*}A) + M_v/W_v$$

τ : shear stress
V : shear force
M_v: torsional moment
W_v: torsional resistance

7.1 Load case A

This load case is described in Chapter 6.6 and contains dead load and snow load.

7.1.1 Deformations

Deformations are shown and discussed for the two different sets of boundary conditions, when the wall is able and unable to carry the horizontal reaction forces respectively, see Chapter 6.3.

Deformations when the wall supports the horizontal reaction forces

Since the roof is symmetric and close to symmetrically loaded, the deformations will be close to symmetric as well. The structure is stiff, which explains the fact that most of it is almost undeformed, that is deformations less than 1 mm. The global behaviour is shown in Figure 7.1.

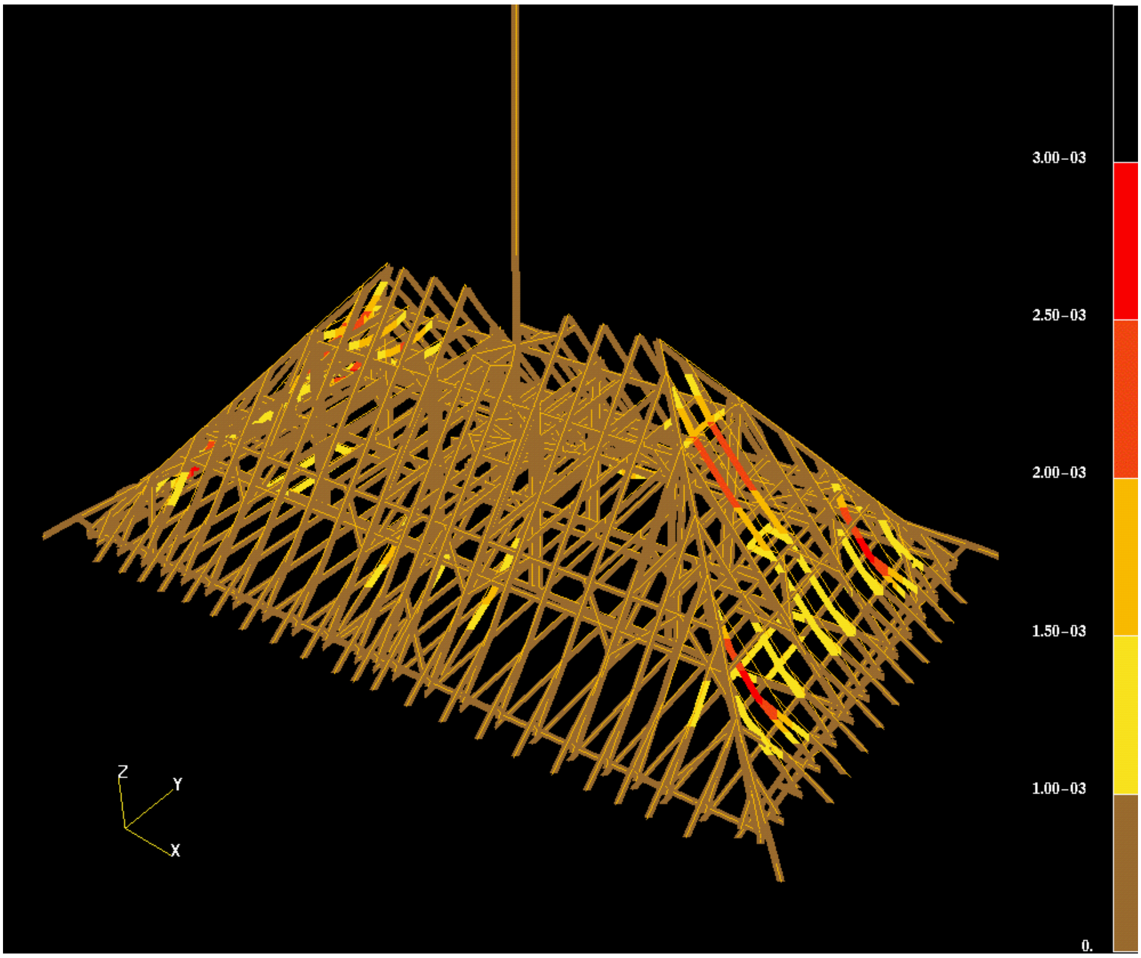

Figure 7.1 The deformed roof loaded with snow and dead load (boundary condition case A). Deformation scale from 0 to 3 mm.

The largest deformations, 3 mm, occur in the rafters located in the short sides close to the corners. When the structure was first loaded the rafters in the short sides as well as in the long sides became deformed, about 1.2 cm. Due to this deformation the outer rafters would pass the inner rafters when bending. This movement is impossible since the distance between the inner and outer rafters is too small. To reduce the deformation the inner and the outer rafters are connected in some places, which forces them to translate more similarly. There is enough space between the outer rafters and the lying timberframe to allow this new reduced deformation. Since the deformations are enlarged in the figures the rafters seem to pass the lying timberframe, which however is not the case.

The deformations in the middle rafters in the short sides are still comparatively large. This can be explained by the lack of contact with the remaining structure except for the contact at the first and second level of collar beams.

Deformations when the roof carries its own horizontal reaction forces

The global behaviour of the structure when the roof carries its own horizontal reaction forces is almost the same as when the wall carries these forces. However, the assymmetrical loads from the weights of the church clock appear to be more essential in this case. Except for this assymmetry the appearance of the structure is similar. As in the former boundary condition case most of the deformations are small, less than 1 mm, compare Figure 7.2. Notice that Figure 7.2 is rotated 180° in comparison with Figure 7.1.

The displacements are still large in the rafters at the lower level of collar beams. However, the maximum deformations are now situated in the short side, where the weights of the church clock are placed, and at a higher level. A detail of the most deformed parts is shown in Figure 7.3. These parts are also shown in Picture 7.1.

An explanation for this behaviour can be found when investigating the fictive elements that support the lowered beam, compare Figure 7.3. As described in Chapter 6.2 problems have occurred in some fictive elements. In the fictive elements the translations are connected but the rotations are independent. In some fictive elements this has resulted in large rotations. In these cases the fictive element has been replaced with a simple connection as described in Chapter 6.2.

Since the first order theory is used, the changed boundary conditions should not result in larger deformations in the rafters shown in Figures 7.2 and 7.3. When the connections between fictive and real elements in the concerned places are replaced with a rigid connection, that is, all translational as well as rotational degrees of freedom are dependent, the problem disappears. With the rigid connection the behaviour of the rafters remains the same even if the boundary conditions are changed.

Nevertheless, the rest of the results concern the case with the original connections.

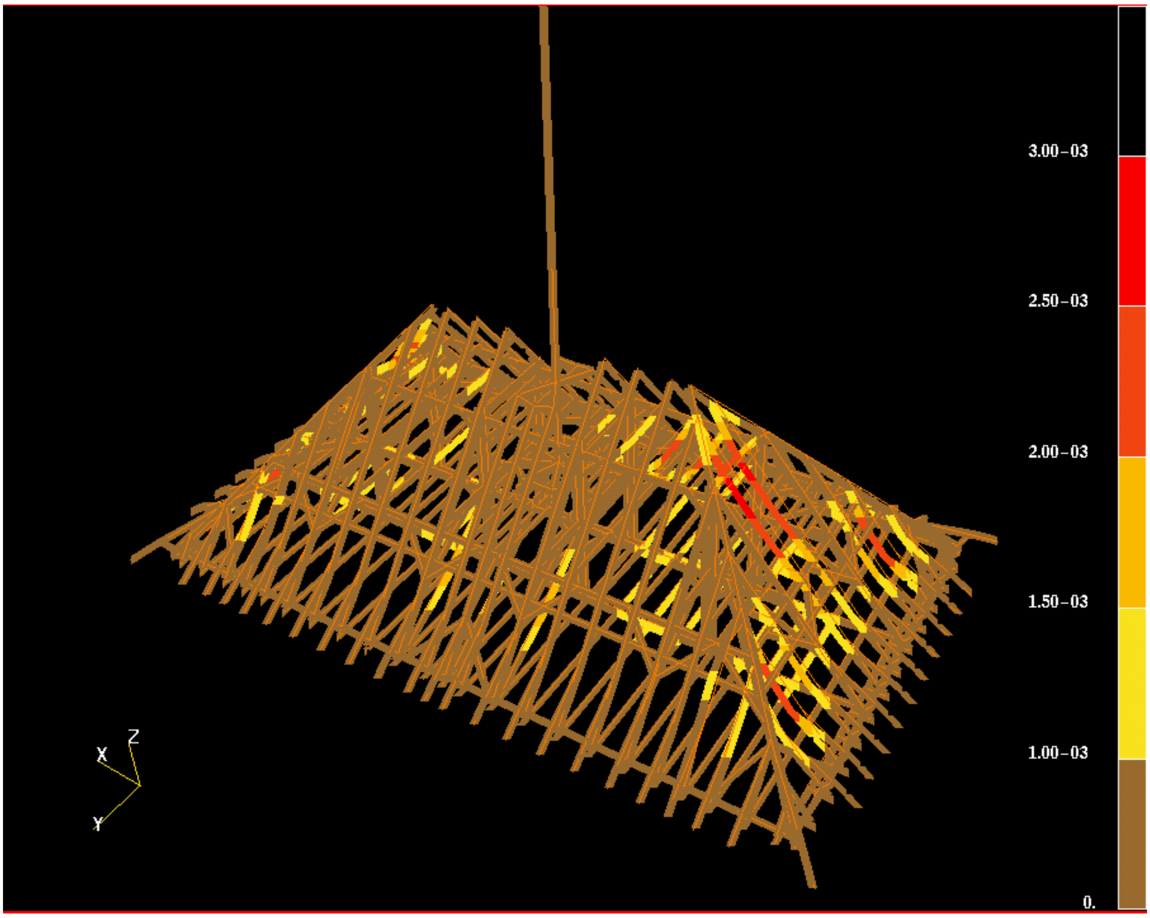

Figure 7.2 The deformed roof loaded with snow and dead load (boundary condition case B). Deformation scale from 0 to 3 mm.

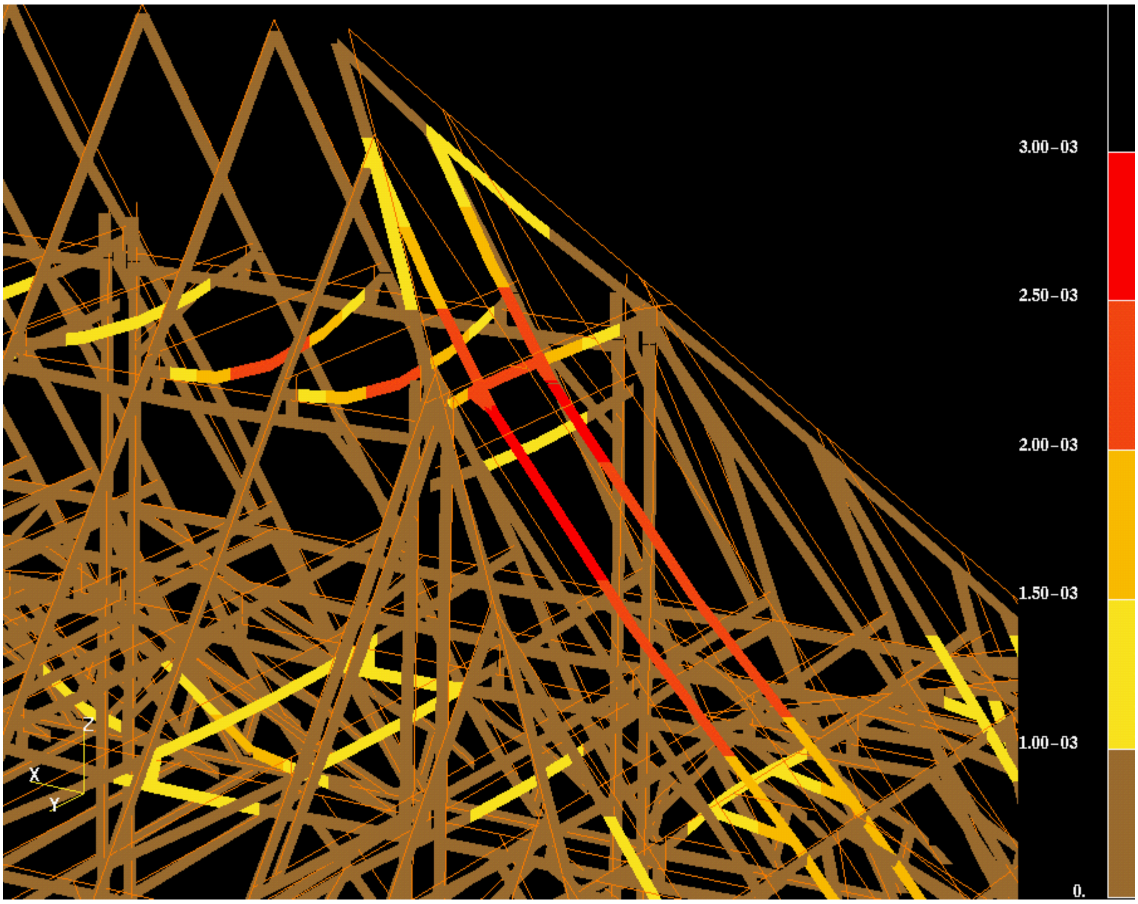

Figure 7.3 *Enlargement of the most deformed parts. Notice how the beam supporting the rafters is lowered. Deformation scale from 0 to 3 mm.*

Picture 7.1 *The beam supporting the rafter, seen from below.*

7.1.2 Normal forces

In this Master's thesis investigation is made concerning the case when the church wall is carrying the horizontal reaction forces, boundary condition case A, if nothing else is mentioned. However, the differences between boundary condition case A and boundary condition case B, compare Chapter 6.3, are local and mostly concern the tie beams, which in reality are tensioned.

The overall behaviour, which can be regarded as symmetric except for the part of the structure where the weights of the church clock are placed, is shown in Figure 7.5. To get more detailed information about the existing load paths it is necessary to show separated parts of the structure. Normal forces occurring in these parts will be explicitly shown and discussed, but first the geometry and the support conditions are further investigated.

- *The outer rafters:*

  The transverse tie beams support all the outer rafters. Some of the outer rafters are at a part of their length parallel and in close contact with the inner rafters in the lying timberframe. Connections between the outer rafters and the collar beams at all levels exist. This, together with the halving joint at the top of the roof, will support the outer rafter.

- *The lying timber frame:*

  The inner rafters in the lying timber frame are supported by the plate, by the braces connecting the inner rafter with the collar beams, see Figure 5.1, and by the collar beams at the first level of collar beams.

- *The queen posts:*

  The queen posts are connected to the longitudinal tie beams with flat iron bars. Connections to the longitudinal collar beams, at both levels, exist through the braces between the queen posts and the longitudinal collar beams, see Figure 7.4, as well as through the direct connections to the longitudinal collar beams. The transverse collar beams and the crossbar, see Figure 5.1, connects the queen posts in the transverse direction.

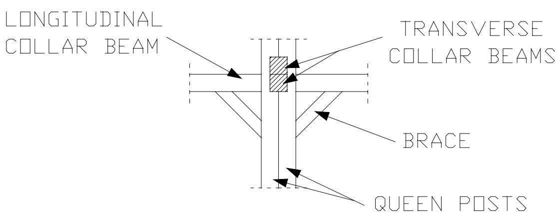

Figure 7.4 Braces connected to the queen posts.

- *The transverse tie beams:*

  The transverse tie beams are supported by the sill beam and the brick wall, compare Figures 5.1 and 6.9. Connections to the outer rafters, the plate and the buttress beam occur.

- *The hipping beams:*

  The hipping beams are supported by the outer rafters through the nogging pieces.

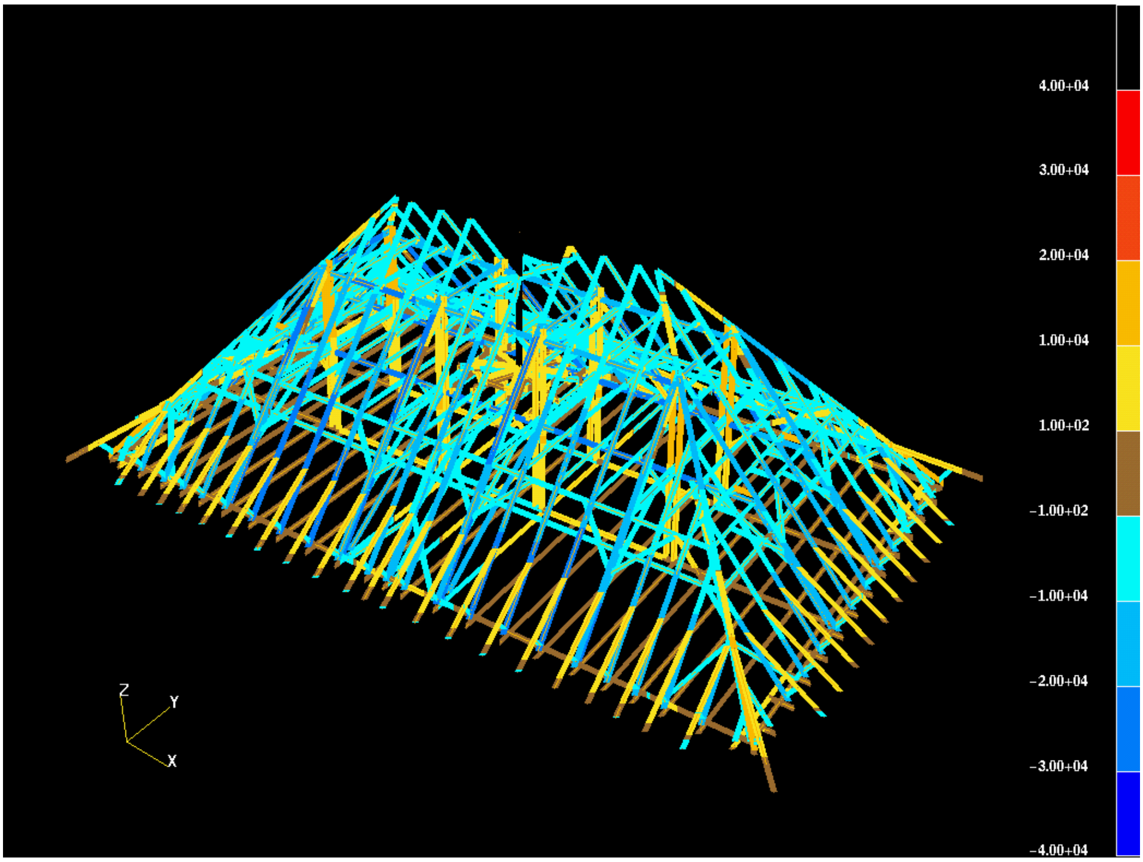

Figure 7.5 Normal forces due to dead load and snow load. Force scale from –40 kN to 40 kN.

A primary load path through the outer rafters is located, compare Figure 7.6. The rafters are generally charged with compressive normal forces of about 10 to 30 kN, naturally caused by snow load and dead load. At the side of the clock weights a slightly larger compressive force occurs, compare Figure 7.6. The yellow coloured parts in Figure 7.6 indicate tension in the lower part of the hip rafters. The first five connecting rafters carry the hip rafter, which explains the tension. Consequently the corners of the church wall are not charged.

An increase in the compressive forces in the outer rafters is also caused since the queen posts are partly suspended from them. This results in a load less than 10 kN, which can be carried by the tenon joint at the top of the queen posts [3].

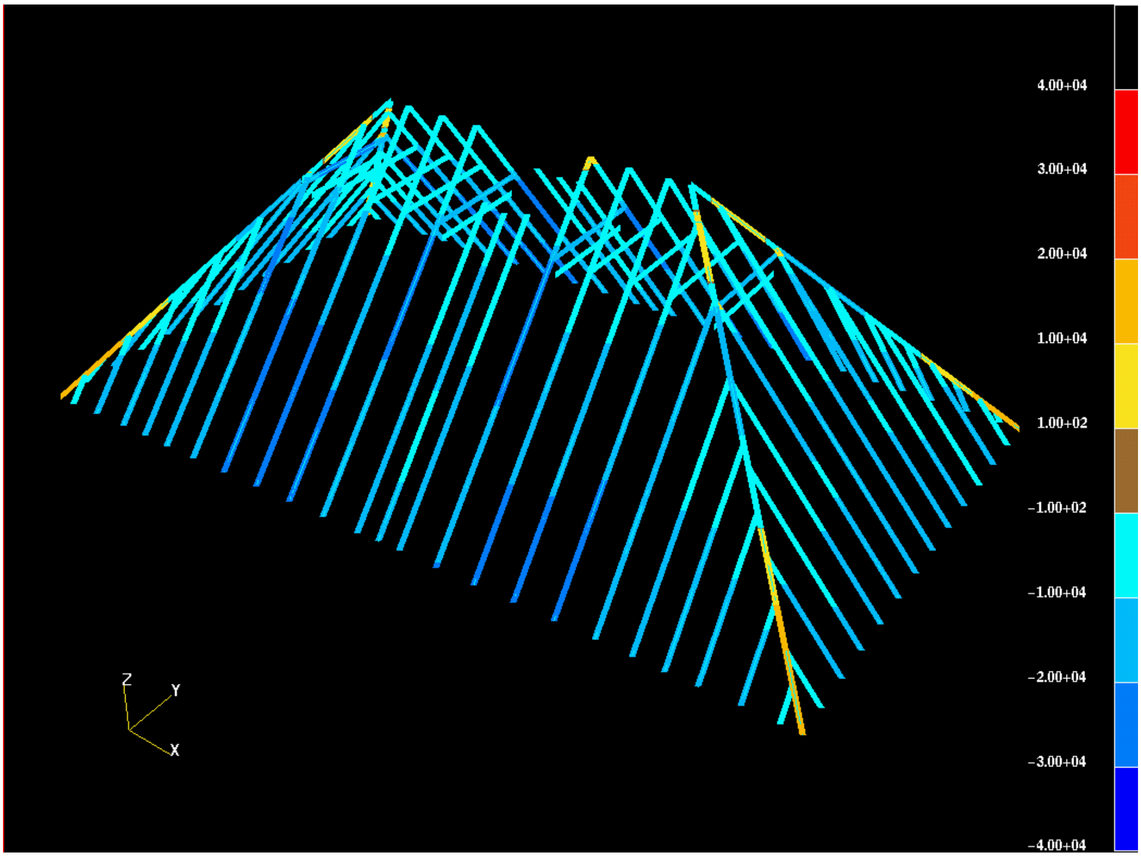

Figure 7.6 The primary load path, represented by the outer rafters. Force scale from –40 kN to 40 kN.

The lying timberframe and the transverse collar beams, at the second level of collar beams, connected to it are compressed but not as much as the outer rafters. The normal forces occurring in the lying timberframe and the collar beams are in compression and between 0 and 10 kN. This structure provides a secondary load path, compare Figure 7.7.

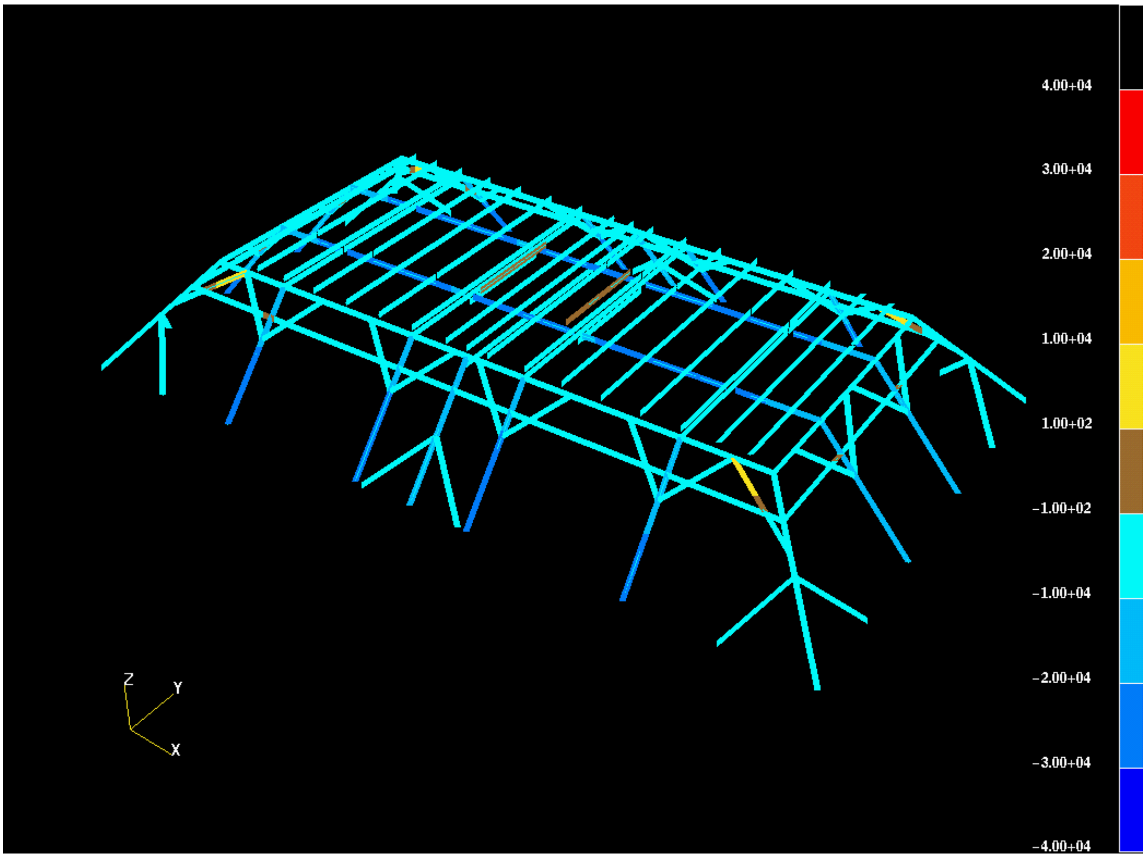

Figure 7.7 The secondary load path, represented by the lying timberframe and the collar beams at the first level of collar beams. Force scale from –40 kN to 40 kN.

The outer rafters act together with the lying timberframe and the transverse collar beams in a way illustrated in Figure 7.8. The main load path will be through the stiffest structure, which is the outer rafters.

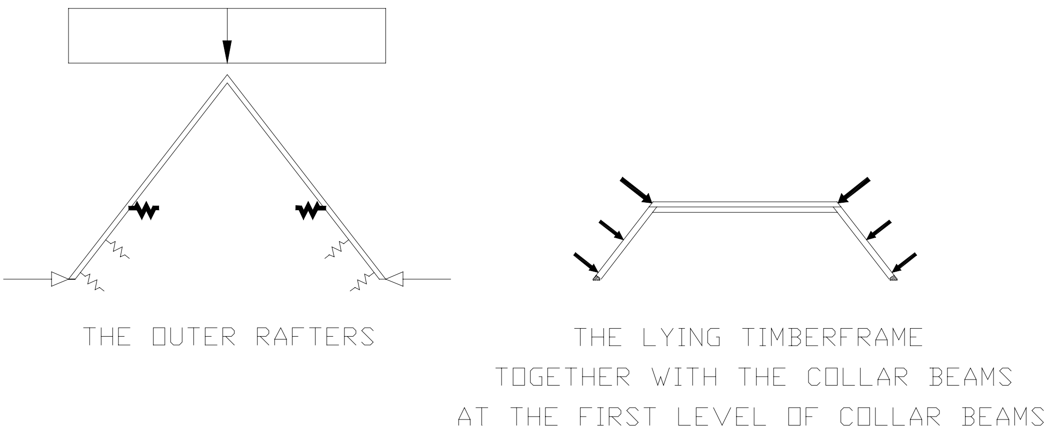

Figure 7.8 Illustration of the co-operation between the primary and the secondary load paths.

The queen posts are tensioned, see Figure 7.9. This is in accordance with our observations when we first inspected the roof structure. A distance was observed between the bottom ends of the queen posts and the longitudinal tie beams. Since the flat iron bars connecting the queen posts and the tie beams were not embossed, it was obvious that they were not compressed.

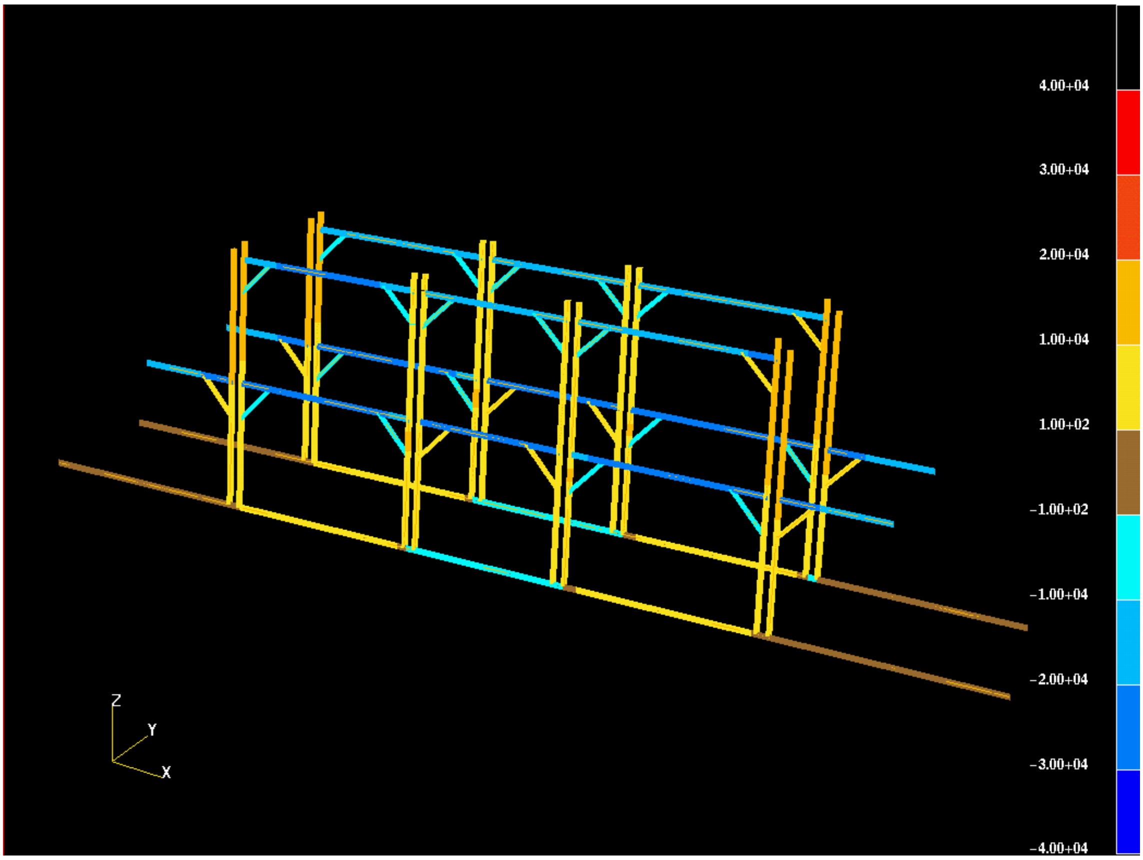

Figure 7.9 The queen posts and the longitudinal tie beams. Force scale from –40 kN to 40 kN.

Since the horizontal reaction forces are supported by the church wall, the transverse tie beams are almost uncharged, see Figure 7.5. The collar beams at higher levels are slightly compressed since the rafters are bending due to the snow load. The longitudinal tie beams connecting the queen posts are both compressed and tensioned, see Figure 7.9. The explanation for this behaviour is that the larger loads, collected from a larger area at the lower parts of the roof, force the queen posts to move as described in Figure 7.10.

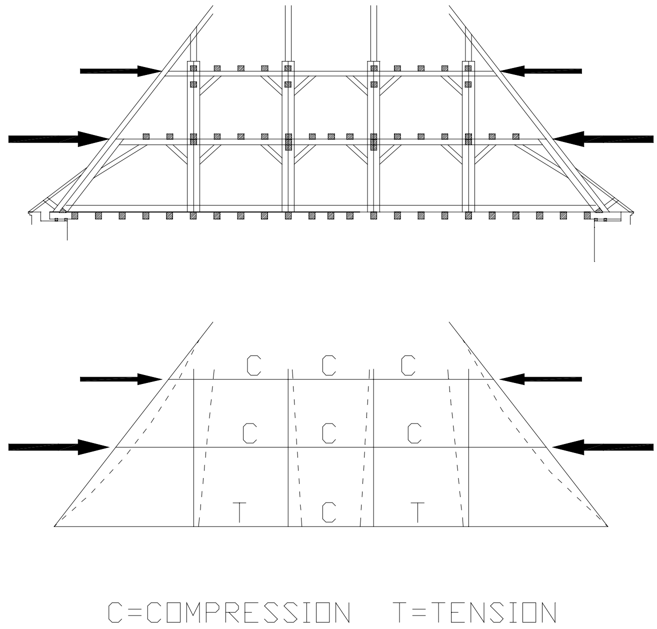

Figure 7.10 The deformation behaviour of the rafters in the short sides results in the variations of the section forces in the longitudinal tie beams.

When the deformation behaviour is strongly enlarged the same result as described in Figure 7.10 can be seen in Figure 7.11.

The hipping beams which provide the roof its hipped shape are insignificantly loaded, compare Figure 7.5.

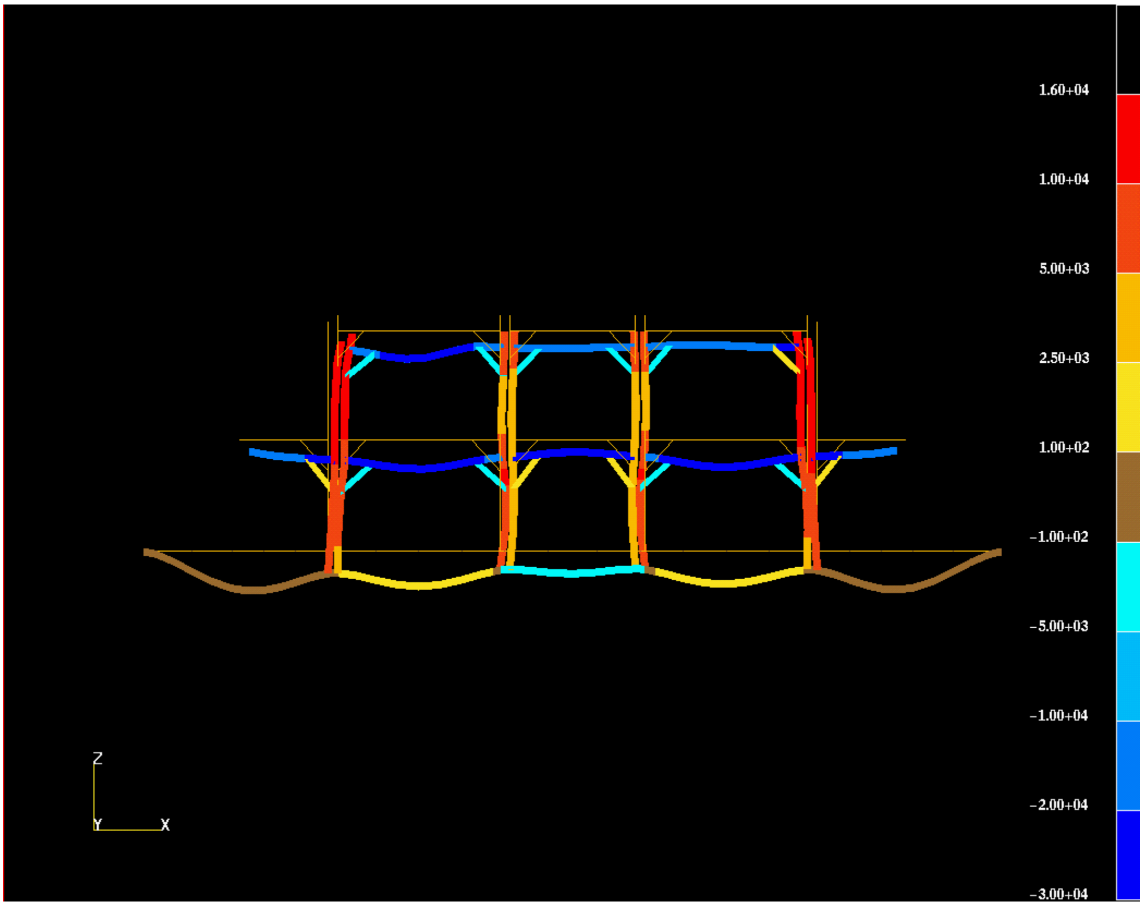

Figure 7.11 Enlarged deformations of the queen posts and the longitudinal tie beams. Force scale from –40 kN to 40 kN.

7.1.3 Normal stresses

The dimensioning strength of wood of a medium quality is about 10.5 MPa [3] [16]. This, however, is the value of timber felled today. When the timber in the Reformed church was felled the value of the strength was probably higher, but since the church is more than 250 years old the value 10 MPa seems reasonable [18]. This value is also in accordance with test results from the restoration of 1987. The normal stresses of the roof are in general very small, even less than 1 MPa, see Figure 7.12. One explanation is that the timber structure is coarse. The extreme value of the stresses is about 3.3 MPa, and occurs in the rafters close to the corners, where the greatest deformations are also found, compare Figure 7.1.

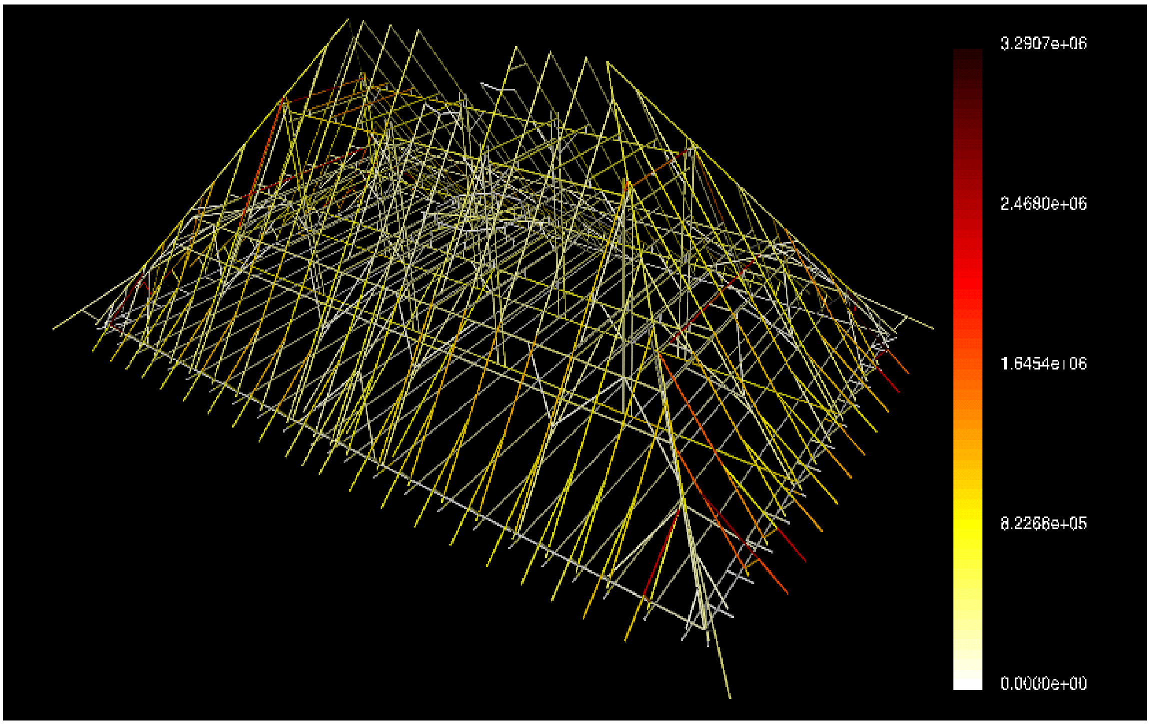

Figure 7.12 Normal stresses due to snow load and dead load. The long side closest to the viewer is the windward side. Stress scale from 0 to 3.3 MPa.

When comparing Figure 7.12 with Figure 7.5 it is found that, since the normal forces are small, the bending moments must contribute to a large part of the normal stresses in the rafters close to the corners. Four possible explanations for the relatively large bending moments are:

- Since calculations of bending moments are based on the equation $C*q*L^2$ (C=constant, compare $1/8*q*L^2$), a relatively large length of the specific rafters can cause large bending moments.

- Based on the same equation as above a relatively large distributed load, q, causes a large bending moment.

- Relatively weak support conditions cause large bending moments, se Figure 7.13.

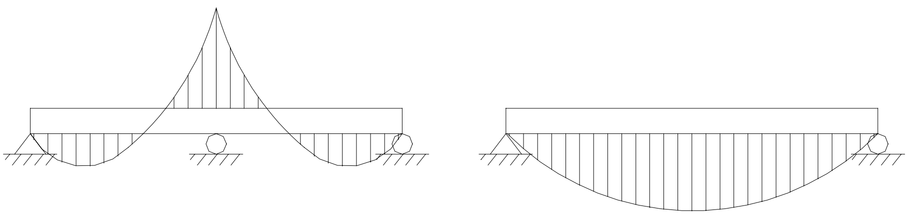

Figure 7.13 Bending moments.

- Constraints from other connected elements, see Figure 7.14.

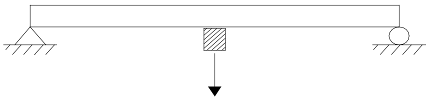

Figure 7.14 Constraints.

The highly stressed rafter closest to the corner, coloured red in Figure 7.12, is charged with a much higher distributed load than the rafters close to it. The reason is that the width of the loaded roof area supported by the highly stressed rafter is larger than the widths of the areas corresponding to the rafters close to it.

The following two rafters are loaded by the same distributed load, which is less than the one charging the rafter closest to the corner. However, the support conditions differ, see Figure 7.15.

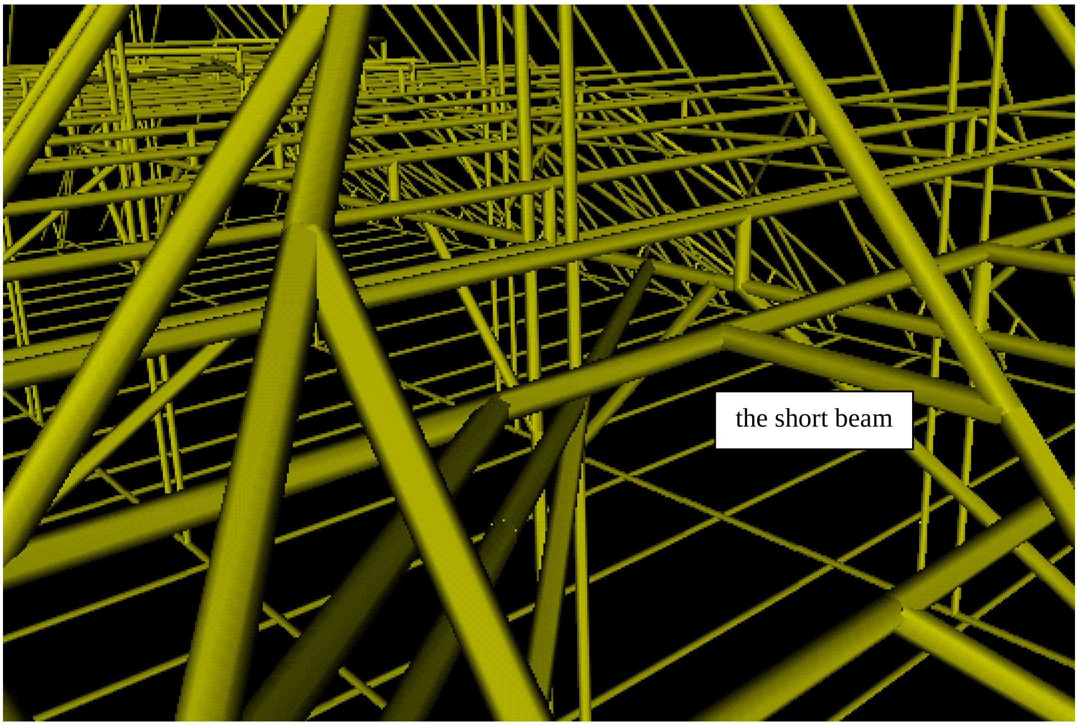

Figure 7.15 Support conditions for the rafters close to a corner.

The short beam connecting the third rafter from the corner with the transverse collar beam constitutes a stiffer support for this rafter than the second rafter's connection to the hip rafter in the corner does. Since the cross section of the short beam is of a relatively large dimension and since it is, from the third rafter, loaded in the stiff direction it can be regarded as a stiff support. That is why the stresses are smaller in the third rafter compared to the second rafter.

Comparatively high stresses, naturally caused by the weights of the church clock, can be seen in rafters furthest away from the viewer in Figure 7.12.

7.1.4 Shear stresses

Almost no shear stresses can be found in the structure, see Figure 7.16. Since most of the connections are frictionless joints and the cross sections are large, no great shear stresses can be expected. Current shear stresses can be disregarded.

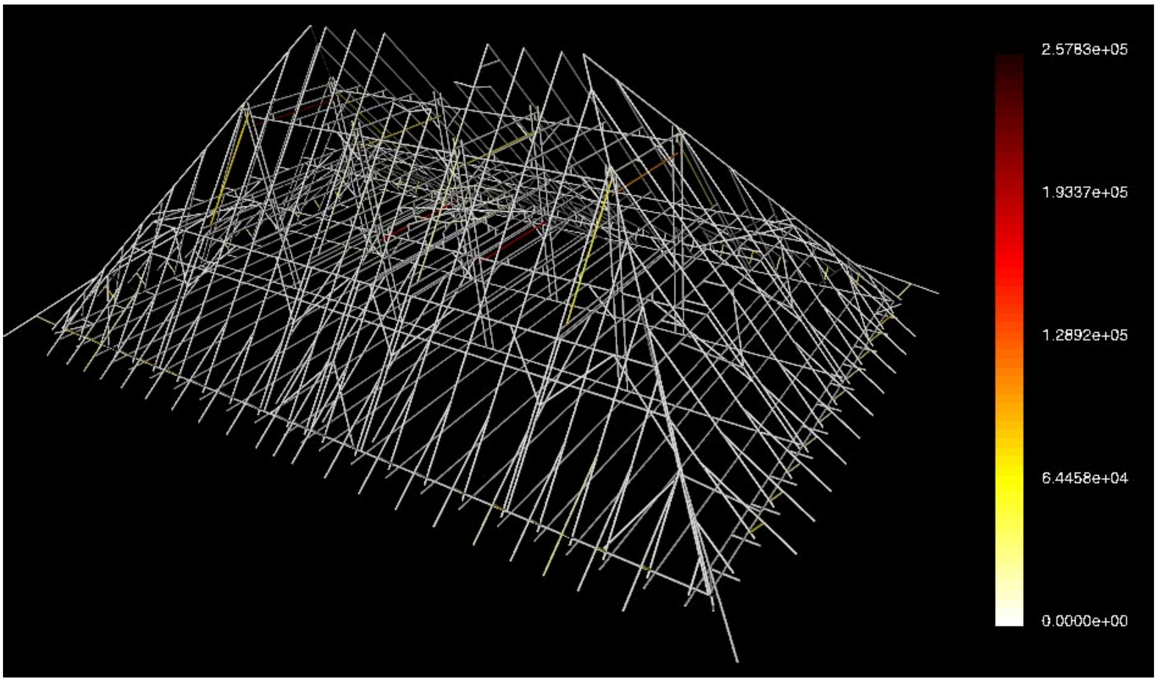

Figure 7.16 Shear stresses due to snow load and dead load. The side closest to the viewer is the windward side. Stress scale from 0 to 0.3 MPa.

7.2 Load case B

This load case is discussed in Chapter 6.6 and contains dead load and wind load.

7.2.1 Deformations

The global deformation behaviour of the structure is that the roof in the windward side is bending into the structure. In a similar way the roof in the leeward side is bending outwards, see Figure 7.17.

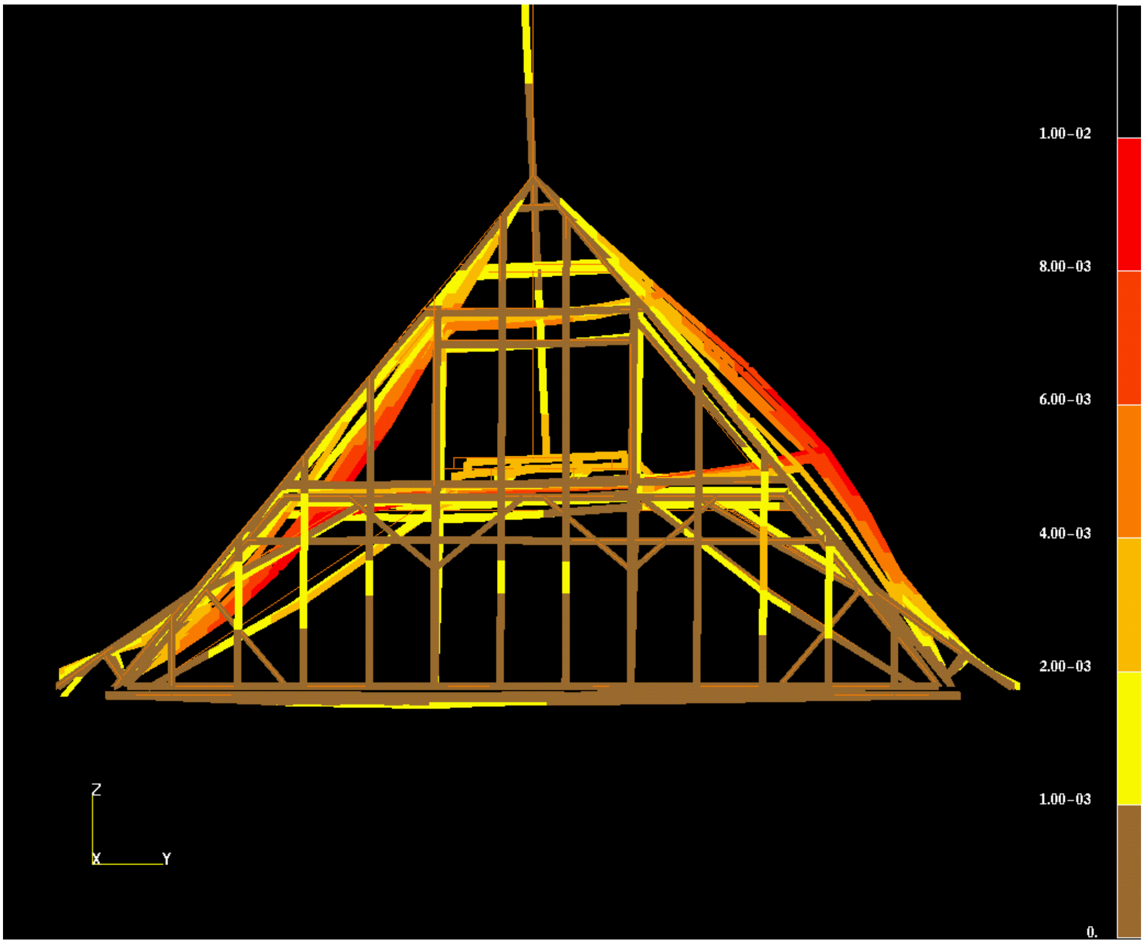

Figure 7.17 Deformations due to wind and dead load, the wind is blowing in positive y-direction. Deformation scale from 0 to 1 cm.

The rafters in the short sides are also slightly bending outwards due to the suction caused by the wind, see Figure 7.18.

Maximum deformation, about 1 cm, occurs in a rafter in the windward side where no connection with the inner rafter in the lying timberframe exists, compare Figure 7.18. Because of the enlarged deformations it looks as if the rafter is passing the lying timberframe when bending inwards. In reality there is enough space between the rafter and the lying timberframe to allow this movement, see Picture 7.2.

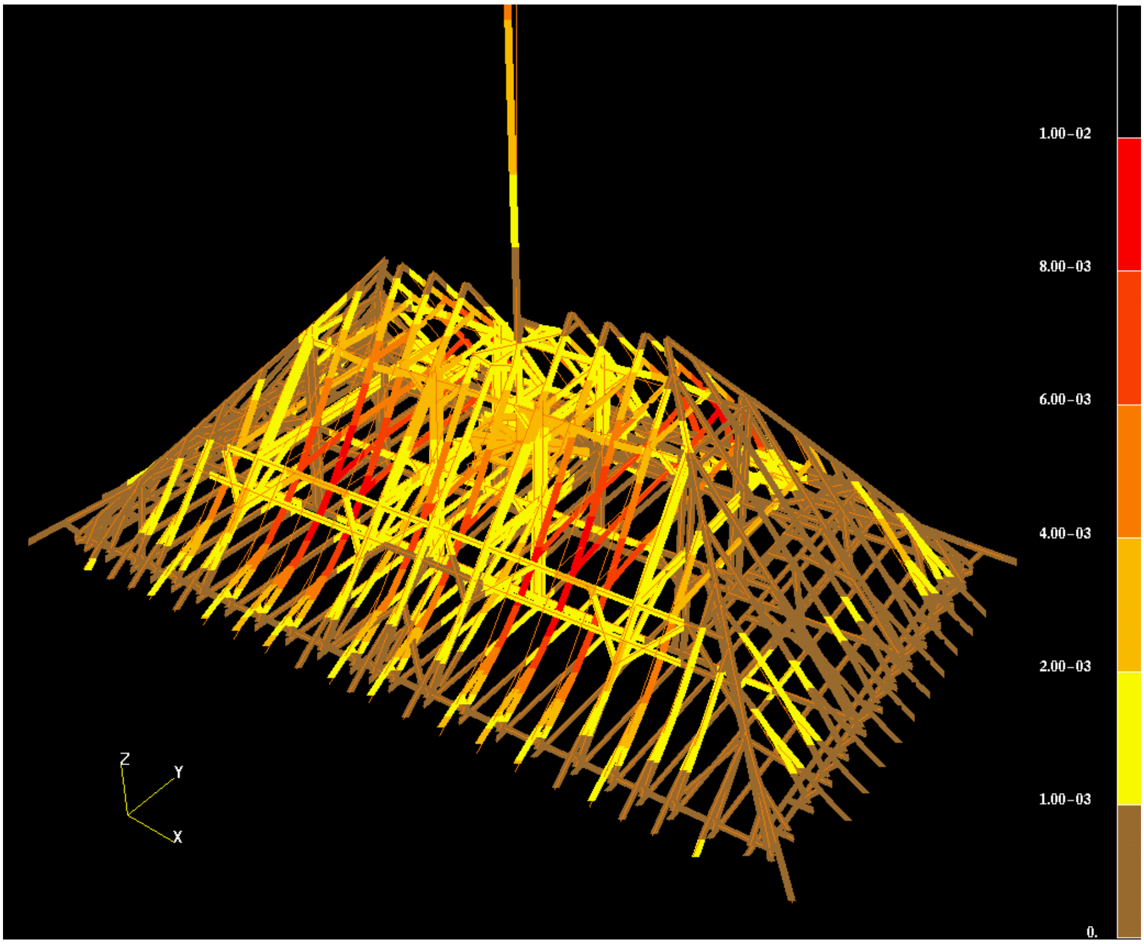

Figure 7.18 Maximum deformation due to wind and dead load. The wind is blowing in positive y-direction. Deformation scale from 0 to 1 cm.

Picture 7.2 The picture shows the space between the rafters and the lying timberframe.

In some parts of the structure the rafters in the lying timberframe are parallel to and in close contact with the outer rafters. This stiffens the structure at these places and reduces the deformations here. The maximum deformation occurs between two such stiff parts, see Figures 7.18 and 7.19, the latter when the structure is loaded with wind alone.

However, the outer rafters and the lying timberframe are not connected, and are therefore allowed to move away from each other. This is why these rafters and the lying timberframe at the leeward side are able to part. The same stiffening effect as described above can also be observed at this side of the roof.

At higher levels the collar beams, due to their connection with the rafters, are forced to follow the movement of the rafters, see Figures 7.17 and 7.18. However the deformations in the collar beams are even in this case reduced because of the stiffening effect caused by the rafters in the lying timber frame.

The spire is modelled as a rigid body, which explains its deformation. It is forced to follow the movement of the structure and will hence bend towards the wind. This means that the spire actually acts as a support to the rest of the structure. If no wind load charges the spire a more dangerous load case will occur.

Deformations when the structure is loaded with nothing but wind

On the way to finding which load paths are primary when the structure is charged with wind, a load case consisting of nothing but wind is studied, that is, no dead loads occur. It is interesting to find out which parts of the structure are closely connected to the wind load.

The deformations of the structure are almost the same as for the former described load case; compare Figure 7.18 and Figure 7.19. However, the deformations in the windward side are slightly less when the wind is not collaborating with the dead load. On the other hand the deformations in the leeward side and in the short sides will for the same reason be slightly increased compared with the former load case.

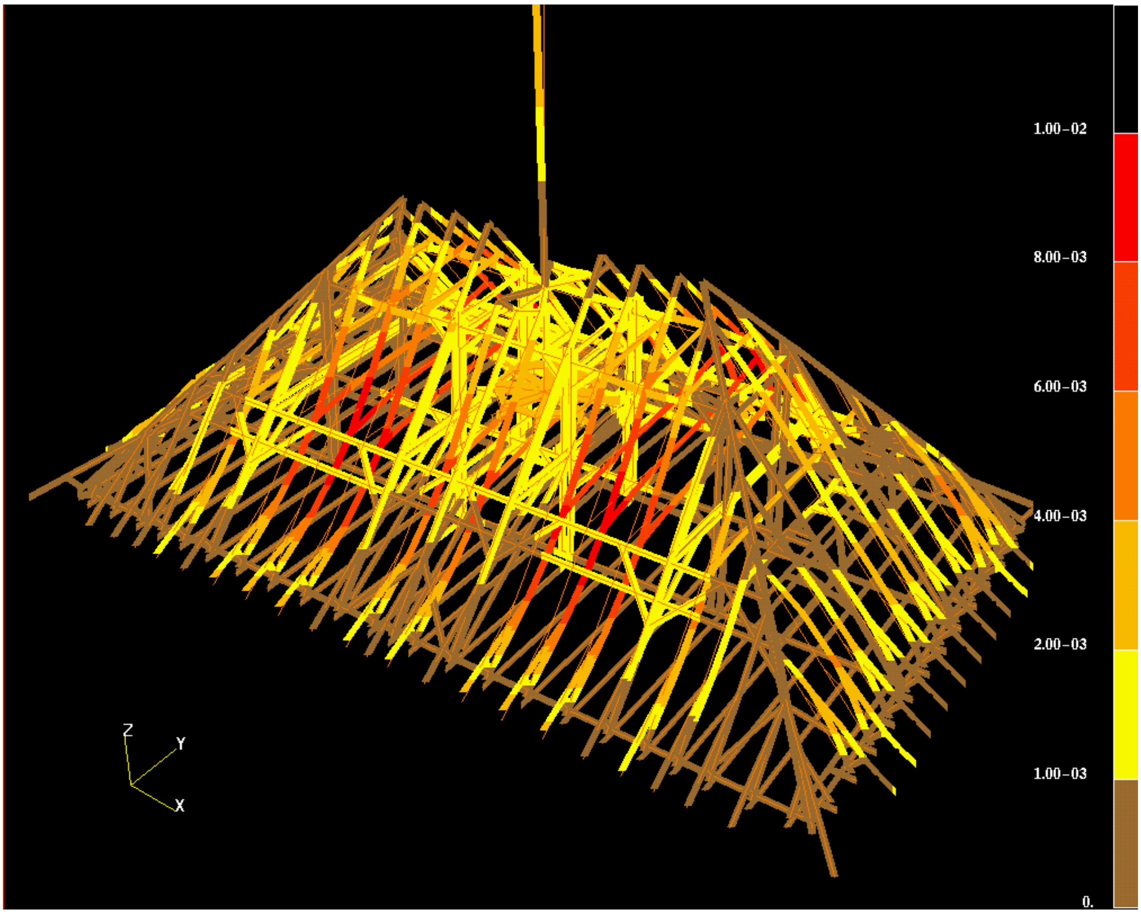

Figure 7.19 Deformations when the structure is loaded with nothing but wind. The wind is blowing in positive y-direction. Deformation scale from 0 to 1 cm.

The conclusion drawn is that it is mainly the wind which causes the deformations of the roof. The dead load is just strengthening or moderating the deformation behaviour.

7.2.2 Normal forces

It is interesting to study the qualitative normal force behaviour of the structure due to wind alone, see Figure 7.20. For reasons explained later, the cornice, in this figure, is unable to carry tensile forces. However, the overall behaviour shown in Figure 7.20 will not be discussed immediately, but is used later on in comparison to the behaviour occurring when the structure is loaded by both wind and dead load. It can be said, however, that the normal forces due to wind alone are relatively small.

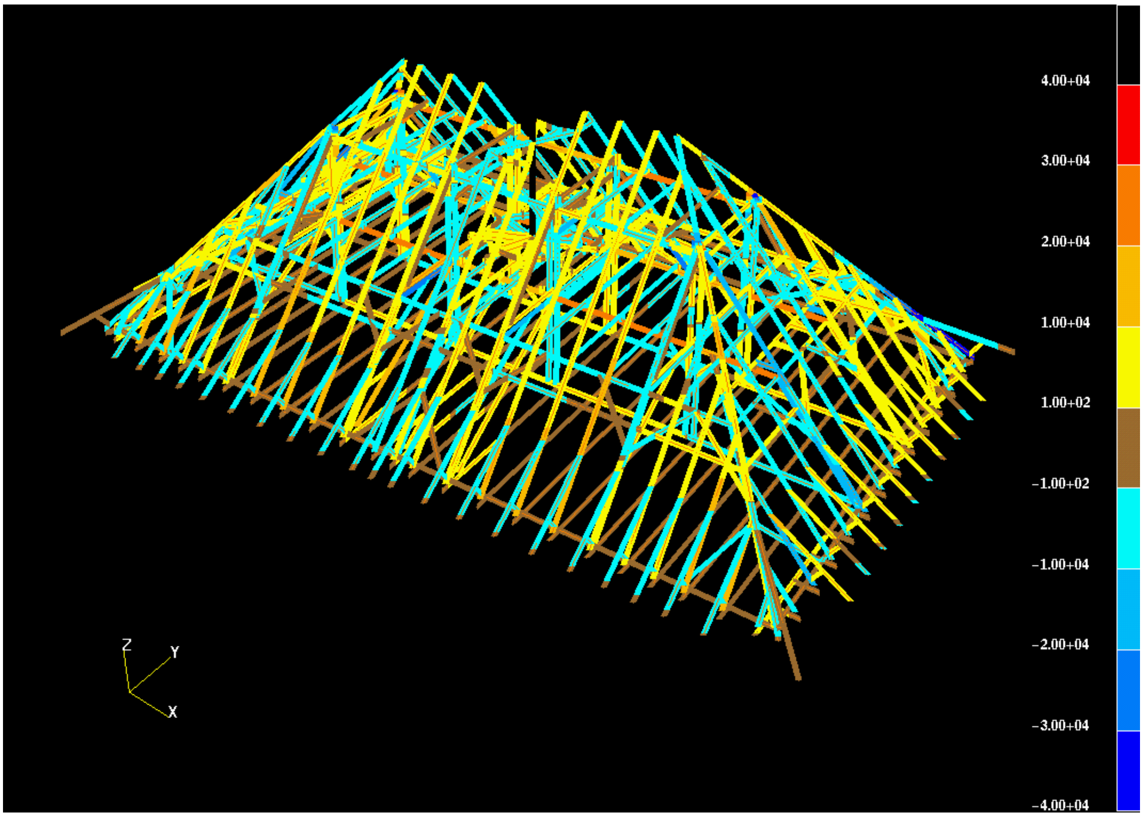

Figure 7.20 Normal forces when the wall is unable to carry tensile forces. The structure is loaded with nothing but wind, blowing in positive y-direction. Force scale from –40 kN to 40 kN.

This chapter is given a structure similar to the one used in Chapter 7.1.2. The overall normal force behaviour will be separated into the same parts and further discussed.

Almost every outer rafter is compressed, see Figure 7.21, but in some of the rafters close to the corner tension occurs. This means that the corresponding supports are also tensioned. Compared to the normal forces due to nothing but wind, see Figure 7.20, the tensile behaviour of the rafters is changed and more rafters are compressed. The wind provides a lifting effect on the roof structure. When the dead load is included this effect is levelled out.

The hip rafters are both tensioned and compressed, see Figure 7.21. The side rafters connected to a hip rafter are a comparatively stiff load path. That is why the nether part of the corner is hanging in the upper part. The same behaviour can be seen in Figure 7.20.

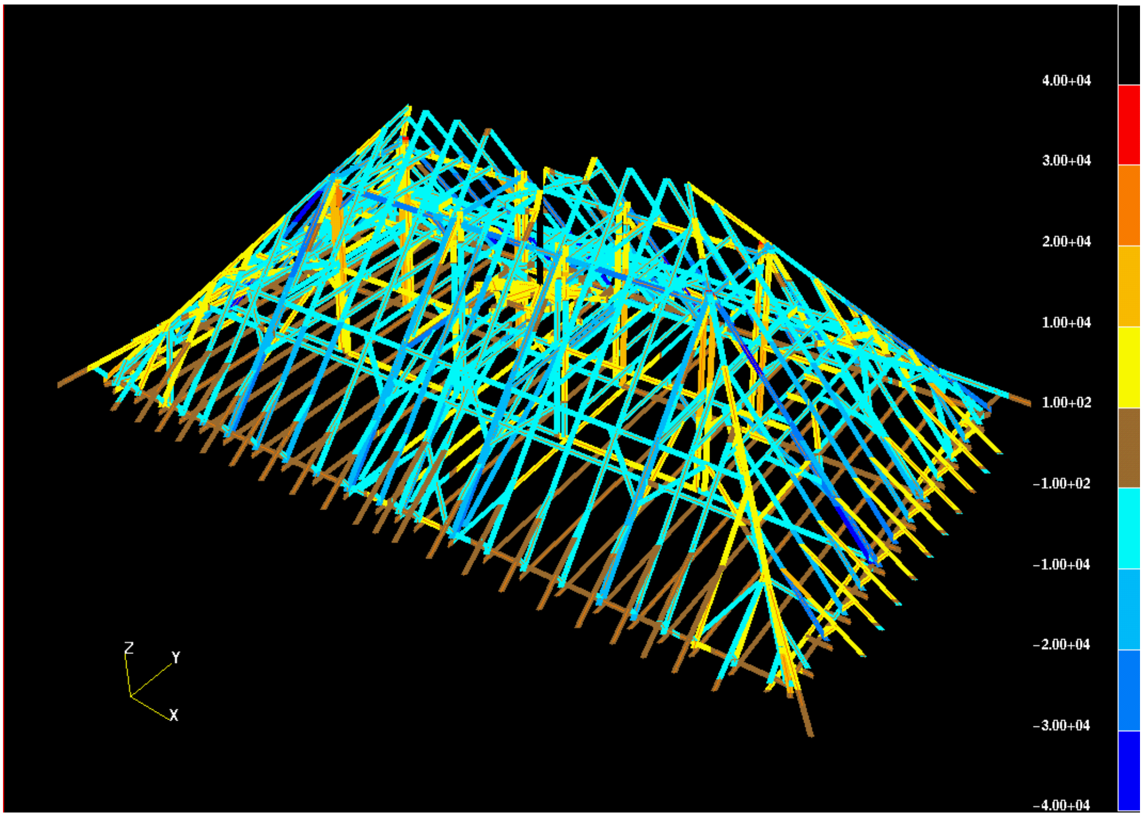

Figure 7.21 Normal forces due to wind and dead load, when the wall is unable to carry tensile forces. The wind is blowing in positive y-direction. Force scale from –40 kN to 40 kN.

A distinct distribution into one primary and one secondary load path cannot be made in this load case. The distribution is, however, still interesting to study, see Figure 7.22 and Figure 7.23.

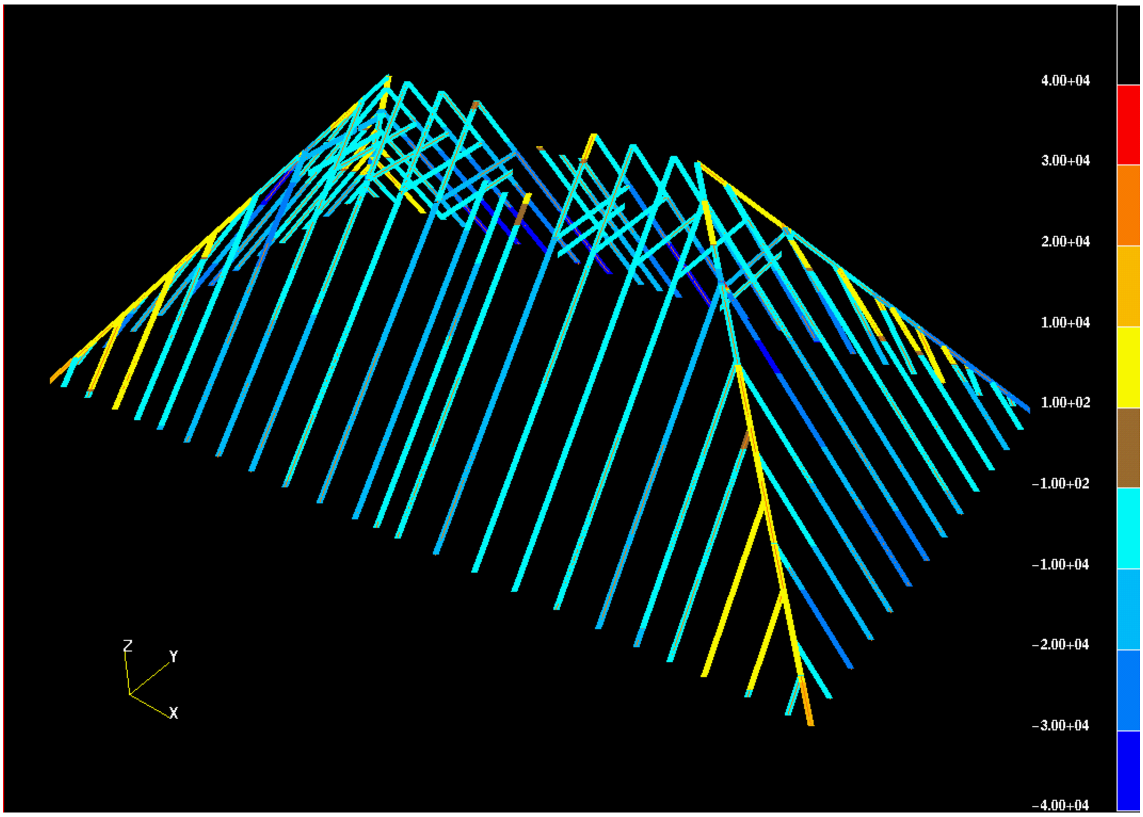

Figure 7.22 The primary load path, when the structure is loaded with wind and dead load, represented by the outer rafters. The wind is blowing in positive y-direction. Force scale from –40 kN to 40 kN.

The lying timber frame is generally compressed and so are the transverse collar beams at the first level of collar beams, compare Figure 7.23. The behaviour of the two longitudinal collar beams at the first level of collar beams differs. The stiffening effect of the rafters in the lying timberframe, earlier discussed in Chapter 7.2.1, makes it possible to see all the collar beams at the first level as one single beam supported by the rafters in the lying timberframe. Due to the wind the described beam bends as in Figure 7.24; this is why tension and compression occur in the way seen in the longitudinal collar beams in Figure 7.23. With a little effort, the same tendency can be observed when the structure is loaded with nothing but wind, compare Figure 7.20.

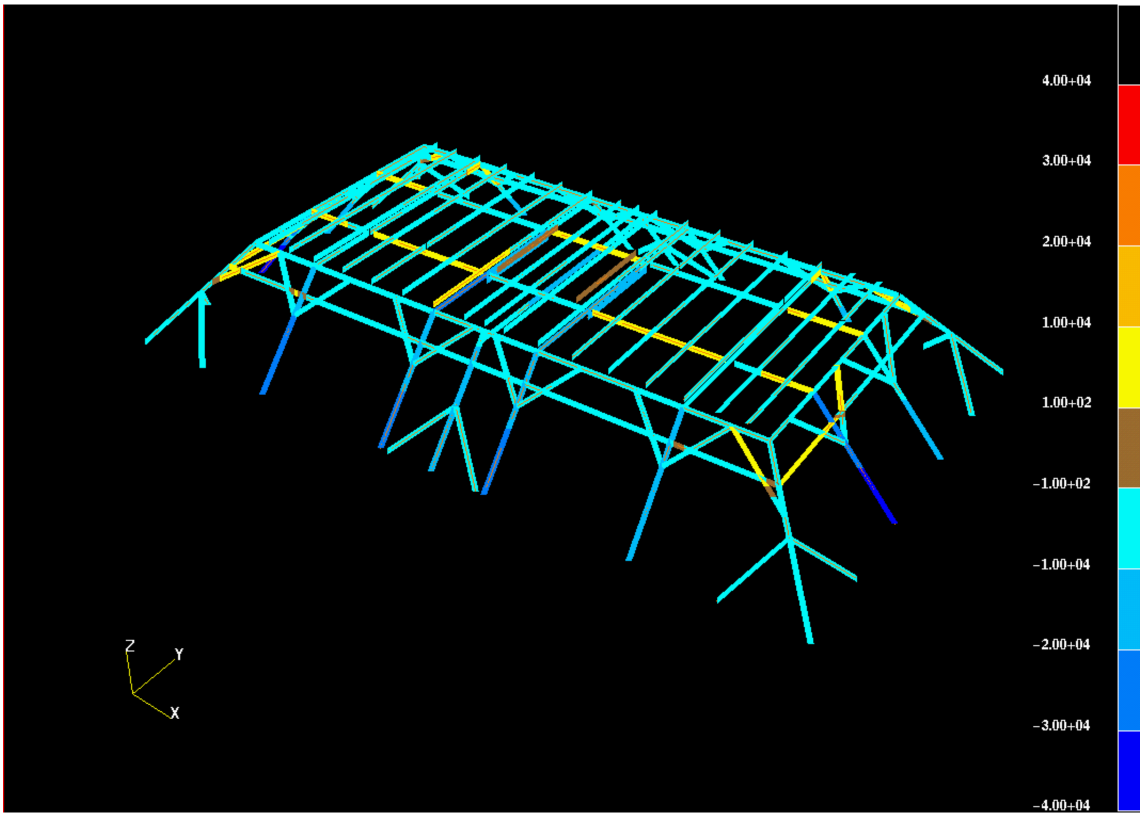

Figure 7.23 The secondary load path, when the structure is loaded with wind and dead load, represented by the lying timberframe and the collar beams at the first level of collar beams. The wind is blowing in positive y-direction. Force scale from–40 kN to 40 kN.

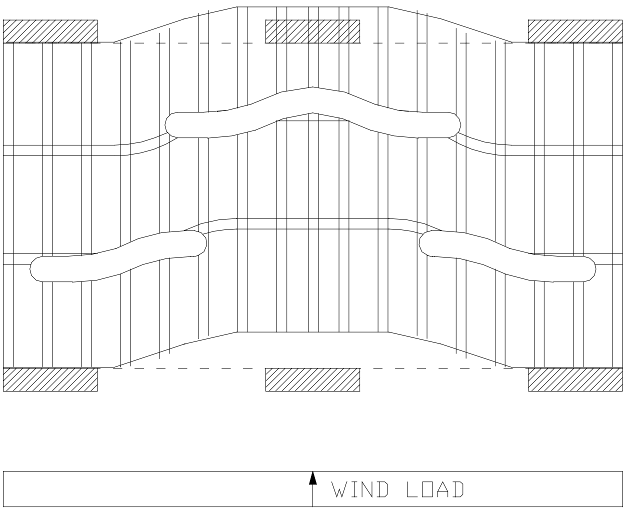

Figure 7.24 A description of the collar beams at the first level of collar beams. The striped rectangles represent the stiffening effect of the lying timberframe, and the blank areas represent the tensioned parts, yellow in Figure 7.23.

The transverse tie beams are almost unloaded, see Figure 7.21, and the transverse collar beams at both levels are only slightly compressed, see Figure 7.22 and Figure 7.23. The compression is mainly caused by the dead load, but also by the wind, since the pressure is larger at the windward side than the suction at the leeward side. The behaviour is similar to the case when the structure is loaded with nothing but wind.

Except for the two columns in the windward side, closest to the symmetry line, all of the queen posts in both load cases are tensioned, see Figure 7.20 and Figure 7.25. The tensile behaviour can be related to the same reasons as discussed earlier in Chapter 7.1.2. One possible explanation of the compressed behaviour in the two queen posts is that the extreme wind forces the roof to bend in so much that it will compress these columns, see Figure 7.26. This explanation is confirmed by the similarity between the two load cases: wind load, with or without dead load, which shows that the wind causes this behaviour.

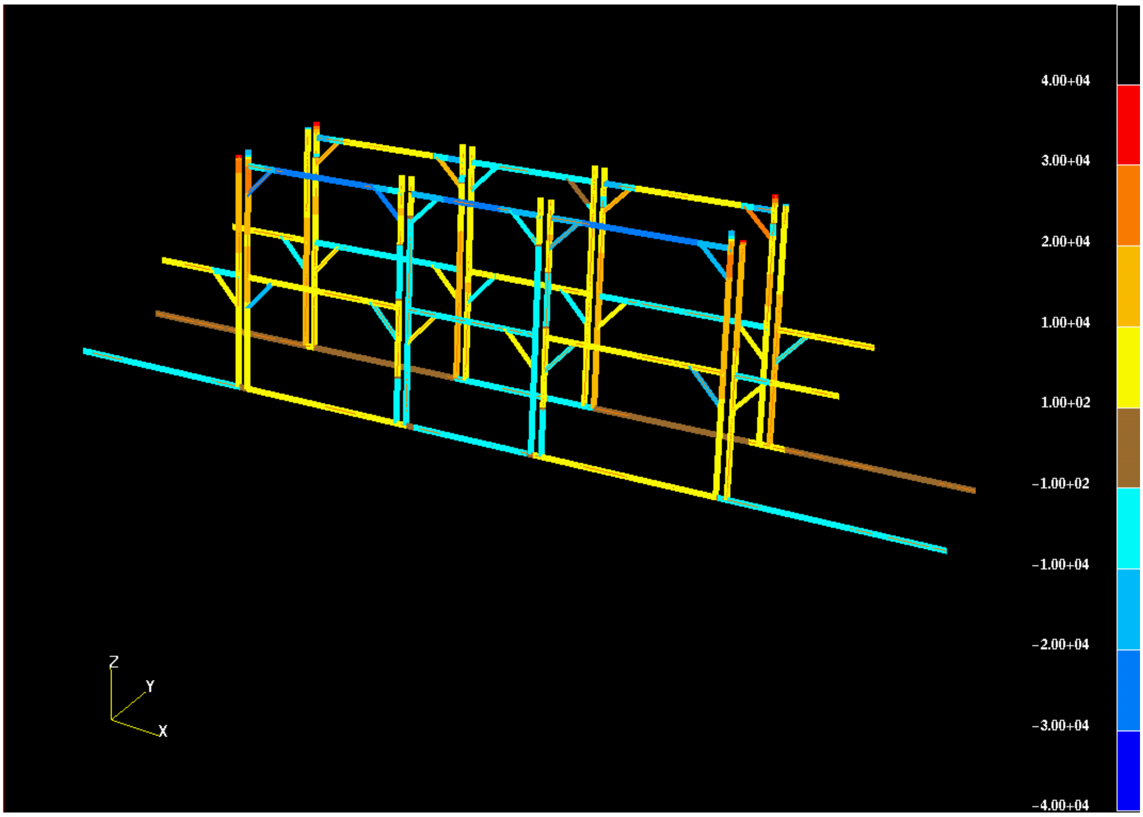

Figure 7.25 The queen posts when the structure is loaded with wind and dead load. The wind is blowing in positive y-direction. Force scale from –40 kN to 40 kN.

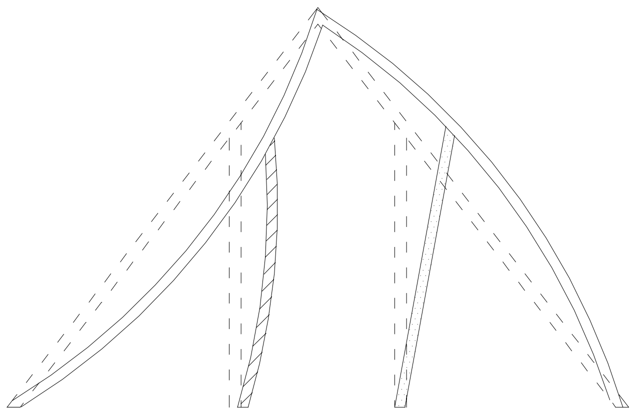

Figure 7.26 The roof compresses the columns closest to the windward side (striped) when bending. The columns closest to the leeward side (dotted) are tensioned.

The longitudinal tie beams and the longitudinal collar beams at both levels, see Figure 7.25, act as in the load case containing snow, see Figure 7.10.

The normal forces were studied for the boundary condition case when the wall is carrying the horizontal reaction forces, see Figure 7.27. The hipping beams then became tensioned at the windward side.

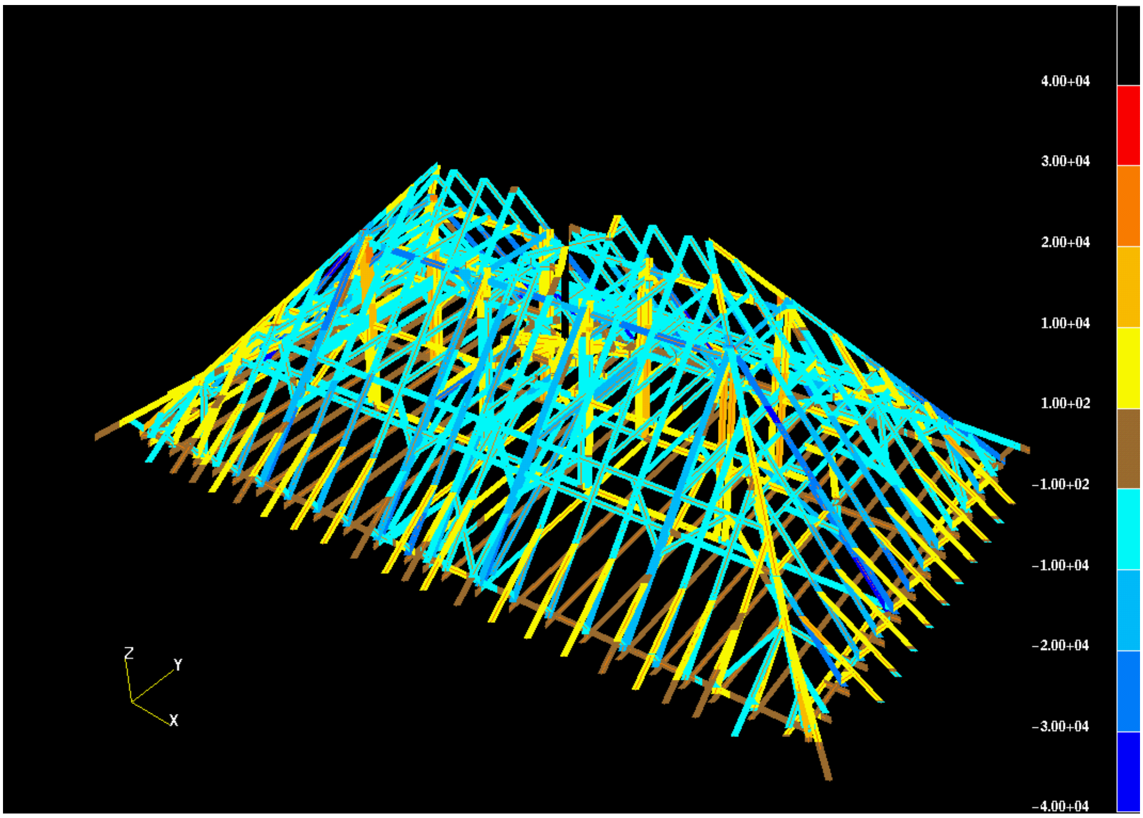

Figure 7.27 Normal forces due to wind and dead load, when the wall is able to carry tensile forces. The wind is blowing in positive y-direction. Force scale from –40 kN to 40 kN.

This is today an impossible behaviour, since there is no longer any contact between the hipping beams and the cornice. The hipping beams are today resting on the outer rafters through the nogging piece. However, before releasing the hipping beams in relation to the restoration of 1989 the cornice was charged with loads as in Figure 7.27.

The only difference observed between the results is that the hipping beams on the windward side in the former case become almost unloaded. This is the result of releasing them.

7.2.3 Normal stresses

When comparing the normal stresses in this load case, where the structure is loaded with wind, with the ones in the former load case, dead load and snow load, it can be observed that the behaviour is almost the same, compare Figure 7.28 with Figure 7.12. One exception is that normal stresses occur to a higher degree in the hipping beams in this load case.

The normal stresses are higher in the hipping beams in the long side on the leeward side than on the windward side, see Figure 7.28. Because of the fact that the boundary conditions on the windward side are released, as described in Chapter 7.2.2, this stress behaviour is conceivable. The wind forces the structure to bend, see Figure 7.17, and this movement creates the normal stresses.

In some outer rafters in the leeward side, see marker A in Figure 7.28, the stresses are a little bit higher than in the opposite part of the structure. One explanation is that the weights corresponding to the church clock are located at this side.

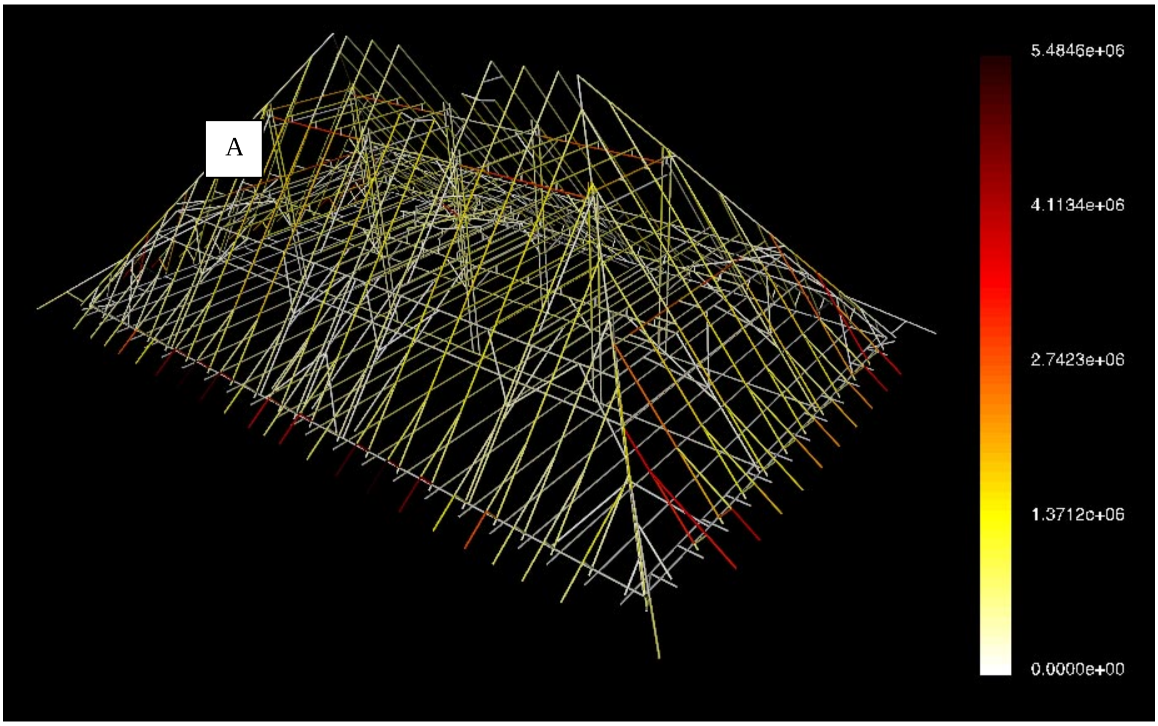

Figure 7.28 Normal stresses due to wind load and dead load. The long side closest to the viewer represents the leeward side. Stress scale from 0 to 5.5 MPa.

The stress behaviour is almost the same regardless of whether the structure is loaded with wind load and dead load or only with wind load, see Figure 7.29. The stress values are smaller in the latter case, however.

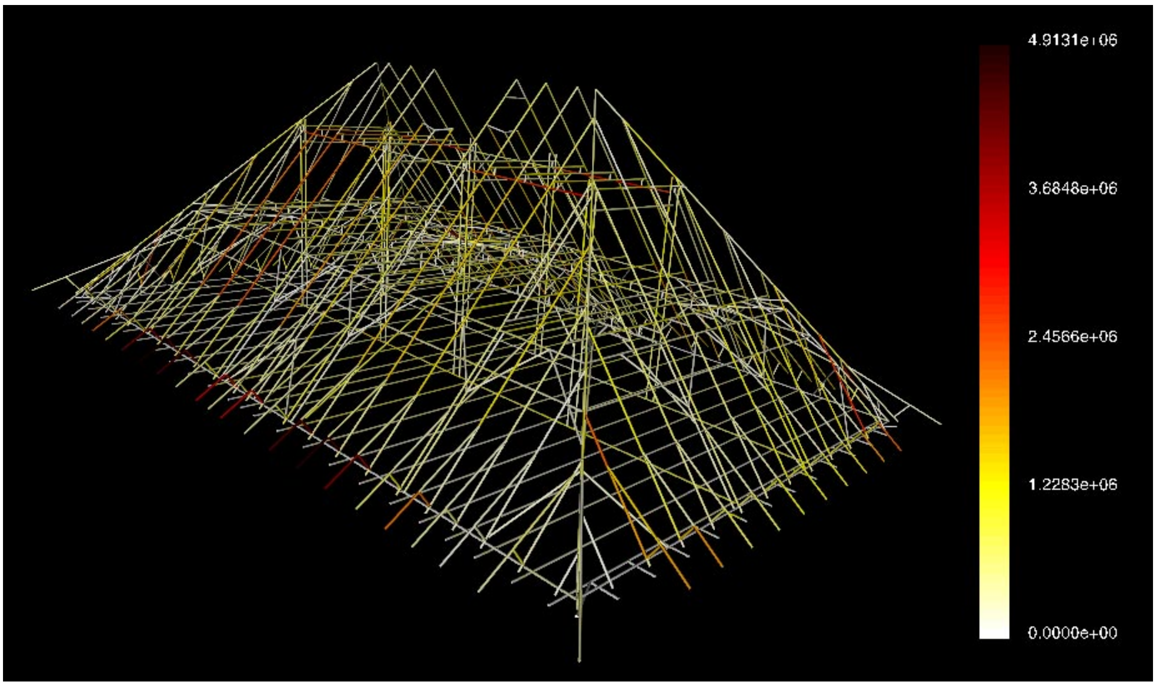

Figure 7.29 Normal stresses due to nothing but wind load. The long side closest to the viewer represents the leeward side. Stress scale from 0 to 4.9 MPa.

The same rafters close to the corner as discussed in Chapter 7.1.3 are also in this load case, and for the same reasons, comparatively highly stressed.

7.2.4 Shear stresses

The shear stresses are also small in this load case, see Figure 7.30. The greatest stresses can be observed in the star structure on which the spire is resting. The whole part of the spire is, however, of no particular interest in this analysis, see Chapter 6.2.

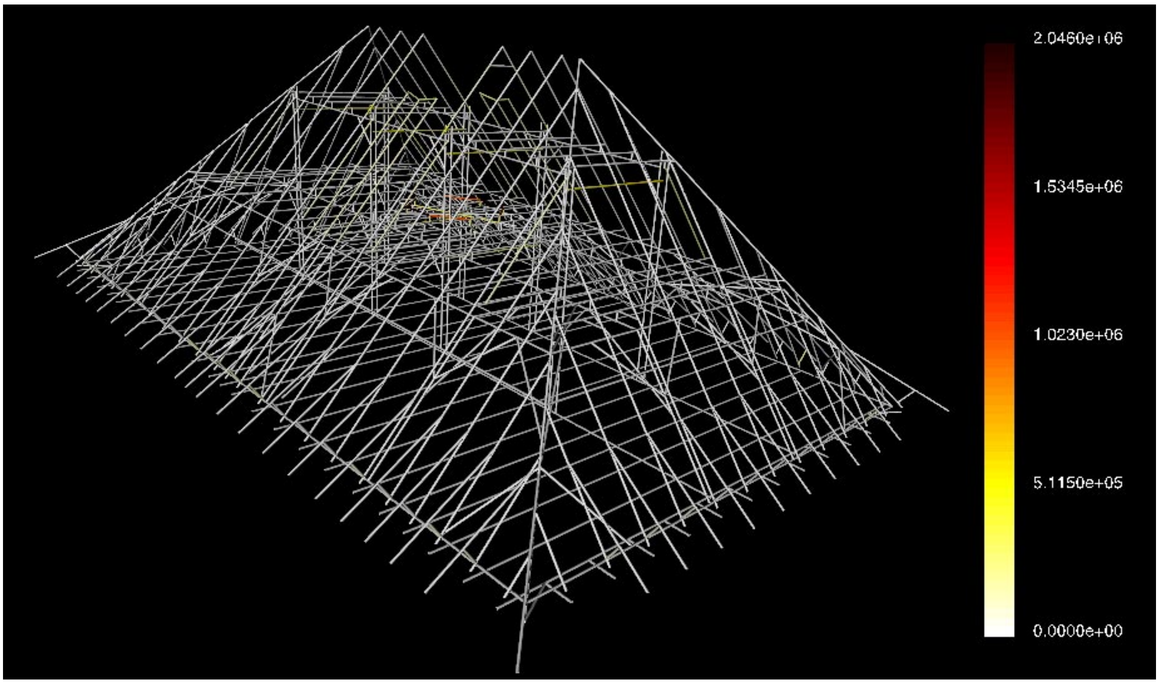

Figure 7.30 Shear stresses due to wind load and dead load. The long side closest to the viewer is the windward side. Stress scale from 0 to 2 MPa.

The shear stresses are located at the same place and their values are almost the same regardless of whether the dead load is included or not. The dead load is actually causing shear stresses, since the rafters bend. In comparison with the stresses caused by the wind these stresses are small and cannot be observed.

7.3 Special study of the corners

Of special interest is the behaviour of the corners, since it might give the answers to the following questions:

- How much of the load goes through the hip rafter in the corner and how much goes through the braces?

- Is there any load carrying need of the braces?

The behaviour of the deformations, normal forces and stresses are the same for the four corners, therefore it is enough to present the quantities for one of them.

To help the reader to see the behaviour in the corners, the geometry and terminology of the corner are shown in Figure 7.31 and in Picture 7.3.

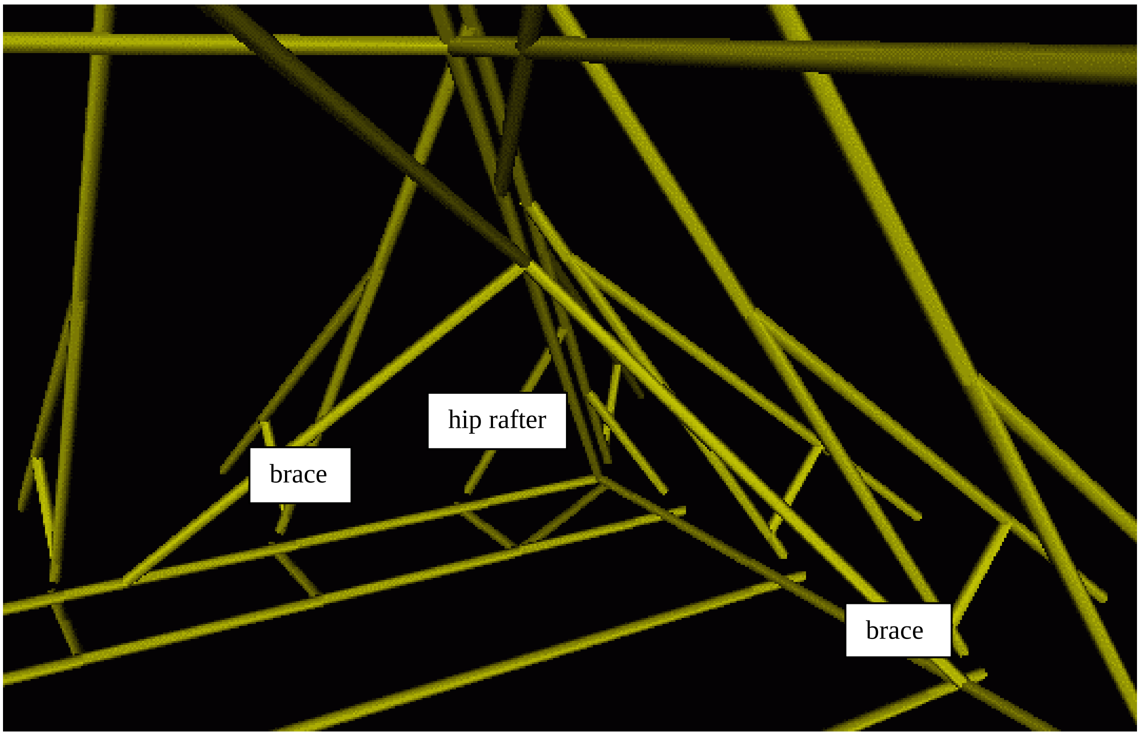

Figure 7.31 Geometry and terminology in the corner.

Picture 7.3 Geometry in the corner.

7.3.1 Deformations

To be able to see the load division between the hip rafter and the braces, the deformation ranges are further enlarged.

Load case: Dead load and snow load

The corner is so little deformed, less than 0.15 mm, that it can be considered as undeformed, see Figure 7.32.

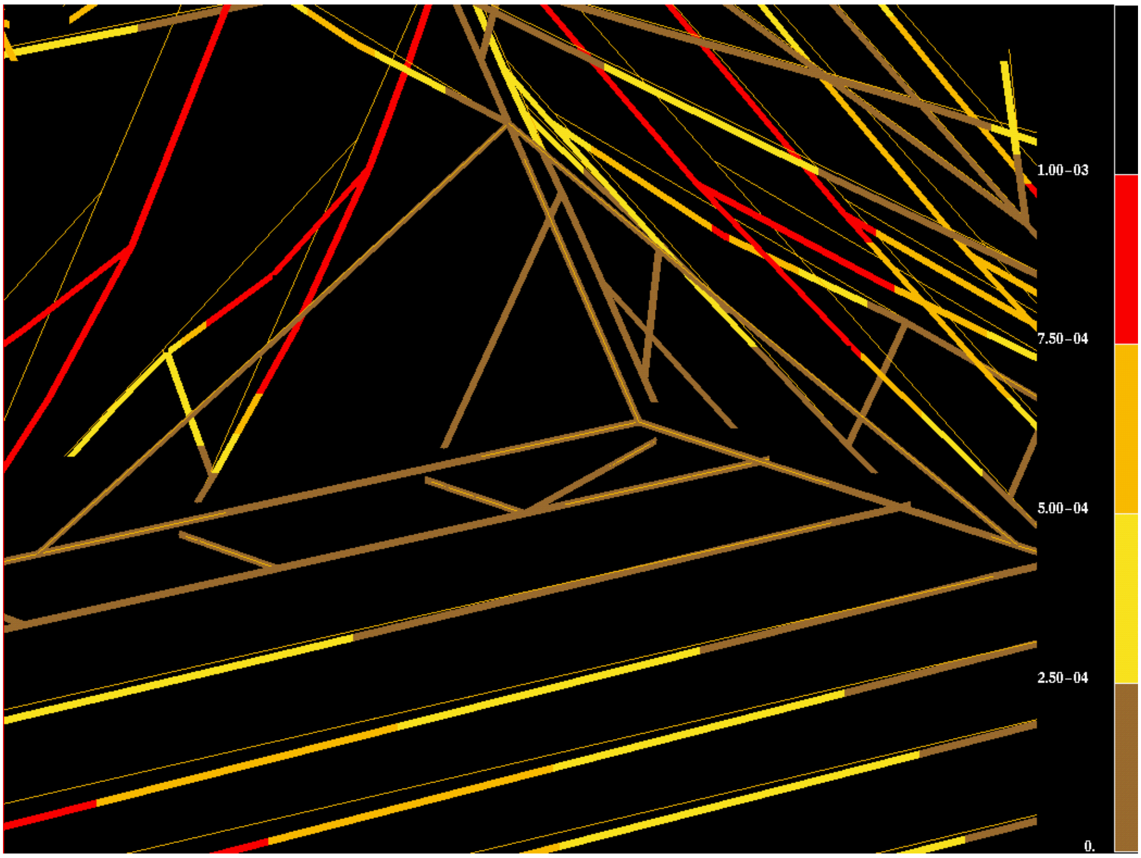

Figure 7.32 Deformations in the corners, seen from inside, due to dead load and snow load. Deformation scale from 0 to 1 mm.

Load case: Dead load and wind load

The deformations in this case are even smaller than in the former case. The greatest deformation observed is about 0.1 mm.

7.3.2 Normal forces

Load case: Dead load and snow load

The corner is barely loaded, see Figure 7.33, which together with the big dimensions of the timber beams can explain the insignificantly small deformations described above. The hip rafter is more loaded than the braces. That is, the primary load path in the corner is through this hip rafter.

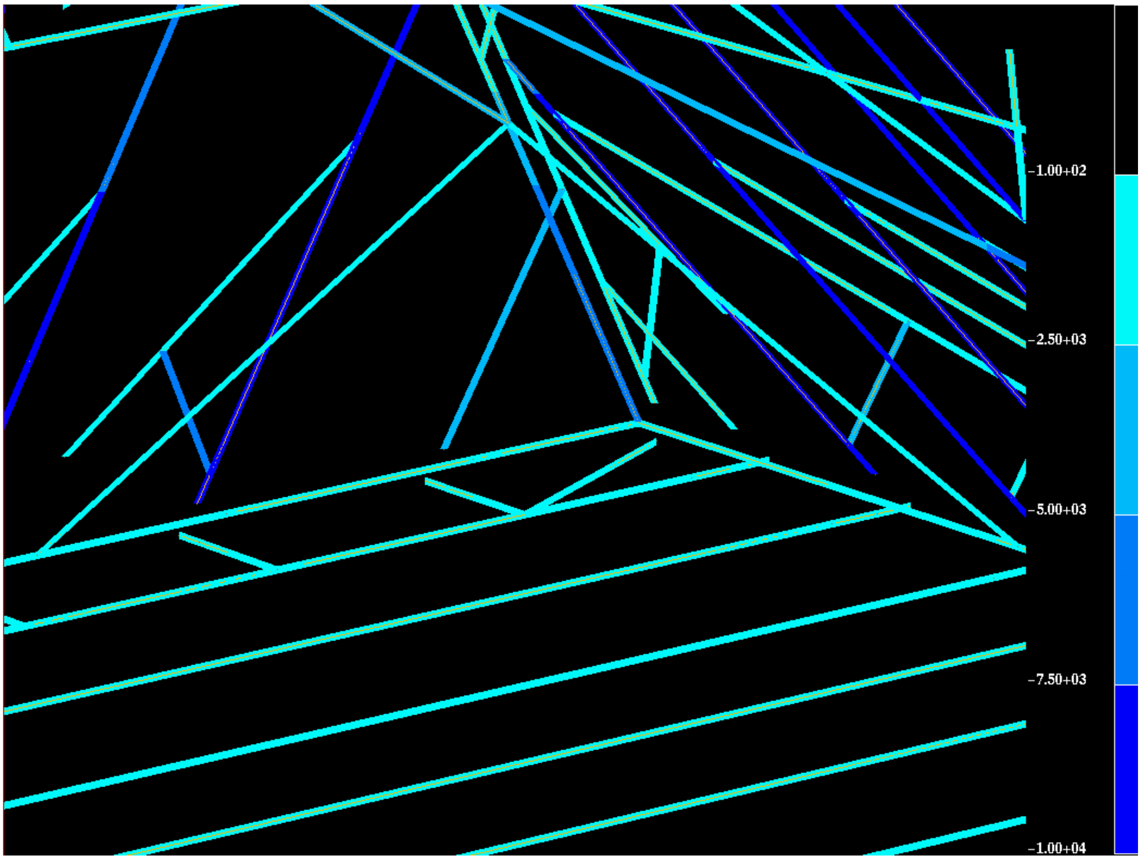

Figure 7.33 Normal forces in the corners due to dead load and snow load. Force scale from –10 kN to –0.1 kN.

Most of the load charging the hip rafters is in axial direction. Because there are small distances between the rafters connected to the hip rafter, any possible bending can be handled. That is, the braces do not seem to have any particular function.

Load case: Dead load and wind load

As in the former load case, the beams in the corner are compressed and the distribution of the loads between the hip rafter and the braces is the same, compare Figure 7.34.

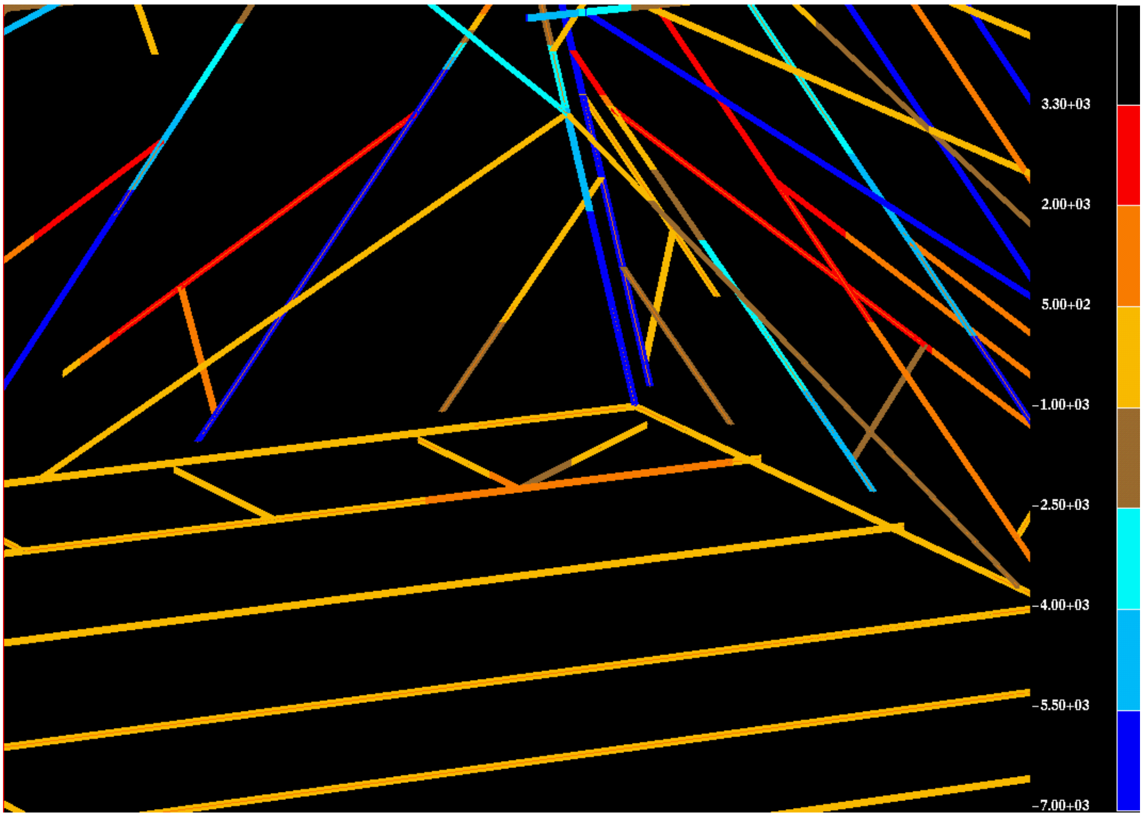

Figure 7.34 Normal forces in the corners due to dead load and wind load. The corner shown is located at the leeward side. Force scale from –7 kN to 3.3 kN.

7.3.3 Normal stresses

Load case: Dead load and snow load

The two braces as well as the hip rafter can be regarded as unstressed, see Figure 7.35. This behaviour is expected since the forces were small in these parts of the structure and since the cross sections are large.

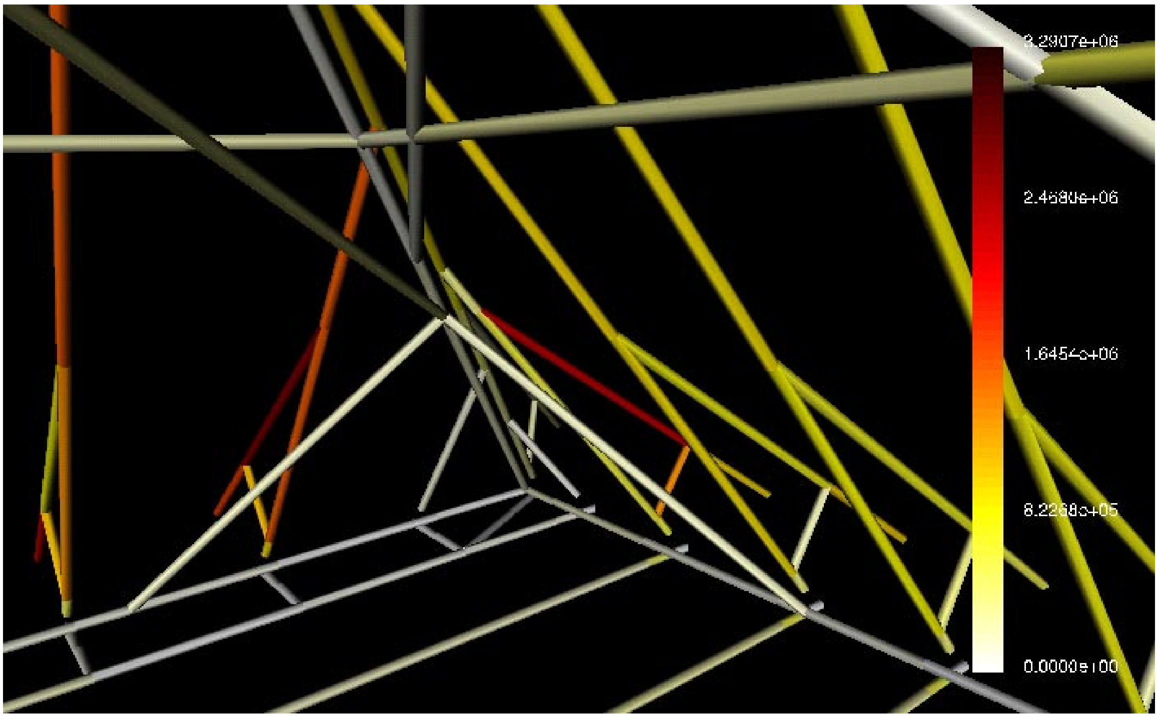

Figure 7.35 Normal stresses in the corners, due to dead load and snow load. Force scale from 0 to 3.3 MPa.

Load case: Dead load and wind load

The behaviour of the corners in the windward side as well as in the leeward side coincides with the behaviour described above for the former load case.

7.3.4 Shear stresses

No shear stresses occur in the hip rafter or in the braces.

From the facts about both the normal and the shear stresses some conclusions about the structure of the corner can be drawn. The braces again do not seem to have any load carrying function and could therefore, from this point of view, be considered as unnecessary. On the other hand they might have had an important task when the roof was constructed. If the hip rafter begins to rot, an extra load carrying function can be found in the braces.

8 Concluding remarks

The overall conclusion drawn from this Master's thesis is that the deformations, no matter which load is acting and no matter which boundary conditions are used, are insignificant. This coincides with our estimations, since the roof structure consists of timber with large dimensions.

Most of the load is carried vertically and two different load paths, the outer rafters and the lying timberframe together with the transverse collar beams at the first level of collar beams, have occurred. The load paths are almost the same for all load cases and boundary condition sets.

Just as assumed, the queen posts are tensioned and suspended from above. This behaviour can also be seen in the queen posts in the Swedish castle of Glimmingehus.

The spire has a perhaps unexpected function, since it actually supports the rest of the structure when loaded by wind. It would be natural to suspect the spire to strengthen the behaviour, but this is not the case.

The special study of the corners shows that there is no load carrying need of the braces. They are undeformed and almost unloaded. When the building was raised they might have had a stabilising function.

Most of the above conclusions could probably not have been made without the use of the structural analysis and the visualisation of its results. By means of our conclusions and the visualised behaviour, the complexity of the structure can be understood. To be able to draw the conclusions, knowledge of structural mechanics is required, but even the less experienced reader is able to get an overall picture of the behaviour.

This Master's thesis can be used as a model to perform and present a mechanical analysis in a way that is more understandable. The results of this method could then be practicable for many actors in a restoration.

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Appendix

Load calculations

Since we are interested in the everyday behaviour of the structure, all calculations are made in the serviceability state.

Two load cases are used for the calculations:

Load case A: Dead load and snow load
Load case B: Dead load and wind load

All loads are calculated as fictive accelerations due to gravity.

Dead load

Distributed dead loads are calculated from the weight of the timber and of the roofing tiles. There are also point loads representing the spire and the weights of the church clock.

Beams:

The computer program does the calculation of dead load. The acceleration due to gravity, $g = 9.81\ m/s^2$, the density of wood and the dimensions of the beams in the structure are used as input to PATRAN

ρ_{wood} : 690 kg/m^3 [4]
$A_{section}$: the beams are divided into groups depending on the approximate size of their cross sections. These vary from about0.15 * 0.15 to about 0.30 * 0.30 m^2, but will not always be quadratic.

Roofing tiles:

ρ_{tile} : 1800 kg/m^3, [18]
w : the widths of the roof area from which the specific beam is carrying the loads, see Figure A1.

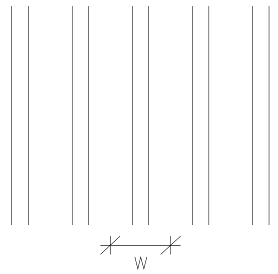

Figure A1 Widths of the roof area from which a beam is loaded.

Spire:

The spire is modelled as a rigid body. To be able to use the real acceleration due to gravity, the dimensions of the beams in the rigid body are calculated to fit.

Snow load

Snow load calculations are based on BKR 94 [7]

$$s = \Psi * s_k$$

$$s_k = \mu * C_t * s_0$$

s : normal value of snow load [N/m^2]
Ψ : load reducing factor, which depends on which snow zone the building is situated in
s_k : characteristic value of snow load on roof [N/m^2]
μ : shape factor, which depends on roof shape and on risk of snow accumulating
C_t : thermal coefficient, normally given the value 1.0
s_0 : basic value of snow load on ground [N/m^2].

Copenhagen is approximately in the same snow zone as Malmoe, which is snow zone 1.

Wind load

Wind load calculations are based on BKR 94 [7].

Topography II is used. If the wind is blowing from topography II to topography III, the velocity profile does not change until after 12 km to what concerns topography III.

$$q_k = C_{dyn} * C_{exp} * q_{ref}$$

$$C_{dyn} = 1 + 6 / (\ln (h / z_0))$$

$$C_{exp} = (\beta * \ln (z / z_0))^2 \qquad z \geq z_{min}$$

q_k : the characteristic value of the velocity power of the wind [N/m^2]
C_{dyn} : the wind gust factor
C_{exp} : exposure factor
q_{ref} : reference velocity power of the reference wind velocity [N/m^2]
h : the height of the building [m]
z_0 : roughness parameter
β : topography parameter
z : height above the ground to the point on, or surface of, the building for which the wind load shall be calculated