DYNAMIC BEHAVIOUR OF FOOTBRIDGES SUBJECTED TO PEDESTRIAN-INDUCED VIBRATIONS

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Preface

The work presented in this master's thesis was carried out at the Division of Structural Mechanics, Lund Institute of Technology, Lund University, Sweden from May to November 2005.

I would like to express my gratitude to my supervisors, Prof. Göran Sandberg and Ph.D. Per-Erik Austrell at the Division of Structural Mechanics and M.Sc. Håkan Camper at Skanska Teknik AB, Malmö for their guidance during this work. I would also like to thank Fridberg Stefánsson at Hönnun Consulting Engineers for supporting me with thoughts and ideas.

A special thanks to the staff and my fellow students at the Division of Structural Mechanics for their help and interesting conversations during coffee breaks.

I would also like to thank my fiancée Ása for moving to Sweden with me and for standing by me throughout my masters education. Finally, I want to express my deepest gratitude to my parents for their support throughout my education.

Lund, November 2005

Fjalar Hauksson

Abstract

Over the last years, the trend in footbridge design has been towards greater spans and increased flexibility and lightness. As a consequence, stiffness and mass have decreased which has lead to smaller natural frequencies and more sensitivity to dynamic loads. Many footbridges have natural frequencies that coincide with the dominant frequencies of the pedestrian-induced load and therefore they have a potential to suffer excessive vibrations under dynamic loads induced by pedestrians.

The main focus of this thesis was on the vertical and horizontal forces that pedestrians impart to a footbridge and how these loads can be modelled to be used in the dynamic design of footbridges. The work was divided into four subtasks. A literature study of dynamic loads induced by pedestrians was performed. Design criteria and load models proposed by four widely used standards were introduced and a comparison was made. Dynamic analysis of the London Millennium Bridge was performed using both an MDOF-model and an SDOF-model. Finally, available solutions to vibration problems and improvements of design procedures were studied.

The standards studied in this thesis all propose similar serviceability criteria for vertical vibrations. However, only two of them propose criteria for horizontal vibrations. Some of these standards introduce load models for pedestrian loads applicable for simplified structures. Load modelling for more complex structures, on the other hand, are most often left to the designer.

Dynamic analysis of the London Millennium Bridge according to British and International standards indicated good serviceability. An attempt to model the horizontal load imposed by a group or a crowd of pedestrians resulted in accelerations that exceeded serviceability criteria.

The most effective way to solve vibration problems is to increase damping by installing a damping system. Several formulas have been set forth in order to calculate the amount of damping required to solve vibration problems. However, more data from existing lively footbridges is needed to verify these formulas.

Contents

Chapter 1

Introduction

1.1 Background

Over the last years, the trend in footbridge design has been towards greater spans and increased flexibility and lightness. Improved construction materials can be more highly stressed under static loads which leads to more slender structures, smaller cross sectional dimensions and greater spans. As a consequence, stiffness and mass decrease leading to smaller natural frequencies resulting in more sensitivity to dynamic loads.

Many footbridges have natural frequencies with the potential to suffer excessive vibrations under dynamic loads induced by pedestrians. Excessive vibrations can be caused by resonance between pedestrian loading and one or more natural frequencies of the structure. The reason for this is that the range of footbridge natural frequencies often coincides with the dominant frequencies of the pedestrian-induced load [38].

It is obvious that if footbridges are designed for static loads only they may be susceptible to vertical as well as horizontal vibrations. Further, recent experiences, for example with the London Millennium Bridge, have shown just how important subject the dynamics of footbridges is.

1.2 Cases

Several cases of footbridges experiencing excessive vibrations due to pedestrian-induced loading have been reported in the last years. The one case that has attracted the most attention is the London Millennium Bridge.

The London Millennium Bridge is located across the Thames River in Central London, see Fig. 1.1. The bridge was opened to the public on 10 June 2000 and during the first day between 80.000 and 100.000 people crossed the bridge, resulting in a maximum crowd density of between 1,3 - 1,5 persons per square meter at any one time [27]. On the first day, the Millennium Bridge experienced horizontal vibrations induced by a synchronised horizontal pedestrian load. The horizontal vibrations took place mainly on the south span, at a frequency of around 0,8 Hz and on the central span, at frequencies of just under 0,5 Hz and 0,9 Hz, the first and second lateral modes respectively [15]. Observations showed that the center span moved by 70 mm. Two days after the opening, the bridge was closed in order to investigate the cause of the vibrations and to design a solution [8].

Figure 1.1: The London Millennium Bridge [35]

One of the earliest reported incidences of excessive horizontal vibrations due to synchronised horizontal pedestrian load occurred on the Toda Park Bridge (T-Bridge), Toda City, Japan [26]. The T-bridge is a pedestrian cable-stayed bridge which was completed in 1989. It has a main span of 134 meters, a side span of 45 meters, and two cable planes with 11 stays per plane, see Fig. 1.2. During a busy day, shortly after the bridge was opened, several thousand pedestrians crossed the T-Bridge which resulted in a strong lateral vibration. The girder vibrated laterally with amplitude of about 10 mm and a frequency of about 0,9 Hz, the natural frequency of the first lateral mode. Although this amplitude does not seem to be large, some pedestrians felt uncomfortable and unsafe. [20], [21].

By video recording and observing the movement of people's heads in the crowd, and by measuring the lateral response, Fujino et al. [20] concluded that 20% of the people in the crowd perfectly synchronised their walking.

In 1975, the north section of the Auckland Harbour Road Bridge in New Zealand (Fig. 1.3) experienced lateral vibrations during a public demonstration, when the bridge was being crossed by between 2.000 and 4.000 demonstrators. The span of the north section is 190 meters and the bridge deck is made of a steel box girder. Its lowest natural horizontal frequency is 0,67 Hz [7].

In addition, horizontal vibrations were among several reasons behind the closure of the Solferino Bridge in Paris immediately after its opening in December 1999. Also, a 100 year-old footbridge, Alexandra Bridge in Ottawa, experienced strong

Figure 1.2: The Toda Park Bridge [20]

Figure 1.3: Auckland Harbour Bridge [36]

lateral vibrations in July 2000, when subjected to crowd loading by spectators of a fireworks display [8].

In conclusion, it is obvious that the problem of pedestrian-induced lateral vibrations has occurred on a range of different structural types (suspension, cable-stayed and steel girder bridges) as well as on footbridges made of different materials (steel, composite steel-concrete and reinforced and prestressed concrete) [38]. It is therefore stated, that pedestrians can induce strong vibrations on a footbridge of any structural form, only if there is a lateral mode of a low enough natural frequency [8].

1.3 Objective

The main objective of this thesis is to study the vertical and horizontal forces that pedestrians impart to a footbridge. Special attention is given to the responses of a structure due to dynamic loads induced by groups or a crowd of pedestrians which can lead to the synchronisation of a percentage of the persons. The work is divided into four subtasks:

- Literature study of dynamic loads induced by pedestrians.
- Comparison of design criteria and load modelling in international, European, British and Swedish standards.
- Dynamic analysis of the London Millennium Bridge and a parameter study of parameters such as pedestrian synchronisation, bridge mass and structural damping.
- Study of available solutions to vibrations problems and improvements of design procedures.

The aim is to study how dynamic loads induced by pedestrians can be modelled to be used in the dynamic design of footbridges.

1.4 Contributions

The main contributions of this thesis can be summerized as follows:

- Comparison of design criteria and load modelling in the standards ISO 10137, Eurocode, BS 5400 and Bro 2004.
- Calculation of dynamic response of the London Millennium Bridge when subjected to dynamic loading according to the standards ISO 10137, BS 5400 and Bro 2004.
- An effort is made to generalize load models for one pedestrian as a load model for a group of people and for a crowded bridge.
- Comparison of the two load models proposed by Dallard et al. [15] and Nakamura [21].

1.5 Disposition

This thesis consists of four main parts. First there is a theoretical study of structural dynamics and dynamic loads induced by pedestrians. Chapter 2 covers these subjects and includes formulation of the equation of motion and the eigenvalue problem. Solution methods used to solve these equation both for systems with single and multiple degrees of freedom are introduced. Chapter 2 also includes a literature study of dynamic loads induced by pedestrians.

In Chapter 3, design criteria for footbridges and models for dynamic pedestrian loads set forth in four widely used standards are compared. This chapter includes a discussion on how current standards and codes of practice deal with vibration problems of footbridges.

A dynamic analysis of the London Millennium Bridge is performed in Chapter 4. This chapter includes a general description of the bridge structure as well as a description of the finite element modelling of the bridge. Chapter 4 also includes a description of the dynamic analysis performed both using an MDOF finite element model and an SDOF model.

Chapter 5 discusses different solution techniques to vibration problems and improved design guidelines for dynamic design of footbridges.

Finally, conclusions are summarized in Chapter 6

Matlab and ABAQUS codes used in this thesis are provided in Appendix A and Appendix B respectively.

Chapter 2

Theory

2.1 Structural dynamics

Structural dynamics describe the behaviour of a structure due to dynamic loads. Dynamic loads are applied to the structure as a function of time, resulting in time varying responses (e.g. displacements, velocities and accelerations) of the structure.

To obtain the responses of the structure a dynamic analysis is performed with the objective to solve the equation of equilibrium between the inertia force, damping force and stiffness force together with the externally applied force:

$$f_I + f_D + f_S = f(t) \tag{2.1}$$

where f_I is the inertial force of the mass and is related to the acceleration of the structure by $f_I = m\ddot{u}$, f_D is the damping force and is related to the velocity of the structure by $f_D = c\dot{u}$, f_S is the elastic force exerted on the mass and is related to the displacement of the structure by $f_S = ku$, where k is the stiffness, c is the damping ratio and m is the mass of the dynamic system. Further, $f(t)$ is the externally applied force [18].

Substituting these expressions into Eq. 2.1 gives the equation of motion

$$m\ddot{u} + c\dot{u} + ku = f(t) \tag{2.2}$$

Pedestrian induced vibrations are mainly a subject of serviceability [38]. In this thesis, it is therefore assumed, that the structures respond linearly to the applied loads and the dynamic response can be found by solving this equation of motion.

Two different dynamic models are presented in the following sections. First the structure is modelled as a system with one degree of freedom (an SDOF-model) and a solution technique for the equations of the system is presented. Then the structure is modelled as a multi-degree-of-freedom system (an MDOF-model). Modal analysis is then presented as a technique to determine the basic dynamic characteristics of the MDOF-system.

2.1.1 SDOF model

In this section the analysis of generalized SDOF systems is introduced. First the equation of motion for a generalized SDOF system with distributed mass and stiffness is formulated. Then a numerical time-stepping method for solving this equation is presented. It is noted, that the analysis provides only approximate results for systems with distributed mass and stiffness.

Equation of motion

A system consisting of a simple beam with distributed mass and stiffness can deflect in an infinite variety of shapes. By restricting the deflections of the beam to a single shape function $\psi(x)$ that approximates the fundamental vibration mode, it is possible to obtain approximate results for the lowest natural frequency of the system. The deflections of the beam are then given by $u(x,t) = \psi(x)z(t)$, where the generalized coordinate $z(t)$ is the deflection of the beam at a selected location.

It can be shown (see for example [5]) that the equation of motion for a generalized SDOF-system is of the form

$$\tilde{m}\ddot{z} + \tilde{c}\dot{z} + \tilde{k}z = \tilde{f}(t) \tag{2.3}$$

where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$ and $\tilde{f}(t)$ are defined as the generalized mass, generalized damping, generalized stiffness and generalized force of the system. Further, the generalized mass and stiffness can be calculated using the following expressions

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 \; dx \tag{2.4}$$

$$\tilde{k} = \int_0^L EI(x)[\psi''(x)]^2 \; dx \tag{2.5}$$

where $m(x)$ is mass of the structure per unit length, $EI(x)$ is the stiffness of the structure per unit length and L is the length of the structure [5].

Damping is usually expressed by a damping ratio, ζ, estimated from experimental data, experience and/or taken from standards. The generalized damping can then be calculated from the expression

$$\tilde{c} = \zeta \; (2\tilde{m}\omega) \tag{2.6}$$

where ω is the natural frequency of the structure.

Once the generalized properties $\tilde{m}$, $\tilde{c}$, $\tilde{k}$ and $\tilde{f}(t)$ are determined, the equation of motion (Eq. 2.3) can be solved for $z(t)$ using a numerical integration method. Finally, by assuming a shape function $\psi(x)$, the displacements at all times and at all locations of the system are determined from $u(x,t) = \psi(x)z(t)$ [5].

Response analysis

The most general approach for the solution of the dynamic response of structural systems is to use numerical time-stepping methods for integration of the equation of motion. This involves, after the solution is defined at time zero, an attempt to satisfy dynamic equilibrium at discrete points in time [34].

One method commonly used for numerical integration is the central difference method, which is an explicit method. Explicit methods do not involve the solution of a set of liner equations at each step. Instead, these methods use the differential equation at time t_i to predict a solution at time t_{i+1} [34].

The central difference method is based on a finite difference approximation of the velocity and the acceleration. Taking constant time steps, $\Delta t_i = \Delta t$, the central difference expressions for velocity and acceleration at time t_i are

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \quad \text{and} \quad \ddot{u}_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} \tag{2.7}$$

Substituting these approximate expressions for velocity and acceleration into the equation of motion, Eq. 2.2, gives

$$m\,\frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} + c\,\frac{u_{i+1} - u_{i-1}}{2\Delta t} + k u_i = f_i \tag{2.8}$$

where u_i and u_{i-1} are known from preceding time steps.

The unknown displacement at time t_{i+1} can now be calculated by

$$u_{i+1} = \frac{\hat{f}_i}{\hat{k}} \tag{2.9}$$

where

$$\hat{k} = \frac{m}{(\Delta t)^2} + \frac{c}{2\Delta t} \tag{2.10}$$

and

$$\hat{f}_i = f_i - \left[\frac{m}{(\Delta t)^2} - \frac{c}{2\Delta t}\right] u_{i-1} - \left[k - \frac{2m}{(\Delta t)^2}\right] u_i \tag{2.11}$$

This solution at time t_{i+1} is determined from the equilibrium condition at time t_i, which is typical for explicit methods [5].

2.1.2 MDOF model

All real structures have an infinite number of degrees of freedom (DOF's). It is, however, possible to approximate all structures as an assemblage of finite number of massless members and a finite number of node displacements. The mass of the structure is lumped at the nodes and for linear elastic structures the stiffness properties of the members can be approximated accurately. Such a model is called a multi-degree-of-freedom (MDOF) system.

In this section the analysis of MDOF systems is introduced. First the equation of motion for a MDOF system is formulated. Then the concept of modal analysis is presented. Modal analysis includes the formulation of the eigenvalue problem and a solution method for solving the eigenvalue problem. Finally, modal analysis can be used to compute the dynamic response of an MDOF system to external forces.

Equation of motion

As mentioned above, a structure can be idealized as an assemblage of elements connected at nodes. The displacements of the nodes are the degrees of freedom. By discretizing the structure in this way, a stiffness matrix $\mathbf{K}$, a damping matrix $\mathbf{C}$ and a mass matrix $\mathbf{M}$ of the structure can be determined, see for example [5]. Each of these matrices are of order $N \; x \; N$ where N is the number of degrees of freedom.

The stiffness matrix for a discretized system can be determined by assembling the stiffness matrices of individual elements. Damping for MDOF systems is often specified by numerical values for the damping ratios, as for SDOF systems. The mass is idealized as lumped or concentrated at the nodes of the discretized structure, giving a diagonal mass matrix.

The equation of motion of a MDOF system can now be written on the form:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) \tag{2.12}$$

which is a system of N ordinary differential equations that can be solved for the displacements $\mathbf{u}$ due to the applied forces $\mathbf{f}(t)$. It is now obvious that Eq. 2.12 is the MDOF equivalent of Eq. 2.3 for a SDOF system [5].

Modal analysis

Modal analysis can be used to determine the natural frequencies and the vibration mode shapes of a structure. The natural frequencies of a structure are the frequencies at which the structure naturally tends to vibrate if it is subjected to a disturbance. The vibration mode shapes of a structure are the deformed shapes of the structure at a specified frequency.

When performing modal analysis, the free vibrations of the structure are of interest. Free vibration is when no external forces are applied and damping of the structure is neglected. When damping is neglected the eigenvalues are real numbers. The solution for the undamped natural frequencies and mode shapes is called real eigenvalue analysis or normal modes analysis. The equation of motion of a free vibration is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0 \tag{2.13}$$

This equation has a solution in the form of simple harmonic motion:

$$\mathbf{u} = \phi_n \sin \omega_n t \quad \text{and} \quad \ddot{\mathbf{u}} = -\omega_n^2 \phi_n \sin \omega_n t \tag{2.14}$$

Substituting these into the equation of motion gives

$$\mathbf{K}\phi_n = \omega_n^2 \mathbf{M}\phi_n \tag{2.15}$$

which can be rewritten as

$$[\mathbf{K} - \omega_n^2 \mathbf{M}]\phi_n = \mathbf{0} \tag{2.16}$$

This equation has a nontrivial solution if

$$\det\ [\mathbf{K} - \omega_n^2 \mathbf{M}] = 0 \tag{2.17}$$

Equation 2.17 is called the system characteristic equation. This equation has N real roots for ω_n^2, which are the natural frequencies of vibration of the system. They are as many as the degrees of freedom, N. Each natural frequency ω_n has a corresponding eigenvector or mode shape ϕ_n, which fulfills equation 2.16. This is the generalized eigenvalue problem to be solved in free vibration modal analysis.

After having defined the structural properties; mass, stiffness and damping ratio and determined the natural frequencies ω_n and modes ϕ_n from solving the eigenvalue problem, the response of the system can be computed as follows. First, the response of each mode is computed by solving following equation for $q_n(t)$

$$M_n \ddot{q_n} + C_n \dot{q_n} + K_n q_n = f_n(t) \tag{2.18}$$

Then, the contributions of all the modes can be combined to determine the total dynamic response of the structure

$$\mathbf{u}(t) = \sum_{n=1}^{N}\ \phi_n\ q_n(t) \tag{2.19}$$

The parameters M_n, K_n, C_n and $f_n(t)$ are defined as follows

$$M_n = \phi_n^T\ \mathbf{M}\ \phi_n \quad , \quad K_n = \phi_n^T\ \mathbf{K}\ \phi_n \quad , \quad C_n = \phi_n^T\ \mathbf{C}\ \phi_n \ \text{ and } \ f_n(t) = \phi_n^T\ \mathbf{f}(t) \tag{2.20}$$

and they depend only on the n th-mode ϕ_n, and not on other modes. Thus, there are N uncoupled equations like Eq. 2.18, one for each natural mode [5].

In practice, modal analysis is almost always carried out by implementing the finite element method (FEM). If the geometry and the material properties of the structure are known, an FE model of the structure can be built. The mass, stiffness and damping properties of the structure, represented by the left hand side of the equation of motion (Eq. 2.12), can then be established using the FE method. All that now remains, in order to solve the equation of motion, is to quantify and then to model mathematically the applied forces $\mathbf{f}(t)$. This will be the subject of the next two sections.

2.2 Dynamic loads induced by pedestrians

During walking on a structure, pedestrians induce dynamic time varying forces on the surface of the structure. These forces have components in all three directions, vertical, lateral and longitudinal and they depend on parameters such as pacing frequency, walking speed and step length. Dynamic forces induced by humans are therefore highly complex in nature [38].

Several studies have been performed in order to quantify pedestrian walking forces. These studies have paid more attention to the vertical component of the dynamic force than the horizontal component. This is because until the opening of the Millennium Bridge, almost all documented problems with pedestrian-induces vibrations were associated with vertical forces and vibrations [8].

The typical pacing frequency for walking is around 2 steps per second, which gives a vertical forcing frequency of 2 Hz. Slow walking is in the region of 1,4 - 1,7 Hz and fast walking in the range of 2,2 - 2,4 Hz. This means that the total range of vertical forcing frequency is 1,4 - 2,4 Hz with a rough mean of 2 Hz. Since the lateral component of the force is applied at half the footfall frequency, the lateral forcing frequencies are in the region of 0,7 - 1,2 Hz, see Fig. 2.1 [2].

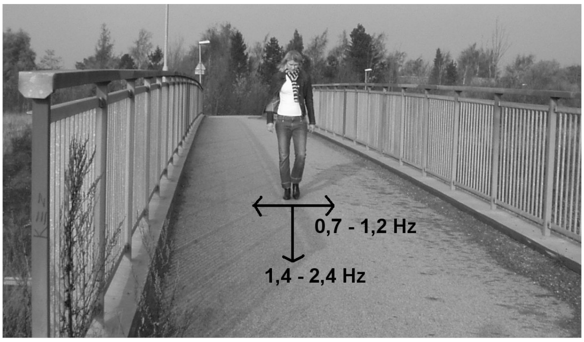

Figure 2.1: Vertical and horizontal forcing frequencies

Many footbridges have natural vertical and lateral frequencies within the limits mentioned above (1,4 - 2,4 Hz vertical and 0,7 - 1,2 Hz horizontal). They have therefore the potential to suffer excessive vibrations under pedestrian actions. The necessity to consider horizontal as well as vertical pedestrian excitation is therefore obvious [18].

This section, which is merely a literature review, focuses on dynamic loads induced by pedestrians. First, the vertical forces induced by a single person are looked at. This is the part that most work has been laid on and therefore these forces are fairly well quantified. Next, the focus will be on the horizontal forces induced by a single person. Finally there is a section on the synchronisation phenomenon of people walking in groups and crowds. This phenomena has only recently been discovered and it is therefore not well understood.

2.2.1 Vertical loads

Several measurements have been conducted to quantify vertical loads imposed by pedestrians on structures. Most measurements indicate that the shape of the vertical force produced by one person taking one step is of the kind shown in Fig. 2.2.

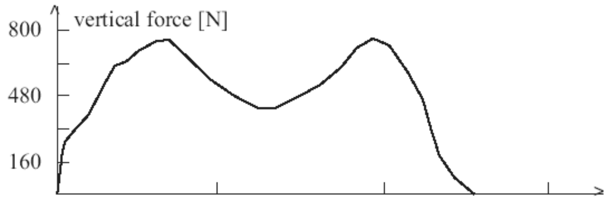

Figure 2.2: Vertical force produced by one person taking one step [38]

Measurements of continuous walking has also been made. The measured time histories were near periodic with an average period equal to the average step frequency. General shapes for continuous forces in both vertical and horizontal directions have been constructed assuming a perfect periodicity of the force, see Fig. 2.3 [38]

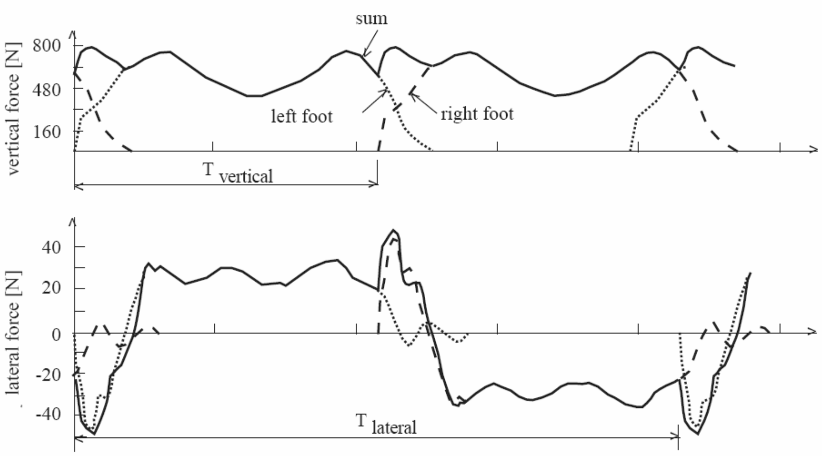

Figure 2.3: Periodic walking time histories in vertical and horizontal directions [38]

As mentioned in the previous section, the vertical forcing frequency is generally in the region of 1,4 - 2,4 Hz [8]. This has been confirmed with several experiments, for example by Matsumoto who investigated a sample of 505 persons. He concluded that the pacing frequencies followed a normal distribution with a mean of 2,0 Hz and a standard deviation of 0,173 Hz, see Fig. 2.4 [38].

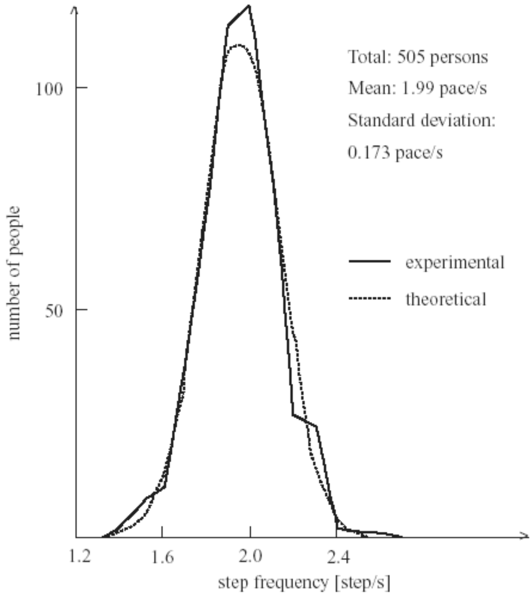

Figure 2.4: Pacing frequencies for normal walking according to Matsumoto [38]

2.2.2 Horizontal loads

When walking on a structure, pedestrians produce horizontal dynamic forces on the surface of the structure. These forces are a consequence of a lateral oscillation of the gravity center of the body and the lateral oscillations are a consequence of body movements when persons step with their right and left foot in turn. The amplitudes of these lateral oscillations are, in general, of about 1 - 2 cm, see Fig. 2.5 [20].

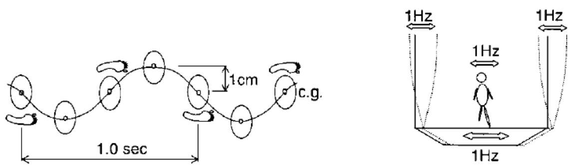

Figure 2.5: Mechanism of lateral vibration [20]

The frequency of the horizontal force is half the pacing frequency and hence lies in the region of 0,7 to 1,2 Hz for a pacing frequency of 1,4 to 2,4 Hz [2]. On a stationary surface this force has been found to be about 10% of the vertical loading, which is about 4% of the pedestrian's weight [21].

It should be noted that the horizontal loading parameters are not well quantified. Few measurements of the magnitude of horizontal loading due to walking have been made and, in addition, they have almost all been made on unmoving surfaces [38].

2.2.3 Loads due to groups and crowds

Having described both vertical and horizontal forces produced by a single pedestrian it is of major interest to look at forces produced by both a group of people walking at the same speed and a crowd of walking people. It is under such circumstances that the phenomenon of human-structure synchronisation has been discovered.

During footbridge vibration some kind of human-structure interaction occurs. A human-structure synchronisation is when the pedestrians adapt their step to the vibrations of the structure [38]. For example, the movements of the Millennium Bridge (see Section 1.2) were caused by a lateral loading effect that has been found to be due to such human-structure synchronisation [15].

Vertical synchronisation

When walking over a bridge, pedestrians are more tolerant of vertical vibration than horizontal. In a study reported by Bachmann and Ammann in 1987, it is suggested that vertical displacements of at least 10 mm are required to cause disturbance to a natural footfall rate. This corresponds to accelerations of at least 1,6 m/s^2 at 2 Hz. Also, a group test with 250 people on the London Millennium Bridge revealed no evidence of synchronisation to vertical acceleration amplitudes of up to 0,4 m/s^2 [33]. Further, these tests provided no evidence that the vertical forces generated by pedestrians are other than random. It is therefore most probable that existing vibrations limits presented in standards (see Chapter 3) are sufficient to prevent vertical synchronisation between structure and pedestrians.

Horizontal synchronisation

It is known that pedestrians are sensitive to low frequency lateral motion on the surface on which they walk. The phenomenon of horizontal synchronisatoin can be described in the following way:

First, random horizontal pedestrian walking forces, combined with the synchronisation that occurs naturally within a crowd, cause small horizontal motion of the bridge and perhaps, walking of some pedestrians becomes synchronized to the bridge motion.

If this small motion is perceptible, it becomes more comfortable for the pedestrians to walk in synchronisation with the horizontal motion of the bridge. Because lateral motion affects balance, pedestrians tend to walk with feet further apart and attempt to synchronise their footsteps with the motion of the surface. The pedestrians find this helps them maintain their lateral balance.

This instinctive behaviour of pedestrians ensures that the dynamic forces are applied at a resonant frequency of the bridge and consequently, the bridge motion increases. The walking of more pedestrians is synchronized, increasing the lateral bridge motion further.

As the amplitude of the motion increases, the lateral dynamic force increases, as well as the degree of synchronisation between pedestrians. In this sense, the vibration has a self-excited nature and it takes some time before the vibration is fully developed. However, because of the human behaviour of pedestrians, they reduce walking speed or stop walking when the vibration becomes uncomfortable. Therefore, the vibration amplitude does not become infinitely large [7], [20], [33].

Observations indicate that a significant proportion of pedestrians can start to synchronize when the amplitude of the walkway motion is only a few millimeters [15].

In 2002, Willford [33] reported tests that were undertaken shortly after the opening of the London Millennium Bridge. These tests were performed with a single walking person on a platform moving horizontally. The objective was to investigate the phenomenon of human-structure interaction and synchronisation [19]. Willford's results showed that as the horizontal movement increased so did the lateral pedestrian force. As the amplitude of the deck increased from 0 to 30 mm, the horizontal dynamic load increased from being 5% of the pedestrians vertical static load to 10%. These tests also indicated that at 1 Hz, amplitudes of motion as low as 5 mm caused a 40% probability of synchronisation between pedestrian and structure [33].

In December 2000, controlled test were performed on the Millennium Bridge. A group of people were instructed to walk in a circulatory route on one span of the bridge. The number of people in the group was gradually increased and the lateral motion of the bridge observed. This test showed that the phenomenon of synchronisation is highly non-linear, see Fig. 2.6. The dynamic response of the bridge was stable until a critical number of people were on the bridge. Thereafter the people tended to walk in synchronisation with the swaying of the bridge resulting in a rapid increase in the amplitude of the dynamic response [27]. The tests also showed that the lateral forces are strongly correlated with the lateral movement of the bridge [15].

Now that loads induced by pedestrians have been described the next step is to model these loads mathematically in order to solve the equation of motion, Eq. 2.2. This will be the subject of the next section.

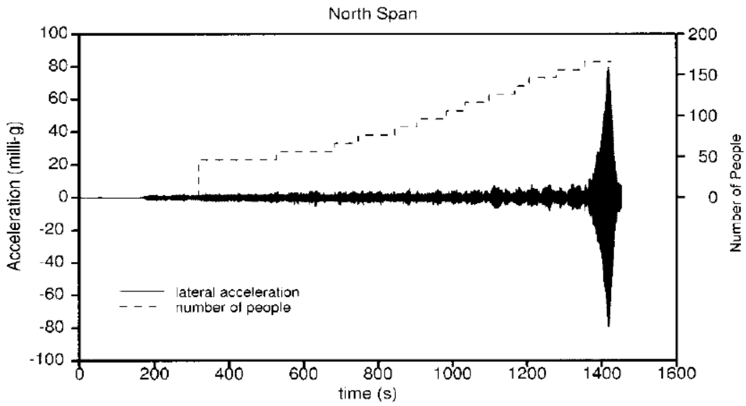

Figure 2.6: Lateral acceleration of the Millennium Bridge and number of pedestrians [8]

2.3 Load models

To be able to perform a dynamic analysis of a structure, a mathematical model of the pedestrian dynamic forces is needed. It is important to model mathematically the dynamic forces due to both a single pedestrian and a crowd of people traversing the structure.

In this section, three different mathematical models for describing the dynamic pedestrian force will be introduced. First the load from a single pedestrian is approximated as a periodic force which can be represented as a Fourier series. An attempt is then made to model loads from a group or a crowd using the same principles. Finally, two different load models modelling the synchronised loads from crowds are presented. These models are both results from observations and measurements of the phenomenon of horizontal human-structure synchronisation.

2.3.1 Periodic load model

Periodic load models are based on an assumption that all pedestrians produce exactly the same force and that this force is periodic [38]. It is also assumed that the force produced by a single pedestrian is constant in time.

One person

Dynamic loading due to a moving pedestrian may be considered to be a periodic force, see for example Fig. 2.3. This force $f_p(t)$ can be represented as a Fourier series in which the fundamental harmonic has a frequency equal to the pacing rate

[25]:

$$f_p(t) = Q + \sum_{n=1}^{k} Q\ \alpha_n\ \sin(2\pi n f t + \phi_n)) \tag{2.21}$$

where Q is the pedestrian's weight, α_n is the load factor of the nth harmonic, f is the frequency of the force, ϕ_n is the phase shift of the nth harmonic, n is the number of the harmonic and k is the total number of contributing harmonics [38].

Several measurements have been made in order to quantify the load factor α_n which is the basis for this load model. The results from three such measurements are shown in Table 2.1.

Table 2.1: *Dynamic load factors after different authors*

Author	Dynamic load factor	Direction
Blanchard, 1977	$\alpha_1 = 0,257$	Vertical
Bachmann et al., 1987	$\alpha_1 = 0,37$ $\alpha_2 = 0,10$ $\alpha_3 = 0,12$ $\alpha_4 = 0,04$ $\alpha_5 = 0,08$	Vertical
Bachmann et al., 1987	$\alpha_1 = 0,039$ $\alpha_2 = 0,010$ $\alpha_3 = 0,043$ $\alpha_4 = 0,012$ $\alpha_5 = 0,015$	Lateral
Young, 2001	$\alpha_1 = 0,37(f - 0,92)$ $\alpha_2 = 0,054 + 0,0044f$ $\alpha_3 = 0,026 + 0,0050f$ $\alpha_4 = 0,010 + 0,0051f$	Vertical

In 1977, Blanchard et al. proposed a vertical dynamic load factor of 0,257. Ten years later, Bachmann and Ammann reported the first five harmonics of the vertical as well as the horizontal force. They found the first harmonic of the vertical dynamic load to be 37% of the vertical static load and the first harmonic of the horizontal dynamic load to be 3,9% of the vertical static load, Table 2.1.

In 2001, a year after the opening of the Millennium Bridge, Young presented the work of some researchers. The principles of this work are now used by Arup Consulting Engineers when modelling walking forces and the corresponding structural responses. Young proposed the first four harmonics of the vertical force as a function of the walking frequency f, see Table 2.1 [38].

It is noted that all these tests, performed in order to quantify the load factors, were obtained by direct or indirect force measurements on rigid surfaces [38]. It has already been stated that horizontal movements of the surface seem to increase the horizontal pedestrian force, see Section 2.2.3.

Groups and crowds

It is natural that a first attempt to model loads induced by a group of pedestrians is in terms of multiplying the load induced by a single pedestrian, $f_p(t)$, with some constant. In 1978, Matsumoto tried to defined such a constant [38]. He assumed that pedestrians arrived on the bridge following a Poisson distribution, whereas the phase angle followed a completely random distribution. Based on these assumptions, Matsumoto defined a factor m for multiplying the vibration amplitude calculated for one person

$$m = \sqrt{\lambda T_0} \tag{2.22}$$

where λ is the mean flow rate of persons over the width of the deck [pers/s] and T_0 is the time in seconds needed to cross a bridge of length L [2]. The product λT_0 is equal to the number of pedestrians on the bridge at any time instant, denoted n. The multiplication factor $m = \sqrt{n}$ is therefore equivalent to a load due to absolutely unsynchronised pedestrians [38]. In the same sense, the multiplication factor $m = n$ is equivalent to a load due to absolutely synchronised pedestrians.

It is now clear, that if a crowd traversing the structure is synchronised to some degree, the multiplication factor is on the interval $m = [\sqrt{n} \quad n]$. Finally, having decided the degree of synchronisation, the total load from a group or a crowd, $F_p(t)$ can be calculated using the formula

$$F_p(t) = m \cdot f_p(t) \tag{2.23}$$

2.3.2 Dallard's load model

In December 2000, Dallard et al. [7] performed a test on the Millennium Bridge, see Section 2.2.3. The objective of the test was to provide the data needed to solve the vibration problem on the Millennium Bridge. The test showed that the dynamic force induced by pedestrians was approximately proportional to the lateral velocity of the bridge [7].

According to Dallard et al., the dynamic force per pedestrian, $f_p(t)$, can be related to the local velocity of the bridge, $\dot{u}_{local}$, by

$$f_p(t) = k\dot{u}_{local} \tag{2.24}$$

where k is a constant dependant on the bridge characteristics. The pedestrians' contribution to the modal force is $\phi f_p(t)$, where ϕ is the modeshape. The local velocity is related to the modal velocity by $\dot{u}_{local} = \phi\dot{u}$. The single pedestrians' contribution to the modal force is therefore

$$\phi f_p(t) = \phi k\dot{u}_{local} = \phi^2 k\dot{u} \tag{2.25}$$

Hence the modal excitation force generated by n people uniformly distributed on the span is

$$F_p(t) = \frac{1}{L}\int_0^L \phi^2 \; nk\dot{u} \; dx \tag{2.26}$$

The value of k has to be estimated for each case. Based on the field tests conducted on the London Millennium Bridge, k was found to be 300 Ns/m in the lateral frequency range 0,5 - 1,0 Hz.

For example, if the mode shape of the structure is approximated with the function

$$\phi(x) = \sin\frac{2\pi x}{L} \tag{2.27}$$

the lateral pedestrian force becomes

$$F_p(t) = \frac{1}{L}\int_0^L \phi^2 \; nk \; \dot{u} \; dx = \frac{1}{2} nk \; \dot{u}(t) \tag{2.28}$$

Because the lateral pedestrian force is proportional to the bridge velocity, the moving pedestrians act as negative dampers (or amplifiers) increasing the response of the structure. Based on this force model, Dallard et al. proposed a primary design check requirement that the net damping should remain positive. They also derived an expression for the level of damping needed if the damping force is to exceed the excitation force [7].

According to Dallard et al., required damping is

$$c > \frac{1}{L}\int_0^L \phi^2 \; \frac{nk}{4\pi f M} \; dx \tag{2.29}$$

where L is the length of the bridge, n is the number of pedestrians traversing the bridge and M is the modal mass of the bridge.

For a given level of damping, the limiting number of people, n_L, can be derived from Eq. 2.29

$$n_L = \frac{4\pi c f M}{k \frac{1}{L} \int_0^L \phi^2 dx} \tag{2.30}$$

Now, assuming the same mode shape as before, (Eq. 2.27) the required damping can be calculated as

$$c > \frac{nk}{8\pi f M} \tag{2.31}$$

In the same way, the limiting number of people is

$$n_L = \frac{8\pi c f M}{k} \tag{2.32}$$

The simplicity of this load model is clearly an advantage. The disadvantages are however, that when the lateral force $F_p(t)$ is larger than the damping force $c\dot{u}(t)$, the bridge response increases infinitely [21]. This is not in accordance with observations. Because of the human behaviour of pedestrians, they reduce walking speed or stop walking when the response of the bridge becomes sufficiently large. Therefore, the bridge response does not increase infinitely. Therefore, in 2004, Nakamura [21] proposed modifications to Dallards load model. This model will be dealt with in the next section.

2.3.3 Nakamura's load model

In a paper published in the Journal of Structural Engineering in January, 2004, Nakamura proposes a load model to evaluate pedestrian lateral dynamic forces [21]. His work is based on observations and calculations of the T-bridge in Japan, which experienced strong lateral vibrations induced by pedestrians, see Chapter 1.2.

The basic equation in Nakamura's model is the equation of motion

$$M_B\ddot{u}(t) + C_B\dot{u}(t) + K_B u(t) = F(t) \tag{2.33}$$

where M_B is the modal mass, C_B is the modal damping and K_B is the modal stiffness of the bridge. Further, $u(t)$ is the modal displacement of the girder, $\dot{u}(t)$ is the modal velocity of the girder and $\ddot{u}(t)$ is the modal acceleration of the girder. $F(t)$ is the modal lateral dynamic force applied by all the pedestrians to the bridge, given by

$$F(t) = k_1 k_2 \; \frac{\dot{u}(t)}{k_3 + \mid \dot{u}(t) \mid} \; G(f_B) \; M_P \; g \tag{2.34}$$

The coefficient k_1 is a ratio of the lateral force to the pedestrian's weight. The coefficient k_2 is the percentage of pedestrians who synchronize to the girder vibration. $M_P g$ is the modal self weight of pedestrians. $G(f_B)$ is a function to describe how pedestrians synchronize with the bridge's natural frequency. The worst case scenario is obviously when $G(f_B) = 1,0$.

As can be seen, Nakamura assumes that the pedestrians synchronize proportionally with the girder velocity $\dot{u}(t)$ at low velocities. However, when the girder velocity becomes large, the pedestrians feel uncomfortable or unsafe and stop or decrease their walking pace. Therefore, the girder response does not increase infinitely but is limited at a certain level. This limitation depends on the coefficient k_3 [21].

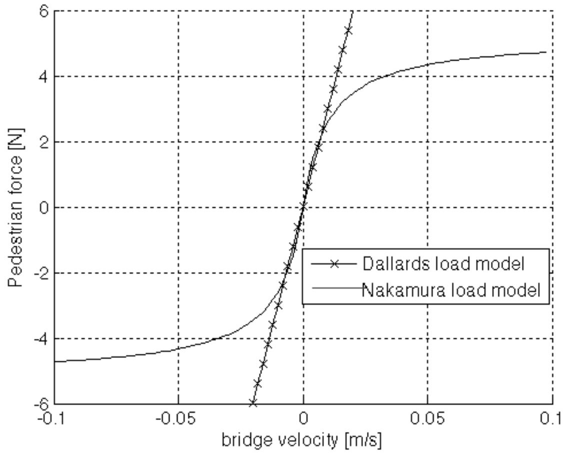

Figure 2.7: Comparison of Dallard's and Nakamura's load models

Figure 2.7 compares the two load models proposed by Dallard and Nakamura. Both models assume that the pedestrian force is a function of bridge velocity. However, the force proposed by Dallard increases linearly with bridge velocity whereas the force proposed by Nakamura increases linearly at low velocities but its increase rate becomes smaller at higher velocities.

Chapter 3

Standards

New lightweight and high-strength structural materials, longer spans and greater slenderness of footbridges have in the past years caused several problems with vibration serviceability, see Chapter 1.2. This chapter will discuss how these problems are dealt with in current standards and codes of practice.

The main focus in this chapter will be on the serviceability criteria and the load models proposed by four widely used standards. Since the dynamic design of the Millennium Bridge was according to the British standard BS 5400, this code will be looked at first. Other codes of practice and design guidelines used internationally are Eurocode and ISO 10137. Also, since this thesis is carried out in Sweden, it is of interest to see how the Swedish standard Bro 2004 deals with dynamic problems in bridges. Finally, there will be a comparison of these four standards and a discussion on the similarities and the differences in vibration criteria and load models.

3.1 BS 5400

The British Standard BS 5400 [28] applies to the design and construction of footbridges. Each of the parts of BS 5400 is implemented by a BD standard, and some of these standards vary certain aspects of the part that they implement. There are two BD standards that relate to the design of footbridges. Design criteria for footbridges are given in BD 29/04 and loads for footbridges are given in BD 37/01 [31].

The BS 5400 standard is one of the earliest codes of practice which dealt explicitly with issues concerning vibrations in footbridges. In BS 5400: Appendix C there is defined a procedure for checking vertical vibrations due to a single pedestrian for footbridges having natural vertical frequencies of up to 5 Hz [28]. Based on experience with lateral vibrations of the London Millennium Bridge, an updated version of BS 5400, BD 37/01 [10], requires check of the vibration serviceability also in the lateral direction. For all footbridges with fundamental lateral frequencies lower than 1,5 Hz a detailed dynamic analysis is now required. However, the procedure for that is not given [38] [25].

The BD 29/04 standard, which deals with design criteria for footbridges, states that the designer should consider the susceptibility of any footbridge to vibrations induced by pedestrians. Particular consideration shall be given to the possibility that the passage of large numbers of people may unintentionally excite the structure into motion. It is noted that designers should be aware that footbridges having modes of oscillation with frequencies less than 5 Hz involving vertical motions of the deck, and/or less than 1,5 Hz involving horizontal motions of the deck, are particularly susceptible to unacceptably large oscillations caused by the passage of large groups of people who may synchronize their walking patterns [9].

The BD 29/04 further states that all footbridges shall satisfy the vibration serviceability requirements set out in BD 37/01: Appendix B5.5. There it is stated that if the fundamental natural frequency of vibration exceeds 5 Hz for the unloaded bridge in the vertical direction and 1,5 Hz for the loaded bridge in the horizontal direction, the vibration serviceability requirement is deemed to be satisfied.

If the fundamental frequency of vertical vibration, on the other hand, is less than, or equal to 5 Hz, the maximum vertical acceleration of any part of the bridge shall be limited to $0,5\sqrt{f_0}$ m/s^2. The maximum vertical acceleration can be calculated either with a simplified method or a general method.

The simplified method for deriving the maximum vertical acceleration given in BD 37/01 is only valid for single span, or two-or-three-span continuous, symmetric, simply supported superstructures of constant cross section. For more complex superstructures, the maximum vertical acceleration should be calculated assuming that the dynamic loading applied by a pedestrian can be represented by a pulsating point load F, moving across the main span of the bridge at a constant speed v_t as follows:

$$F = 180 \sin(2\pi\ f_0\ t)\ [\mathrm{N}] \tag{3.1}$$

$$v_t = 0,9\ f_0\ [\mathrm{m/s}] \tag{3.2}$$

where f_0 is the fundamental natural frequency of the bridge and t is the time.

If the fundamental frequency of horizontal vibration is less than 1,5 Hz, special consideration shall be given to the possibility of excitation by pedestrians of lateral movements of unacceptable magnitude. Bridges having low mass and damping and expected to be used by crowds of people are particularly susceptible to such vibrations. The method for deriving maximum horizontal acceleration is, however, not given [10].

3.2 Eurocode

In EN1990: Basis of Structural Design [11], it is stated that pedestrian comfort criteria for serviceability should be defined in terms of maximum acceptable acceleration of any part of the deck. Also, recommended maximum values for any part of the deck are given, see Table 3.1 [11].

Table 3.1: *Maximum acceptable acceleration, EN1990.*

-	Maximum acceleration
Vertical vibrations	$0,7$ m/s^2
Horizontal vibrations, normal use	$0,2$ m/s^2
Horizontal vibrations, crowd conditions	$0,4$ m/s^2

The standard Eurocode 1: Part 2, defines models of traffic loads for the design of road bridges, footbridges and railway bridges. Chapter 5.7 deals with dynamic models of pedestrian loads. It states that, depending on the dynamic characteristics of the structure, the relevant natural frequencies of the main structure of the bridge deck should be assessed from an appropriate structural model. Further, it states that forces exerted by pedestrians with a frequency identical to one of the natural frequencies of the bridge can result into resonance and need be taken into account for limit state verifications in relation with vibrations. Finally, Eurocode 1 states that an appropriate dynamic model of the pedestrian load as well as the comfort criteria should be defined [12]. The method for modelling the pedestrian loads are, however, left to the designer.

Eurocode 5, Part 2 [13] contains information relevant to design of timber bridges. It requires the calculation of the acceleration response of a bridge due to small groups and streams of pedestrians in both vertical and lateral directions. The acceptable acceleration is the same as in EN1990, $0,7$ and $0,2$ m/s^2 in the vertical and the horizontal directions, respectively. A verification of this comfort criteria should be performed for bridges with natural frequencies lower than 5 Hz for the vertical modes and below 2,5 Hz for the horizontal modes [38]. A simplified method for calculating vibrations caused by pedestrians on simply supported beams is given in Eurocode 5: Annex B [13]. Load models and analysis methods for more complex structures are, on the other hand, left to the designer.

In Eurocode 5, it is also noted that the data used in the calculations, and therefore the results, are subject to very high uncertainties. Therefore, if the comfort criteria are not satisfied with a significant margin, it may be necessary to make provision in the design for the possible installation of dampers in the structure after its completion [13].

3.3 ISO 10137

The ISO 10137 guidelines [16] are developed by the International Organization for Standardization with the objective of presenting the principles for predicting vibrations at the design stage. Also, to assess the acceptability of vibrations in structures [16].

ISO 10137 defines the vibration source, path and receiver as three key issues which require consideration when dealing with the vibration serviceability of structures. The vibration source produces the dynamic forces or actions (pedestrians). The medium of the structure between source and receiver constitutes the transmission path (the bridge). The receiver of the vibrations is then again the pedestrians of the bridge. According to ISO 10137, the analysis of response requires a calculation model that incorporates the characteristics of the source and of the transmission path and which is then solved for the vibration response at the receiver [16].

ISO 10137 states that the designer shall decide on the serviceability criterion and its variability. Further, ISO 10137 states that pedestrian bridges shall be designed so that vibration amplitudes from applicable vibration sources do not alarm potential users. In Annex C, there are given some examples of vibration criteria for pedestrian bridges. There it is suggested to use the base curves for vibrations in both vertical and horizontal directions given in ISO 2631-2 (Figures 3.1 and 3.2), multiplied by a factor of 60, except where one or more persons are standing still on the bridge, in which case a factor of 30 should be applicable. This is due to the fact that a standing person is more sensitive to vibrations than a walking one [38].

However, according to Zivanovic [38], these recommendations are not based on published research pertinent to footbridge vibrations.

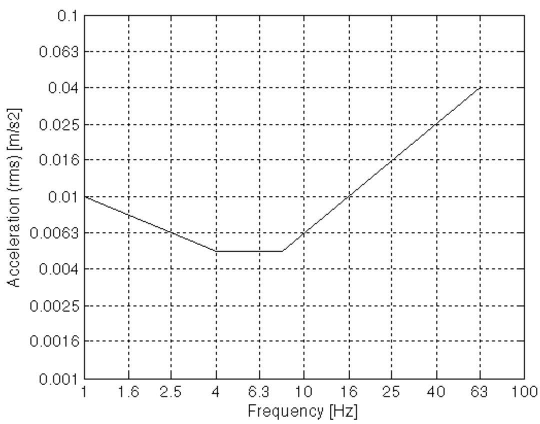

Figure 3.1: Vertical vibration base curve for acceleration

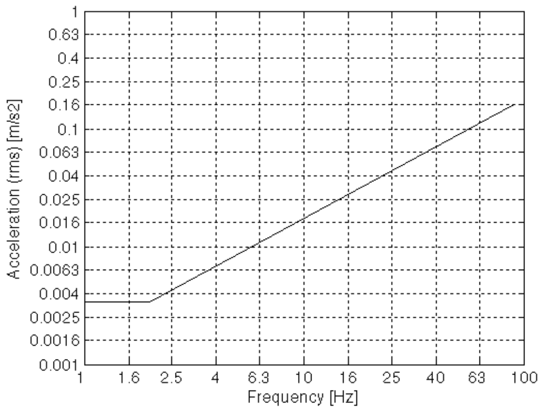

Figure 3.2: Horizontal vibration base curve for acceleration

According to ISO 10137, the dynamic actions of one or more persons can be presented as force-time histories. This action varies with time and position as the persons traverse the supporting structure.

The design situation should be selected depending on the pedestrian traffic to be admitted on the footbridge during its lifetime. It is recommended to consider the following scenarios:

- One person walking across the bridge
- An average pedestrian flow (group size of 8 to 15 people)
- Streams of pedestrians (significantly more than 15 persons)
- Occasional festive of choreographic events (when relevant)

According to ISO 10137: Annex A, the dynamic force $F(t)$ produced by a person walking over a bridge can be expressed in the frequency domain as a Fourier series, Eq. 3.3 and 3.4 [16].

$$F_v(t) = Q(1 + \sum_{n=1}^{k} \alpha_{n,v} \ \sin(2\pi n f t + \phi_{n,v}))) \text{ vertical direction} \tag{3.3}$$

and

$$F_h(t) = Q(1 + \sum_{n=1}^{k} \alpha_{n,h} \ \sin(2\pi n f t + \phi_{n,h}))) \text{ horizontal direction} \tag{3.4}$$

where

$\alpha_{n,v}$ = numerical coefficient corresponding to the n^{th} harmonic, vertical direction
$\alpha_{n,h}$ = numerical coefficient corresponding to the n^{th} harmonic, horizontal dir.

Q = static load of participating person
f = frequency component of repetitive loading
$\phi_{n,v}$ = phase angle of n^{th} harmonic, vertical direction
$\phi_{n,h}$ = phase angle of n^{th} harmonic, horizontal direction
n = integer designating harmonics of the fundamental
k = number of harmonics that characterize the forcing function in the frequency range of interest

Some examples of values for the numerical coefficient α_n are given in ISO 10137: Annex A.

Dynamic action of groups of participants depends primarily on the weight of the participants, the maximum density of persons per unit floor area and on the degree of coordination of the participants.

The coordination can be represented by applying a coordination factor $C(N)$ to the forcing function:

$$F(t)_N = F(t) \cdot C(N) \tag{3.5}$$

where N is the number of participants. For example, if the movements of a group of people are un-coordinated, the coordination factor becomes:

$$C(N) = \sqrt{N}/N \tag{3.6}$$

3.4 Bro 2004

Bro 2004 [4] is a general technical standard, which applies to the design and construction of bridges in Sweden. Bro 2004 is published by the Swedish Road Administration (SRA). The SRA is the national authority assigned the overall sectoral responsibility for the entire road transport system. The SRA is also responsible for the planning, construction, operation and maintenance of the state roads [37].

Bro 2004 states that footbridges should have fundamental frequencies of vertical modes of vibration above 3,5 Hz. Alternatively, the bridge should be checked for vibration serviceability. If any natural frequency of vertical vibration is less, or equal to 3,5 Hz, the root-mean-square vertical acceleration (a_{RMS}) of any part of the bridge shall be limited to $a_{RMS} \leq 0,5$ m/s^2. The vertical acceleration can be calculated from dynamic analysis. The dynamic analysis can be performed either with a simplified method or a general method.

The simplified method given in Bro 2004 is only applicable to simply supported beam bridges. For more complex superstructures, a detailed analysis using handbooks or computer programs is required [4].

The RMS-vertical acceleration should be calculated assuming that the dynamic loading applied by a pedestrian is represented by a stationary pulsating load

$$F = k_1\ k_2\ \sin(2\pi\ f_F\ t)\ [\mathrm{N}] \tag{3.7}$$

where $k_1 = \sqrt{0,1BL}$ and k_2=150 N are loading constants, f_F is the frequency of the load, t is the time, B is the breath of the bridge and L is the length of the bridge between supports.

Bro 2004 speaks only of vertical accelerations and no requirements or precautions regarding horizontal vibrations are set forth in the code [4].

3.5 Comparison

Table 3.2 compares the serviceability criteria set forth in the four standards discussed in this chapter. A comparison of the vertical and the horizontal vibration criteria are presented in Fig. 3.3 and Fig. 3.4 respectively. The ISO 10137 and Bro 2004 curves are obtained by converting the RMS acceleration to the maximum value by multiplying by the factor $\sqrt{2}$.

A comparison of the vertical vibration criteria show that Eurocode and Bro 2004 present a frequency independent maximum acceleration limit of 0,7 m/s^2. For a footbridge with a natural vertical frequency of 2 Hz, which is the mean pacing rate of walking, the BS 5400 criteria also gives $a_{max} \leq 0,5\sqrt{2 \text{ Hz}} = 0{,}7$ m/s^2. ISO 10137 gives, on the other hand, a slightly lower value, $a_{max} \simeq 0,6$ m/s^2.

Table 3.2: *Acceleration criteria.*

Standard	Vertical acceleration	Horizontal acceleration
BS 5400	$a_{max} \leq 0,5\sqrt{f}$ m/s^2	No requirements
Eurocode	$a_{max} \leq 0,7$ m/s^2	$a_{max} \leq 0,2$ m/s^2
ISO 10137	60 times base curve, Figure 3.1	60 times base curve, Figure 3.2
Bro 2004	$a_{RMS} \leq 0,5$ m/s^2	No requirements

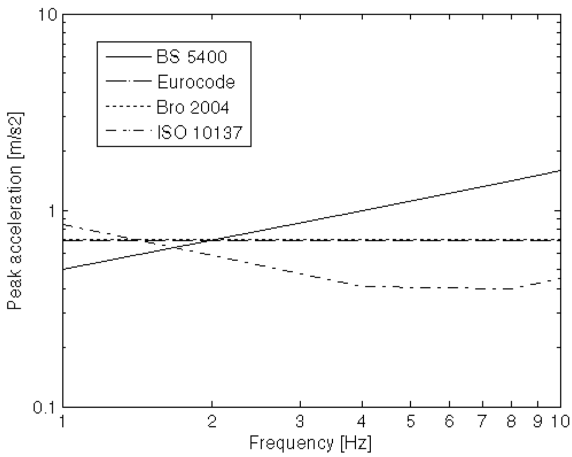

Figure 3.3: Comparison of acceptability of vertical vibration

A comparison of the horizontal vibration criteria show that Eurocode presents a frequency independent maximum acceleration limit of 0,2 m/s^2. ISO 10137 gives a frequency independent maximum acceleration of $a_{max} \simeq 0,31$ m/s^2 up to a frequency of 2 Hz. Neither BS 5400 nor Bro 2004 present a numerical acceleration criteria for horizontal vibration. However, BS 5400 states that if the fundamental frequency of horizontal vibration is less than 1,5 Hz, the designer should consider the risk of lateral movements of unacceptable magnitude.

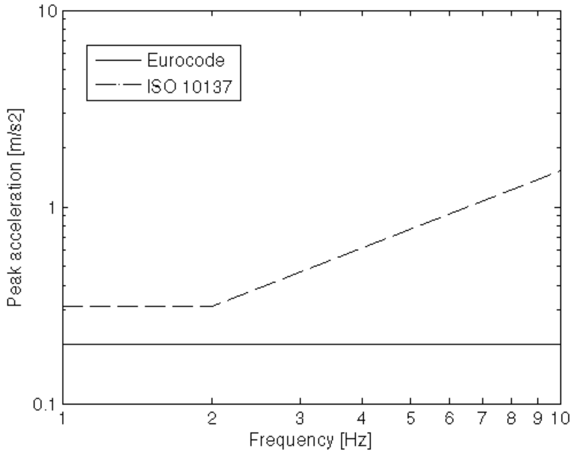

Figure 3.4: Comparison of acceptability of horizontal vibration

The British standard BS 5400 proposes a pedestrian load model only in the vertical direction and not in the horizontal. ISO 10137 models both vertical and horizontal loads imposed by one pedestrian. It is noted that the modelling of the horizontal pedestrian load assumes that the static weight of the pedestrian, Q, acts in the horizontal direction. Eurocode proposes load models for both vertical and horizontal loads only for simplified structures. For more complex structures, the modelling of pedestrian loads are left to the designer. The Swedish standard Bro 2004 proposes a load model for calculations of vertical vibrations. However, it proposes neither a load model nor a design criteria for horizontal vibrations.

The load models proposed by these standards are all based on the assumptions that pedestrian loads can be approximated as periodic loads.

Chapter 4

The London Millennium Bridge

The London Millennium Bridge is a footbridge across the River Thames in London. It was opened on the 10th June 2000 and during the opening day it suffered excessive lateral vibrations induced by pedestrians.

In this chapter, a dynamic analysis of the London Millennium Bridge will be performed. The chapter is divided into three sections. In the first section, there is a general description of the bridge structure. The second chapter describes the finite element modelling of the bridge and the dynamic analysis performed using this model. The last chapter describes a dynamic analysis performed using a SDOF model of the bridge.

4.1 Bridge structure

The London Millennium Bridge is a shallow suspension bridge in three spans, see Figure 4.1. The lengths of the three spans are 81 meters for the north span, 144 meters for the center span between the piers and 108 meters for the south span. The sag of the cable profile is 2,3 meters in the center span, resulting in a sag-span ratio of 1,6:100 (a conventional suspension bridge has a sag-span ratio of 1:10).

The cables form the primary structure of the bridge. They consist of two groups of four 120 mm diameter locked coil cables that span from bank to bank over two river piers, see Figure 4.2. The cables are fixed against translation in any direction at each bank and locked longitudinally at the top of each pier, see Figure 4.3.

As said before, the cable profile is very shallow and such a shallow cable profile results in large cable tensions. According to the designers [15] the deadload of the bridge is 2000 kg/m along the bridge axis and the resulting total dead load cable tension is 22,5 MN.

The bridge deck is 4,0 meters wide and it is made up of extruded aluminium box sections. To support the bridge deck, fabricated steel box sections span between the two cable groups every 8 meters. Then, two steel edge tubes span onto the steel box sections. The aluminium bridge deck spans between these edge tubes on each side. The deck edge members are free to slide in the direction of the bridge span at 8,0 meter intervals but fixed against translation in the other two directions.

Figure 4.1: The London Millennium Bridge [36]

Figure 4.2: There are four cables on each side [36]

The abutments on each bank are stiff in order to limit the live load deflections of the bridge. The river piers are, however, relatively slender which means that the spans interact. The bridge deck is articulated at 16 meter long intervals and therefore it provides no lateral stiffness. Finally, it is concluded, that the structural system both in vertical and horizontal directions derives its stiffness from the cables [7].

4.2 FEM-Model

A dynamic analysis of the London Millennium Bridge was performed using the Finite Element Method. The objective of the analysis was to investigate the response of the bridge structure due to dynamic loads applied by pedestrians.

Figure 4.3: The cables are locked at each pier [36]

In order to analyse the structure dynamically, a 3-dimensional finite element (FE) model of the central span of the Millennium Bridge was established. It was decided only to look at the central span as this was the part of the bridge that experienced the largest vibrations on the opening day, see Chapter 1.2.

This section follows the modelling process as well as the dynamic analysis. First, the selection of software is discussed, then the structural geometry is described, followed by a description of the material and section properties and finally, there is a brief discussion on the boundary conditions. This is then followed by a description of the dynamic analysis and its results. The section ends with a parametric study of the synchronisation of pedestrians.

4.2.1 FEM software

In this thesis the computer software ABAQUS was used for FE modelling and dynamic analysis of the Millennium Bridge. An FE analysis using ABAQUS consists of three distinct stages: preprocessing, simulation and postprocessing.

In the preprocessing stage a model of the structure is established and an ABAQUS input file is created. Here, the model of the Millennium Bridge was created graphically using ABAQUS/CAE which then generates an ABAQUS input file.

The simulation is the stage in which ABAQUS/Standard solves the numerical problem defined in the model. The output from the simulation is stored in a binary file ready for postprocessing.

Once the simulation was completed, postprocessing was performed with ABAQUS/CAE. The results were evaluated using the Visualization module which has a variety of options for displaying the results, including animations, deformed shape plots, and XY plots.

4.2.2 Geometric model

The first step in an FE modelling is to consider how to represent the characteristics of the bridge structure. The FE model of the Millennium Bridge consists of two cables, secondary beams that span between the cables every 8 meters and two edge beams which span onto the secondary beams, see Figure 4.4.

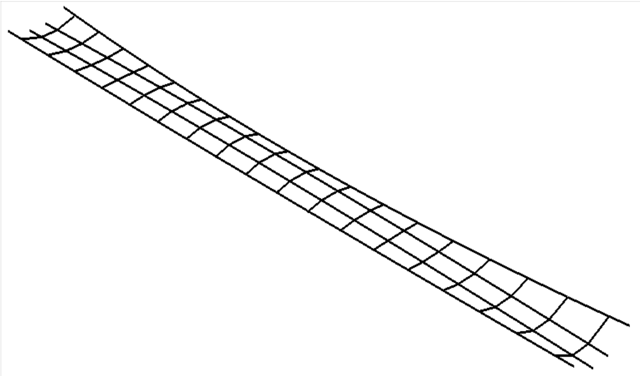

Figure 4.4: FE-model of the center span

The original bridge structure has two groups of four 120 mm diameter cables. However, for the sake of simplicity, the two cable groups of four 120 mm diameter cables have been modelled as two 240 mm diameter cables, which give the same cross-sectional area.

The coordinates of the cables and the deck were taken from structural drawings of the bridge published by the designers [35]. These coordinates were calculated mathematically to the nearest millimeter by the designers. The theoretical geometry is the geometry to be achieved under full dead load.

Deadloads are applied in the FE-model according to data from structural drawings published by the designers [35]. Material density is altered until given weight is achieved. Concentrated forces are used to model weights of cable clamps (cable clamps are used to pin the secondary beams to the cables). The deadloads give a 1,124 m static deflection of the center span when geometric nonlinearity is taken into account. Next, temperature loads are used to model the prestressing of the cables. A difference temperature of $\Delta T = -101,9$ K gives a cable tension of $P = 11,65$ MN and the structure now takes its original form. The tolerances from the coordinates published by the designers are $\Delta x_{max} = 6$ mm.

4.2.3 Material and section model

An important aspect of modelling a structure is the determination of the material and section properties of its components.

All structural elements in the FE model, including secondary beams, edge beams and cables are made of elastic and isotropic steel. The material properties are listed in Table 4.1.

Table 4.1: *Material properties of steel*

Modulus of elasticity	210 GPa
Shear modulus	81 GPa
Poisson ratio	0,30
Thermal expansion coefficient	$12 \cdot 10^{-6}$

The finite elements used for secondary beams and edge beams are 2-node cubic three-dimensional Euler-Bernoulli beams with an average element size of 0,5 m.

The Euler-Bernoulli formulation of beams is based on the assumption that plane sections initially normal to the beam's axis remain plane and normal to the beam axis. Further, it is assumed that the cross-section does not deform in its plane or warp out of its plane. Therefore, Euler-Bernoulli beams provide satisfactory results for slender beams where the beam's cross-sectional dimensions are small compared to distances along its axis [1].

Also, the Euler-Bernoulli beam elements use cubic interpolation functions, which makes them reasonably accurate for cases involving distributed loading along the beam. Therefore, they are well suited for dynamic vibration studies [1].

The section properties of secondary beams, edge beams and cables are listed in Table 4.2. The dimensions of the beams are taken from Arup's structural drawings [35]. The beam elements representing the cables are given a low moments of inertia as the cables have almost no bending stiffness.

Table 4.2: *Section properties*

	Secondary beams	Edge beams	Cables
Cross sectional area	$32,8 \cdot 10^{-3}$ m^2	$7,31 \cdot 10^{-3}$ m^2	$45,2 \cdot 10^{-3}$ m^2
Moment of inertia	$1016 \cdot 10^{-6}$ m^4	$19,6 \cdot 10^{-6}$ m^4	$0,01 \cdot 10^{-6}$ m^4

Finite elements used for modelling the cables are three-dimensional 2-node cubic Euler-Bernoulli beams with hybrid formulation.

The hybrid beam elements in ABAQUS are designed to handle very slender situations, where the axial stiffness of the beam is very large compared to the bending stiffness. This problem arises most commonly in geometrically nonlinear analysis when the beam undergoes large rotations and is very rigid in axial and transverse shear deformation, such as a flexing long cable. The problem in such cases is that slight differences in nodal positions can cause very large forces, which, in turn, cause large motions in other directions. The hybrid elements overcome this difficulty by using a more general formulation in which the axial and transverse shear forces in the elements are included, along with the nodal displacements and rotations, as primary variables. Although this formulation makes these elements more expensive, they generally converge much faster when the beam's rotations are large and, therefore, are more efficient overall in such cases [1].

4.2.4 Boundary conditions

A key for a successful dynamic analysis is a proper modelling of the boundary conditions of the structural system.

As mentioned above (see Section 4.1) the cables are free to rotate, but are fixed against translation in any direction at the river piers. Because the bridge deck is continuous over the piers, there are no restrictions of movements of the deck. The boundary conditions are illustrated in Fig. 4.5.

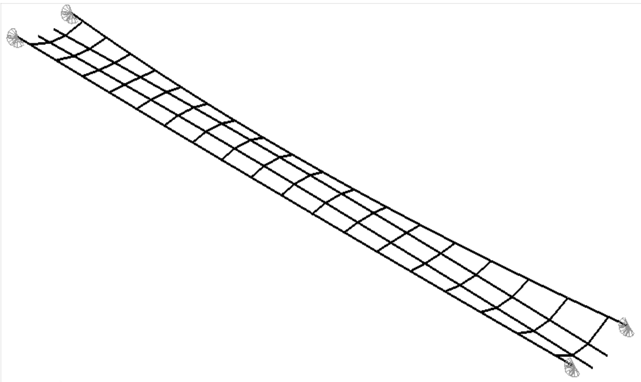

Figure 4.5: Boundary conditions of the FE-model

4.2.5 Dynamic analysis

A frequency extraction procedure was used to determine the first 6 natural frequencies and the corresponding modeshapes of the structure. The frequency extraction procedure in ABAQUS uses eigenvalue techniques to calculate the natural frequencies and the corresponding mode shapes of the structure. The eigenvalue problem for the natural frequencies of an undamped finite element model is described in Section 2.1.2. ABAQUS includes initial stress and load stiffness effects due to preloads when geometric nonlinearity is accounted for in the base state.

Table 4.3: *Frequency extraction*

Nr. of mode	Calculated frequency	Type of mode	Measured frequency
1	0,517 Hz	1st horizontal	0,48 Hz
2	0,570 Hz	1st vertical	-
3	0,794 Hz	2nd vertical	-
4	0,923 Hz	1st torsional	-
5	0,971 Hz	2nd horizontal	0,95 Hz
6	1,241 Hz	2nd torsional	-

The results from the frequency extraction procedure are presented in Table 4.3. The first two modeshapes are shown in Fig. 4.6 and Fig. 4.7 respectively. The modeshapes are close to sinusodal and they tend to only have a significant response in one span, allowing them to be characterised as the first and the second horizontal modes. As can be seen, the calculations for the first two horizontal eigenfrequencies are close to those measured on the real structure.

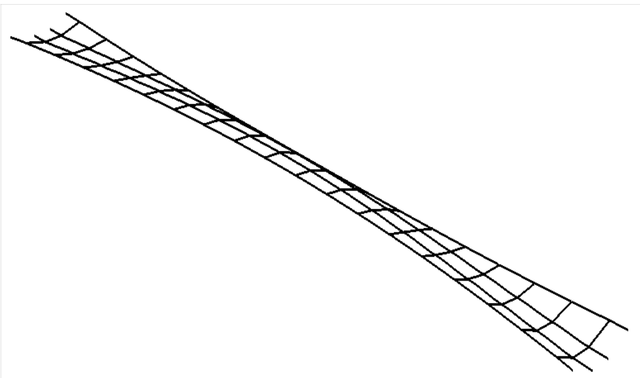

Figure 4.6: 1st horizontal mode, $f_1 = 0,517$ Hz

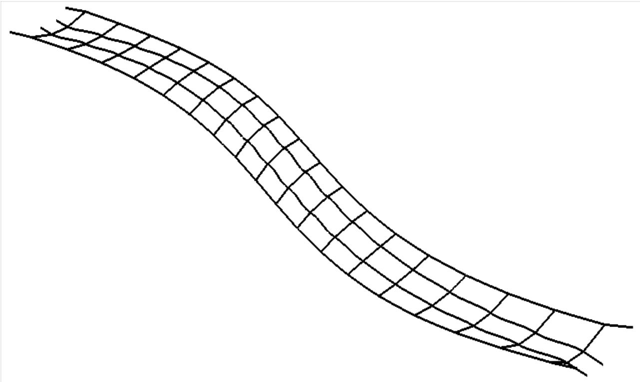

Figure 4.7: 2nd horizontal mode, $f_2 = 0,971$ Hz

The FE-model was also used to calculate the dynamic response of the bridge when subjected to dynamic loading according to the standards BS 5400, ISO 10137 and Bro 2004.

BS 5400

First, the maximum vertical acceleration of the bridge was calculated assuming a pedestrian moving across the main span at a constant speed. The dynamic load applied by the pedestrian was assumed to be represented by a pulsating load $F_p(t)$, moving across the main span of the bridge at a constant speed $v(t)$:

$$F_p(t) = 180 \sin(2\pi \; f_n \; t) \; [\mathrm{N}] \tag{4.1}$$

$$v_t = 0,9 \; f_n \; [\mathrm{m/s}] \tag{4.2}$$

Vertical accelerations were calculated for the first two vertical eigenfrequencies, $f_1 = 0,570$ Hz and $f_2 = 0,794$ Hz.

Next, the maximum horizontal acceleration was calculated assuming a group of 15 people, with no synchronisation, moving at constant speed across the bridge. The dynamic load applied by the group of pedestrians was represented by a pulsating load

$$F_p(t) = \sqrt{n} \; \alpha \; Q \; \sin(2\pi \; f_n \; t) \; = \sqrt{15} \cdot 0,04 \cdot 750\mathrm{N} \cdot \sin(2\pi \; f_n \; t) \tag{4.3}$$

moving across the main span of the bridge at a constant speed

$$v_t = 0,9 \; f_n \; [\mathrm{m/s}] \tag{4.4}$$

Fig. 4.8 shows the acceleration response of the Millennium Bridge subjected to a dynamic loading from a group of 15 people, with no synchronisation, moving across the main span of the bridge at a constant speed $v_t = 0,9 \cdot f_1 = 0,9 \cdot 0,517$ Hz $= 0{,}47$ m/s.

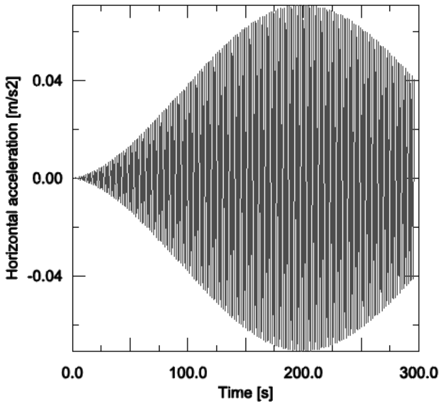

Figure 4.8: Acceleration response, group of non-synchronised people, 1st horizontal natural frequency $f_1 = 0,517$ Hz

Horizontal acceleration was also calculated assuming a fully synchronised group of 15 people. The pulsating load was then assumed to be

$$F_p(t) = n\ \alpha\ Q\ \sin(2\pi\ f_n\ t)\ = 15 \cdot 0,04 \cdot 750\text{N} \cdot \sin(2\pi\ f_n\ t) \tag{4.5}$$

Horizontal accelerations were calculated for the first two horizontal eigenfrequencies, $f_1 = 0,517$ Hz and $f_2 = 0,971$ Hz.

It should be noted that the load models presented in Eq. 4.3 and 4.5 are not taken from the standard BS 5400 as a method for calculating the maximum horizontal acceleration is not given. These load models are merely a proposition from the author of this thesis.

Fig. 4.9 shows the acceleration response of the Millennium Bridge subjected to a dynamic loading from a fully synchronised group of 15 people, moving across the main span of the bridge at a constant speed $v_t = 0,9 \cdot f_2 = 0,9 \cdot 0,971$ Hz = 0,87 m/s.

Table 4.4: *Results from dynamic analysis according to BS 5400*

Nr. of mode	f [Hz]	T / dt [s]	a_{max} [m/s^2]	$a_{criteria}$ [m/s^2]
1st vertical	0,570	266 / 0,01	0,130	0,377
2nd vertical	0,794	294 / 0,01	0,096	0,446
1st horizontal, group of 15	0,517	191 / 0,005	0,071	-
2nd horizontal, group of 15	0,971	157 / 0,005	0,062	-
2nd horizontal, fully synchronised	0,971	157 / 0,005	0,242	-

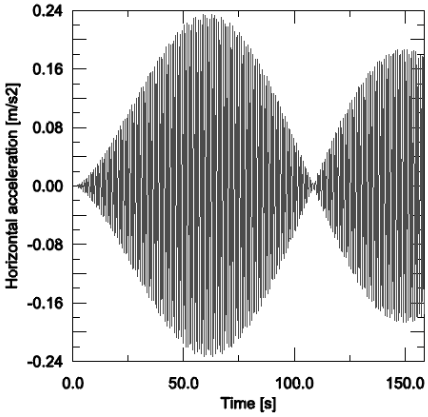

Figure 4.9: Acceleration response, group of fully synchronised people, 2nd horizontal natural frequency $f_2 = 0,971$ Hz

Table 4.4 lists the results from the dynamic analysis according to the British standard BS 5400. The criterium for maximum vertical acceleration is $a_{max} \leq 0,5\sqrt{f}$. No criterium for horizontal acceleration is presented in the BS 5400 standard.

ISO 10137

First, the maximum vertical and horizontal accelerations were calculated under a pulsating point load, $F_p(t)$, representing one pedestrian. The load acted on the point of the bridge that gave the highest response. The vertical load was represented as

$$F_p(t) = Q + \alpha_{n,v} \cdot Q \cdot \sin(2\pi f_{n,v}\ t + \phi_{n,v}) \tag{4.6}$$

and the horizontal load was represented as

$$F_p(t) = \alpha_{n,h} \cdot Q \cdot \sin(2\pi f_{n,h}\ t + \phi_{n,h}) \tag{4.7}$$

where

$\alpha_{1,v} = 0,40$ for the first vertical eigenfrequency $f_{1,v} = 0,570$ Hz
$\alpha_{2,v} = 0,10$ for the second vertical eigenfrequency $f_{2,v} = 0,794$ Hz
$\alpha_{1,h} = 0,10$ for the first horizontal eigenfrequency $f_{1,h} = 0,517$ Hz
$\alpha_{2,h} = 0,10$ for the second horizontal eigenfrequency $f_{2,h} = 0,971$ Hz
$Q = 750$ N is the static pedestrian load
$\phi_{n,v} = \phi_{n,h} = 0$ is the phase angle of the n^{th} harmonic

The results from these calculations are shown in Table 4.5.

Table 4.5: *Dynamic response due to loading from one person*

Nr. of mode	f [Hz]	T / dt [s]	a_{rms} [m/s^2]	$a_{criteria}$ [m/s^2]
1st vertical	0,570	200 / 0,01	0,191	0,300
2nd vertical	0,794	200 / 0,01	0,040	0,300
1st horizontal	0,517	200 / 0,01	0,037	0,310
2nd horizontal	0,971	200 / 0,01	0,040	0,310

Next, the maximum vertical and horizontal accelerations were calculated assuming a uniformly distributed load from a crowded bridge (1,0 persons/m^2) where the pedestrians are not synchronized. The vertical loading was presented as

$$F_p(t) = \sqrt{n}/L \cdot (750 \text{ N/pers} + 300 \text{ N/pers} \cdot \sin(2\pi f t) \tag{4.8}$$
$$= 125 \text{ N/m} + 50 \text{ N/m} \cdot \sin(2\pi f t) \tag{4.9}$$

where n is the number of pedestrians on the bridge and L is the length of the bridge. The horizontal loading was presented as

$$F_p(t) = \sqrt{n}/L \cdot (75 \text{ N/pers} \cdot \sin(2\pi f t) \tag{4.10}$$
$$= 12,5 \text{ N/m} \cdot \sin(2\pi f t) \tag{4.11}$$

The results are shown in Table 4.6.

Table 4.6: *Dynamic response due to loading from a non-synchronised crowd*

Nr. of mode	f [Hz]	T / dt [s]	a_{rms} [m/s^2]	$a_{criteria}$ [m/s^2]
1st vertical	0,570	200 / 0,01	2,667	0,600
2nd vertical	0,794	200 / 0,01	0,395	0,600
1st horizontal	0,517	200 / 0,01	0,549	0,310
2nd horizontal	0,971	200 / 0,01	0,609	0,310

Vertical and horizontal accelerations were also calculated assuming a uniformly distributed load from a crowded bridge (1,0 persons/m^2) where the pedestrians are fully synchronized. The vertical load in this case was modelled as

$$F_p(t) = n/L \cdot (750 \text{ N/pers} + 300 \text{ N/pers} \cdot \sin(2\pi f t) \tag{4.12}$$
$$= 3000 \text{ N/m} + 1200 \text{ N/m} \cdot \sin(2\pi f t) \tag{4.13}$$

where n is the number of pedestrians on the bridge and L is the length of the bridge. The horizontal loading was presented as

$$F_p(t) = n/L \cdot (75 \text{ N/pers} \cdot \sin(2\pi f t) \tag{4.14}$$
$$= 300 \text{ N/m} \cdot \sin(2\pi f t) \tag{4.15}$$

The results are shown in Table 4.7.

It should be noted that the load models presented in Eq. 4.8, 4.10, 4.12 and 4.14 are not taken from the standard ISO 10137. These load models are merely a proposition from the author of this thesis.

Table 4.7: *Dynamic response due to loading from a fully synchronised crowd*

Nr. of mode	f [Hz]	T / dt [s]	a_{rms} [m/s^2]	$a_{criteria}$ [m/s^2]
1st vertical	0,570	200 / 0,01	64,0	0,600
2nd vertical	0,794	200 / 0,01	9,48	0,600
1st horizontal	0,517	200 / 0,01	13,2	0,310
2nd horizontal	0,971	200 / 0,01	14,6	0,310

Bro 2004

The maximum vertical accelerations of the bridge were calculated using a stationary pulsating force $F_p(t)$. The force was placed where it gave the highest response. The force was defined as

$$F_p(t) = k_1\ k_2\ \sin(2\pi\ f_n\ t) = 1.140\ \text{N} \cdot\ \sin(2\pi\ f_n\ t) \tag{4.16}$$

where $k_1 = \sqrt{0,1BL} = \sqrt{0,1 \cdot 4\text{m} \cdot 144\text{m}} = 7,6$ and $k_2 = 150$ N are loading constants, f_n is the n^{th} natural frequency of the bridge and t is the time. The acceleration response was calculated for the 5 first vertical eigenfrequencies.

Fig. 4.10 shows the acceleration response of the central span of the Millennium Bridge subjected to a dynamic load according to the Swedish standard Bro 2004, Eq. 4.16. Table 4.8 lists the results from the dynamic analysis. The criterium for maximum vertical acceleration is $a_{rms} \leq 0,5$ m/s^2. No criterium for horizontal acceleration is presented in the Bro 2004 standard.

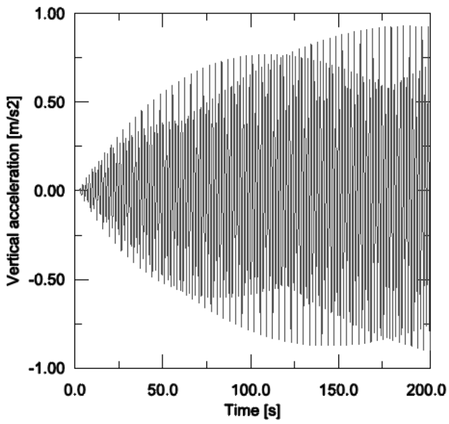

Figure 4.10: Acceleration response, 1st vertical natural frequency $f_1 = 0,570$ Hz

Table 4.8: *Results from dynamic analysis according to Bro 2004*

Nr. of mode	f [Hz]	T / dt [s]	a_{rms} [m/s^2]	$a_{criteria}$ [m/s^2]
1st vertical	0,570	200 / 0,05	0,528	0,500
2nd vertical	0,794	200 / 0,05	0,528	0,500
3rd vertical	1,323	200 / 0,05	0,407	0,500
4th vertical	1,728	200 / 0,05	0,470	0,500
5th vertical	2,149	200 / 0,05	0,398	0,500

Conclusions

A frequency extraction procedure was performed using a three-dimensional FE-model of the Millennium Bridge established in ABAQUS. The calculated eigenfrequencies are close to those measured for the bridge. The calculated values for the first two horizontal eigenfrequencies are 7% and 2% higher than their measured values respectively (see Table 4.3). One characteristic of an FE-model is that the calculated natural frequencies converge from above to the exact solution. In order to provide a more accurate calculations the element size has to be smaller. However, improved accuracy comes at the expense of increased computational costs [5].

The British standard BS 5400 requires check of vibration serviceability in both vertical and horizontal directions. A load model for calculating vertical vibration is proposed in the standard but the modelling of horizontal loads is left the the designer.

The FE-model of the Millennium Bridge was used to calculate vibrations due to the vertical dynamic load proposed by BS 5400. It was found that vertical vibrations did not exceed serviceability criteria set forth in the standard. Acceleration for the two first vertical eigenfrequencies were 34% and 22% of accepted values respectively, indicating that the bridge is not vulnerable to vertical vibrations (see Table 4.4).

The author of this thesis proposed a load model for horizontal load imposed by a group of 15 people traversing the bridge. When assumed that the group was fully synchronised, the maximum horizontal acceleration was calculated to be 0,24 m/s^2 (see Table 4.4). This exceeds the serviceability criteria proposed by Eurocode, $a_{max} \leq 0,2$ m/s^2. It is, however, not likely that a group of 15 people become totally synchronised. Therefore, this model seems to be incapable of modelling the phenomenon of pedestrian-induced vibrations.

The standard ISO 10137 proposes dynamic load models for calculation of vertical and horizontal vibrations. These models are based on the assumption that the load imposed by one pedestrian is periodic. When these loads were used to excite the FE-model of the Millennium Bridge it resulted in vibrations that were only 20% of the acceptance criteria (see Table 4.5).

The author of this thesis then tried to generalize the proposed load model for one pedestrian as a load model for a crowded bridge. When the load imposed by one pedestrian was multiplied by the square root of the number of pedestrians, the calculated response exceeded the serviceability criteria (see Table 4.6). For example, the maximum acceleration for the second horizontal frequency was calculated to be 0,61 m/s^2. This exceeds the criteria given by $a_{max} \simeq 0,31$ m/s^2. Therefore, these calculations indicate that the Millennium Bridge could be vulnerable to horizontal loads imposed by a crowd of pedestrians.

When the load imposed by one pedestrian was multiplied by the number of pedestrians, the calculated response became very high (see Table 4.7). This case was used as an extreme as it is highly unlikely that a crowd of more than 500 people become fully synchronised.

The fact that the Millennium Bridge showed good serviceability when subjected to loads according to BS 5400 and ISO 10137 indicates that the load models proposed by these standards are not adequate to detect a structures vulnerability to horizontal pedestrian-induced vibrations.

The Swedish standard Bro 2004 proposes neither a load model nor a design criteria for horizontal vibrations. However, it requires a check for vertical vibrations. The calculated vertical vibrations exceeded the design criteria for the first two natural frequencies (see Table 4.8). Therefore, according to the Swedish standard, the Millennium Bridge is likely to suffer from large vertical vibrations. It is also probable, that if the Millennium Bridge had been designed according to Bro 2004, some measures to improve the serviceability of the bridge would have been made. However, the Swedish standard Bro 2004 is unable to detect a structures vulnerability to horizontal vibrations.

4.2.6 Parameter studies

In this section, a parameter study made in order to investigate the influence of pedestrian density and the level of synchronisation of pedestrians on the response of the bridge is presented.

The approach of this study was to first vary the pedestrian density, from 0 to 1,5 persons per square meter, and calculate the corresponding maximum acceleration response of the Millennium Bridge. Then, the percentage of synchronised pedestrians was varied from 0% to 100% and the corresponding maximum acceleration response calculated.

It was assumed that the pedestrian loading could be represented as pulsating uniformly distributed load. Further, it was assumed that 4% of the vertical static pedestrian load works laterally when walking and that the pulsating of the load can be represented with the sine-function. The static vertical pedestrian load was assumed to be $Q = 750$ N. The maximum acceleration was calculated for the second horizontal mode, with an eigenfrequency of $f_2 = 0,971$ Hz. The pedestrian load then became:

$$F_p(t) = s \cdot d \cdot B \cdot \alpha \cdot Q \cdot \sin(2\pi\ f_2\ t) \quad [\mathrm{N/m}] \tag{4.17}$$

where s is the percentage of pedestrian synchronisation, d is the density of pedestrians, $B = 4$ m is the breadth of the bridge and $\alpha = 0,04$ is the horizontal load factor.

First, the pedestrian density was varied and the pedestrian synchronisation was kept constant. Then the synchronisation level was varied while the pedestrian density was kept constant.

Results and conclusions

Fig. 4.11 shows the maximum accelerations for different pedestrian densities.

Figure 4.11: Effect of density of pedestrians on bridge response

Fig. 4.12 shows the maximum accelerations for different levels of synchronisation of pedestrians.

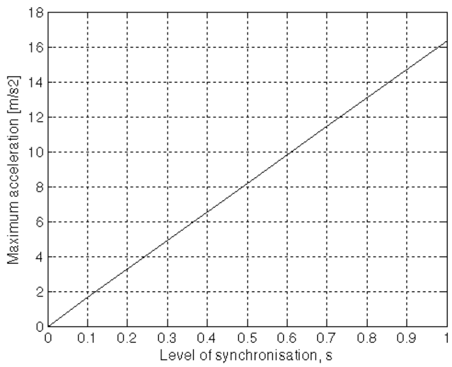

Figure 4.12: Effect of pedestrian synchronisation on bridge response

Fig. 4.11 and 4.12 show that the maximum response of the bridge increases linearly with both pedestrian density and pedestrian synchronisation. This is because the input load $F_p(t)$ is assumed to be linearly proportional to both number of pedestrians as well as their synchronisation.

For example, the maximum horizontal deflection of the Millennium Bridge at the opening day was found to be 70 mm, see Section 1.2. According to the results of this parametric study, this corresponds to a density of pedestrians equal to 1,3 pers/m^2, a load factor for each pedestrian equal to $\alpha = 0,05$ and a pedestrian synchronisation equal to 20%. Alternatively, this correspondes to a density of pedestrians equal to 1 pers/m^2, a load factor equal to $\alpha = 0,04$ and a pedestrian synchronisation of 33%.

This is, however, not in accordance with tests performed on the Millennium Bridge. These tests were described in Section 2.2.3 and Fig. 2.6 shows that relationship between the bridge response and the number of pedestrians is highly non-linear [15].

4.3 SDOF-Model

In this section the Millennium Bridge is modelled as a generalized SDOF system. The objective is to investigate the response of the bridge structure due to dynamic loads applied by pedestrians. This section will focus on the second horizontal modeshape of the central span of the Millennium Bridge. This is because on the opening day, the largest movements of the bridge took place on the central span at a frequency of 0,95 Hz, the second horizontal modeshape.

It is assumed that the central span of the Millennium Bridge can be approximated as a simple beam with distributed mass and stiffness. It is known (see Section 2.1.1) that such a system can deflect in an infinite variety of shapes. However, the deflections of the beam can be restricted to a single shape function that approximates the vibration mode.

Because the first modeshapes of the Millennium Bridge are close to sinusodal it was decided to use the following shape function

$$\psi(x) = \sin \frac{2\pi x}{L} \tag{4.18}$$

This function is plotted in Fig. 4.13 and can be compared with the second horizontal mode shown in Fig. 4.14.

Figure 4.13: The function $\phi(x) = \sin \frac{2\pi x}{L}$

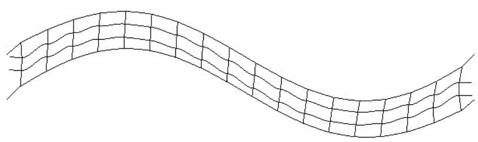

Figure 4.14: The second horizontal modeshape of the Millennium Bridge

The equation of motion for a generalized SDOF-system is

$$\tilde{m}\ddot{z} + \tilde{c}\dot{z} + \tilde{k}z = \tilde{f}(t) \tag{4.19}$$

where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$ and $\tilde{f}(t)$ are defined as the generalized mass, generalized damping, generalized stiffness and generalized force of the system.

The generalized mass is calculated using the following expression

$$\tilde{m} = \int_0^L m(x)[\psi(x)]^2 \, dx = \frac{mL}{2} = \frac{2234 \text{ kg/m} \cdot 144 \text{ m}}{2} = 160.848 \text{ kg} \tag{4.20}$$

where $m(x)$ is mass of the structure per unit length and L is the length of the structure.

The natural frequency of the second horizontal modeshape is given by

$$\omega_2 = 2\pi \cdot f_2 = \sqrt{\frac{\tilde{k}}{\tilde{m}}} \tag{4.21}$$

and by assuming that the second horizontal frequency is $f_2 = 0,971$ Hz (as calculated with ABAQUS, see Section 4.2.5) the stiffness of the structure can be derived as

$$\omega_2 = 2\pi \cdot f_2 = \sqrt{\frac{\tilde{k}}{\tilde{m}}} = 6,098 \Rightarrow \tilde{k} = 5.981.639 \text{ kg/s}^2 \tag{4.22}$$

Damping is expressed by a damping ratio ζ. The estimated ratio used for the design of the Millennium Bridge was $\zeta = 0,005$. After opening of the Millennium Bridge, the damping ratio for each mode was measured at around 0,006 to 0,008. Therefore, a damping ratio of $\zeta = 0,006$ is used here. Once the damping ratio has been established, the generalized damping can then be calculated from the expression

$$\tilde{c} = \zeta \; (2\tilde{m}\omega) = 0,006 \cdot (2 \cdot 160.848 \cdot 6,098) = 11.771 \text{ kg/s} \tag{4.23}$$

where ω is the natural frequency of the structure.

Having determined the generalized properties $\tilde{m}$, $\tilde{c}$ and $\tilde{k}$ the last thing to do before the equation of motion can be solved is to define the dynamic loading. In the next section, three attempts to model pedestrian loading are presented.

When a mathematical expression for the dynamic loading has been established, the equation of motion (Eq. 4.19) is solved for $z(t)$ using the central difference method described in Section 2.1.1. Then, the displacements at all times and at all locations of the system are determined from $u(x,t) = \psi(x)z(t)$ [5]. A routine for solving the equation of motion using the central difference method was written in Matlab, see Appendix A.

4.3.1 Dynamic analysis

The generalized SDOF-model established in the last section is now used to calculate the dynamic response of the Millennium Bridge when subjected to dynamic loading. Three different load models are used. First, the pedestrian load is approximated as a periodic load. This is done in order to compare the solution provided by the SDOF and MDOF-models respectively. Next, a load model proposed by Dallard et al (see Section 2.3.2) is used to calculate the bridge response. Finally, a load model proposed by Nakamura (see Section 2.3.3) is used to calculate the bridge response.

Periodic load

It is assumed that the pedestrian loading can be represented as pulsating uniformly distributed load. Further, it is assumed that the static vertical pedestrian load is $Q = 750$ N, the horizontal load factor is 4%, the density of pedestrians is 1 pers/m^2, the breadth of the bridge is $B = 4$ m, the pedestrian synchronisation is $s = 0,33$ and that the pulsating of the load can be represented with the sine-function.

The horizontal pedestrian load then becomes

$$f(t) = s \cdot d \cdot B \cdot \alpha \cdot Q \cdot \sin(2\pi\ f_2\ t) = 39,6 \text{ N/m} \cdot \sin(2\pi\ f_2\ t) \tag{4.24}$$

and the generalized force of the system is calculated as

$$\tilde{f}(t) = \int_0^{L/2} f(t)\psi(x)\ dx = f(t) \cdot \frac{L}{\pi} = 1815 \text{ N/m} \cdot \sin(2\pi\ f_2\ t) \tag{4.25}$$

Now, the equation of motion, Eq. 4.19 is solved with a Matlab routine (see Appendix A) and the results are shown in Fig. 4.15. Fig. 4.16 shows the displacement response of the bridge due to the same force but calculated with an MDOF-model.

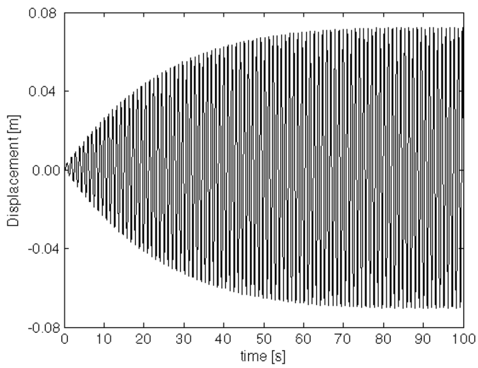

Figure 4.15: Displacement response calculated with the SDOF-model

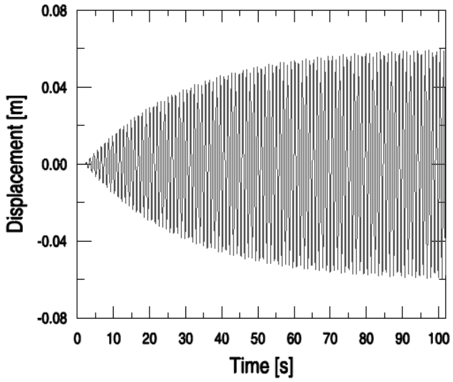

Figure 4.16: Displacement response calculated with the MDOF-model

As can be seen, the displacement histories calculated with the SDOF and the MDOF-model respectively have similar characteristics. However, the maximum displacement calculated with the SDOF-model is somewhat higher than the displacement calculated with the MDOF-model. The values are approximately $u_{max} = 0,072$ m for the SDOF-model and $u_{max} = 0,060$ m for the MDOF-model, a difference of 17%.

Dallard's load model

As described in Section 2.3.2, Dallard et al described the pedestrian force generated by n people uniformly distributed on the span as

$$\tilde{f}(t) = \frac{1}{L}\int_0^L \phi^2 \; nk\dot{u} \; dx = \frac{1}{2}nk \; \dot{u}(t) \tag{4.26}$$

where the value of k has to be estimated for each case. Based on the field tests conducted on the London Millennium Bridge, k was found to be 300 Ns/m in the lateral frequency range 0,5 - 1,0 Hz.

For example, for a pedestrian density of 1 pers/m^2, the force becomes

$$\tilde{f}(t) = \frac{1}{2}nk \; \dot{u}(t) = \frac{1}{2} \cdot 576 \cdot 300 \cdot \dot{u}(t) = 86.400 \cdot \dot{u}(t) \tag{4.27}$$

If the excitation force $\tilde{f}(t) = 86.400\dot{u}(t)$ is compared to the generalized damping force $\tilde{c} = 11.771\dot{u}(t)$ it is obvious that the solution of the equation of motion will not be stable. That is, the bridge response will increase very rapidly towards infinity. Even the smallest initial disturbance will, after some time, cause very large displacements of the bridge. This model is therefore not useful to calculate the response of a structure subjected to a dynamic force. It can, however, be used to evaluate a design criteria for required damping and limiting number of people. This was done in Section 2.3.2.

The required damping ratio was found out to be

$$\zeta > \frac{1}{L} \int_0^L \phi^2 \, \frac{nk}{4\pi f \tilde{m}} \, dx \tag{4.28}$$

where L is the length of the bridge, n is the number of pedestrians traversing the bridge and $\tilde{m}$ is the generalized mass of the bridge.

Similarly, the limiting number of people for a given damping ratio was found out to be

$$n_L = \frac{4\pi\zeta f \tilde{m}}{k \frac{1}{L} \int_0^L \phi^2 dx} \tag{4.29}$$

For the SDOF-model of the Millennium Bridge the required damping is calculated as

$$\zeta > \frac{nk}{8\pi f \tilde{m}} = \frac{576 \cdot 300}{8\pi \cdot 0,971 Hz \cdot 160848} = 0,044 \tag{4.30}$$

In the same way, for the damping ratio $\zeta = 0,006$ the limiting number of people is

$$n_L = \frac{8\pi\zeta f \tilde{m}}{k} = \frac{8\pi \cdot 0,006 \cdot 0,971 Hz \cdot 160848}{300} = 78 \text{ pedestrians} \tag{4.31}$$

Nakamura's load model

Section 2.3.3 describes a load model proposed by Nakamura in 2004. According to Nakamura, the horizontal dynamic force induced by pedestrians can be mathematically modelled as

$$\tilde{f}(t) = k_1 k_2 \, \frac{\dot{u}(t)}{k_3 + \mid \dot{u}(t) \mid} \, G(f_B) \; M_P \; g \tag{4.32}$$

where the coefficient k_1 is a ratio of the lateral force to the pedestrian's weight. The coefficient k_2 is the percentage of pedestrians who synchronize to the girder vibration. $M_P g$ is the modal self weight of pedestrians. $G(f_B)$ is a function to describe how pedestrians synchronize with the bridge's natural frequency. The worst case scenario is when $G(f_B) = 1,0$.

The displacement and the velocity of the Millennium Bridge are simulated using the dynamic load proposed by Nakamura. The equation of motion is solved using the central difference method, which has been implemented in a Matlab routine, see Appendix A.

When applying the load, it is assumed that the horizontal load factor is $k_1 = 0,04$, the pedestrian synchronisation is $k_2 = 0,33$, the static vertical pedestrian load is $Q = 750$ N and the density of pedestrians is 1,5 pers/m^2. The modal self weight of pedestrians can be calculated as

$$M_P g = \frac{144 \text{ m}}{2} \cdot 1,5 \text{ pers/m}^2 \cdot 4 \text{ m} \cdot 75 \text{ kg/pers} \cdot g = 32.400 \text{ N} \tag{4.33}$$

The horizontal pedestrian load then becomes

$$\tilde{f}(t) = 428 \cdot \frac{\dot{u}(t)}{0,01+ \mid \dot{u}(t) \mid} \tag{4.34}$$

The initial velocity is set at $\dot{u}_0 = 0,001$ m/s and the initial displacement at $u_0 = 0$ m.

Fig. 4.17 shows the simulated displacement of the Millennium Bridge and Fig. 4.18 shows the simulated acceleration of the Millennium Bridge.

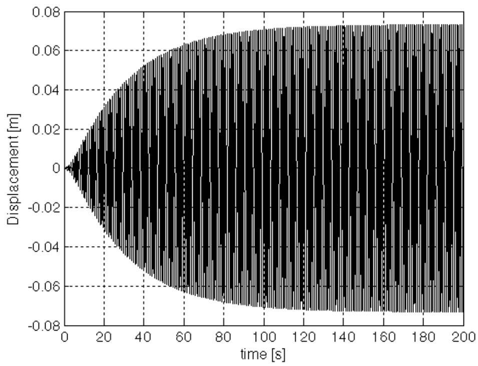

Figure 4.17: Bridge displacements

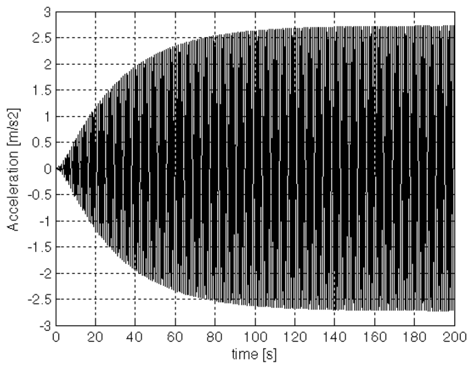

Figure 4.18: Bridge accelerations

Fig. 4.19 shows the horizontal force as a function of the velocity response of the bridge.

As can be seen, the force increases linearly with low velocities but the increase rate becomes smaller at higher velocities.

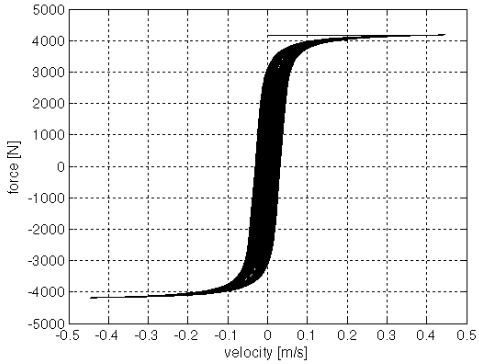

Figure 4.19: Lateral force as a function of bridge velocity

4.3.2 Parameter studies

In this section, a parameter study is performed. The objective is to investigate the influence of several parameters on the response of the bridge. The parameters studied are mass of the girder, structural damping, density of pedestrians and pedestrian synchronisation.

The approach of this study is to calculate the bridge response while one parameter is varied and all the others are kept constant. The load model proposed by Nakamura is used in this study and as before the equation of motion is solved using the central difference method.

Mass of girder

The girder mass is varied and the acceleration response of the bridge is calculated. The stiffness is also varied to keep the natural frequency constant. It is assumed that the horizontal load factor is $k_1 = 0,04$, the pedestrian synchronisation is $k_2 = 0,35$, the static vertical pedestrian load is $Q = 750$ N and the density of pedestrians is 1,0 pers/m^2. The initial velocity is set at $\dot{u}_0 = 0,001$ m/s and the initial displacement at $u_0 = 0$ m.

Fig. 4.20 shows the effect of the bridge mass on the maximum acceleration response.

Damping

The damping ratio, ζ, is varied and the acceleration response of the bridge is calculated. All other parameters are kept constant. It is assumed that the horizontal load factor is $k_1 = 0,04$, the pedestrian synchronisation is $k_2 = 0,35$, the static vertical pedestrian load is $Q = 750$ N and the density of pedestrians is 1,0 pers/m^2. The initial velocity is set at $\dot{u}_0 = 0,001$ m/s and the initial displacement at $u_0 = 0$ m.

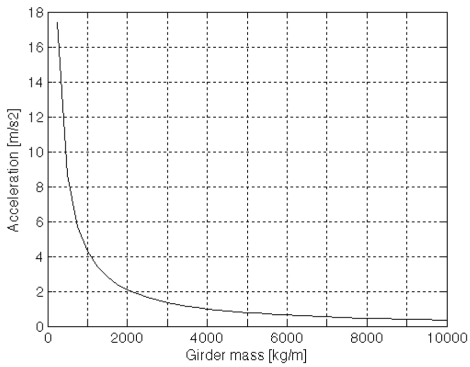

Figure 4.20: Effect of bridge mass on bridge response

Fig. 4.21 shows the effect of bridge damping on the maximum acceleration response.

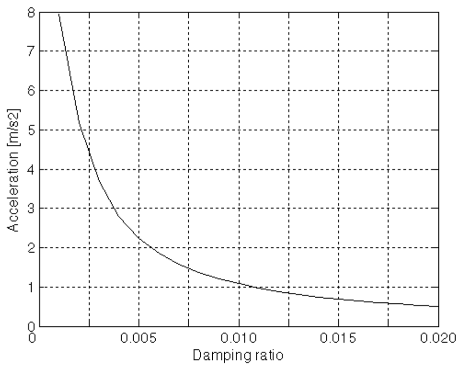

Figure 4.21: Effect of bridge damping on bridge response

Density of pedestrians and pedestrian synchronisation

First, the pedestrian density is varied and the pedestrian synchronisation is kept constant. As before, it is assumed that the horizontal load factor is $k_1 = 0,04$, the pedestrian synchronisation is $k_2 = 0,35$ and the static vertical pedestrian load is $Q = 750$ N. The initial velocity is set at $\dot{u}_0 = 0,001$ m/s and the initial displacement at $u_0 = 0$ m.

Then the synchronisation level is varied while all other parameters are kept constant.

Fig. 4.22 and Fig. 4.23 show the effect of pedestrian density and pedestrian synchronisation on the maximum acceleration response respectively.

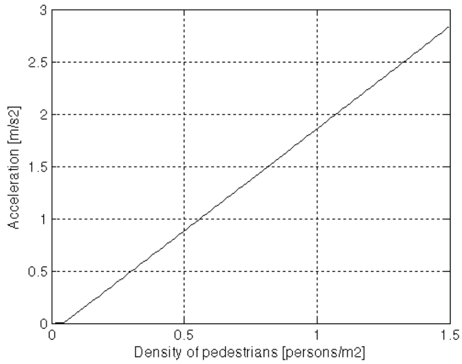

Figure 4.22: Effect of pedestrian density on bridge response

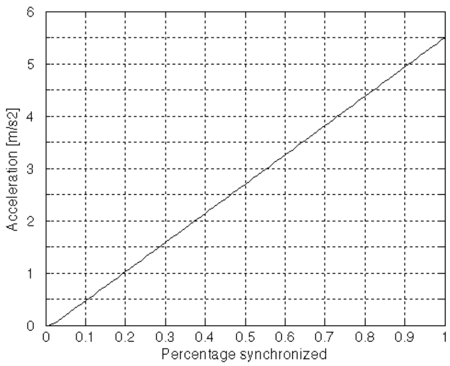

Figure 4.23: Effect of pedestrian synchronisation on bridge response

Conclusions

From Fig. 4.20 it is obvious that lighter bridge structures give higher response to dynamic actions. The bridge mass per unit length for the London Millennium Bridge is approximately 2000 kg/m^2 and it can be read from Fig. 4.20 that to reduce the response by 50% the bridge mass need to be increased to 4000 kg/m^2.

As can be seen in Fig. 4.21, lower damping ratio gives higher response of the bridge structure. The damping ratio, ζ, of the London Millennium Bridge is about 0,006 and it can be read from Fig. 4.21 that the girder response can be halved if the bridge damping ratio is increased to 0,012.

Fig. 4.22 and 4.23 show that after the dynamic force exceeds the bridge damping force, the maximum acceleration response of the bridge increases linearly with both pedestrian density and pedestrian synchronisation. It is noted that this is not at all in accordance with the test performed on the Millennium Bridge, see Section 2.2.3, where Dallard et al found the relationship between the bridge response and the density of pedestrians to be highly non-linear, see Fig. 2.6.

Chapter 5

Solutions and design guidelines

New highly stressed materials, more slender structures, smaller cross-sectional areas and greater spans of footbridges have led to decreasing structural stiffness, damping and mass. Decreasing stiffness leads to smaller natural frequencies resulting in more sensitivity to vibrations. Also, decreased mass and damping means that smaller forces are required to excite the structure and to sustain the vibrations. Further, these changes have moved frequencies of structures into bands which are more easily perceived by users. As a conclusion, footbridges must be designed for dynamic as well as static loads and a dynamic analysis should be a part of the early design stages.

The past years, several examples of excessive vibrations of footbridges have been reported, see Section 1.2. These vibrations have almost all been a matter of serviceability rather than overstressing or fatigue of materials. It has been stated that these excessive vibrations have been caused by resonance between pedestrian loading and a natural vibration frequency of the structure.

Resonance can cause a magnification of the dynamic response which can lead to structural instability, overstressing or fatigue failure of materials. An effort should therefore be made to avoid resonance. Resonance can be avoided by "tuning" the lowest natural frequency of the structure out of the frequency range of the dynamic pedestrian force. Where resonance cannot be avoided, it may be possible to reduce the dynamic response by increasing structural damping.

This chapter deals with both solutions to vibration problems as well as design guidelines for vibration serviceability of footbridges. First, several measures to improve vibration serviceability, including frequency tuning and increased structural damping, are introduced. Then there is a discussion on the most commonly used design guidelines for footbridge vibration serviceability. Finally, an effort is made to improve dynamic design procedures for footbridges.

5.1 Possible solutions

In this section, some solutions to vibration problems due to pedestrian loading are introduced. These solutions include frequency tuning by increasing structural stiffness and measures to increase structural damping.

5.1.1 Increase stiffness

One possible solution to vibration problems due to pedestrian loading is to avoid natural frequencies which are in ranges coinciding with frequencies typical for human-induced dynamic loading. As mentioned in Section 2.2, these frequencies are in the range of 1,4 - 2,4 Hz for the vertical direction and in the range of 0,7 - 1,2 Hz for the horizontal direction. This can be achieved by increasing the stiffness of the footbridge thus moving all its natural frequencies out of the range that can be excited by pedestrians. The stiffness can for example be increased by installing stiffer handrails or adding tie-down cables [38].

In theory, to double the natural frequency of a structure, its stiffness must be increased by a factor of four without increasing the mass at the same time. If the mass increases, the stiffness has to be increased even more. In addition, increasing stiffness is often difficult without considerable structural additions that almost always affect the aesthetics of the bridge [15].

As a conclusion, stiffening is best applied to bridges when the lowest natural frequency of the structure is close to the upper limit of acceptable frequencies [29]. On the other hand, if the lowest natural frequency is a few times lower than the acceptable frequency, stiffening of the structure is not a favourable option.

5.1.2 Increase damping

Another measure against vibration problems of footbridges is to increase the overall damping of the structure. There are several energy absorption mechanisms that contribute to the damping of a structure. For small amplitudes of vibration, damping is mainly provided by material damping due to the viscoelastic behaviour of the material. For higher amplitudes, damping is increased by friction in connections and supports. Also, non-structural elements (pavements and railings) may contribute to the overall damping [32].

Increasing the damping by modifying the structure, connections, supports and non-structural elements may be considered, but often considerable practical problems arise. To increase the damping, it is far more effective, and less expensive, to install a damping system [2].

Damping systems increase the amount of energy that is dissipated by the structure. In this section, three different and commonly used damping systems will be considered. Tuned mass dampers (TMD) can be tuned to specific frequencies and damp out one mode. An alternative to TMDs are tuned liquid dampers (TLD), which are relatively inexpensive and easy to install. Finally, viscoelastic dampers can be added to cover a wider range of frequencies and motions [3].

Tuned mass dampers

A tuned mass damper (TMD) is a passive damping system consisting of a mass and a spring attached to a single point on the bridge. By varying the ratio of the TMD mass to the mass of the bridge, a certain amount of damping can be produced. The TMD can be viewed as an energy sink, where excess energy that is built up in the bridge is transferred to the TMD mass. The energy is then dissipated by some form of viscous damping device that is connected between the bridge and the TMD mass itself [3]. In this way, the natural frequency of the TMD is tuned to one particular frequency resulting in an optimum frequency of the damper. Therefore, TMDs are only effective over a narrow band of frequencies. Also, the smaller the ratio between the mass of the TMD and the mass of the structure, the narrower will be the band of effective frequencies [15].

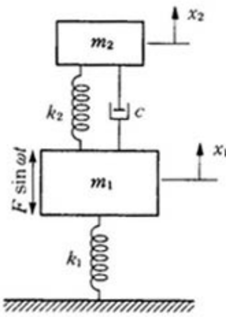

Figure 5.1: TMD attached to an SDF system

The TMD together with the bridge structure can be analysed as a two-degree-of-freedom system, see Fig. 5.1. The lower mass represents the structure while the upper mass models the TMD. The size of the TMD's stiffness and mass depends on the acceptable dynamic response of the structure. The higher the damper mass relative to the structure mass, the lower is the dynamic response. However, for practical reasons the damper mass has an upper limit [32].

Vertical damping of the London Millennium Bridge is provided primarily by vertical tuned mass dampers. After the opening day, see Section 1.2, a total of 26 pairs of vertical TMDs were installed on the Millennium Bridge. This was done although the bridge did not respond excessively in the vertical direction on the

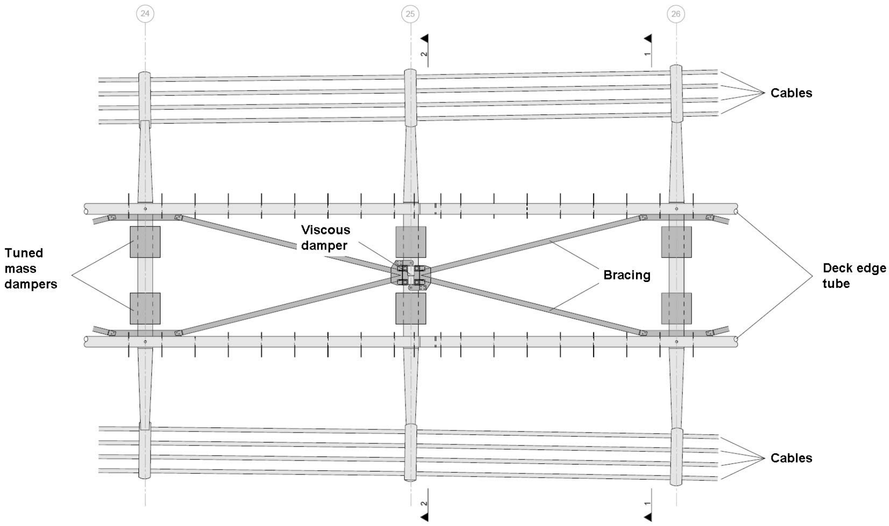

Figure 5.2: Plan of the deck showing placement of TMD and viscous dampers [15]

opening day. The TMDs consist of masses of between 1000 and 3000 kg supported on compression springs and they are situated on top of the transverse arms beneath the deck, see Fig. 5.2 and 5.3. The TMDs are arranged along the length so that they are at or close to the antinodes of the modes that they are damping [7].

Tuned liquid damper

A tuned liquid damper (TLD) is a sloshing type of damper. It consists of a plastic box, filled with water, which is then placed on the bridge. The required height of the liquid is established by nonlinear shallow-water wave theory. The breaking of waves and the viscosity of the water dissipate the vibration energy and generate the required damping. This tuned liquid damper is cost-effective, easy to install and maintain and requires a very low vibration level to which it will respond, which is sometimes a problem with standard mechanical TMDs [38].

Fig. 5.4 shows an idealized model of a TLD attached to a bridge structure. The fundamental frequency of the TLD, according to linear theory, is

$$f = \frac{1}{2\pi}\sqrt{\frac{\pi g}{2L}\tanh\frac{\pi H}{2L}} \tag{5.1}$$

where $2L$ is the length of the box and H is the water height in the box. This value can be used for preliminary design. For more accurate design, numerical and experimental investigations are needed [32].

Figure 5.3: TMD beneath the deck

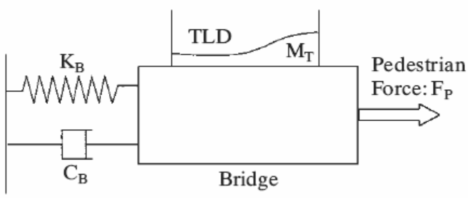

Figure 5.4: TLD attached to an SDOF system [21]

Tuned liquid dampers were used to suppress lateral vibrations of the T-bridge in Japan (see Section 1.2). 600 plastic containers each with size of 360 mm x 290 mm and water depth of 34 mm were placed inside the box girder. Nakamura and Fujino reported that these TLDs were very effective at the time of installation. Ten years after the installation, however, some of the water in the boxes had evaporated resulting in a reduced effectiveness [20].

Viscous dampers

Viscous dampers add energy dissipation to the bridge structure. A fluid viscous damper dissipates energy by pushing fluid through an orifice, producing a damping pressure which creates a force. The fluid viscous damper shown in Fig. 5.5 is similar in action to the shock absorber on an automobile, but it operates at a much higher force level and it is significantly larger than an automobile damper. The construction of a fluid viscous damper is shown in Fig. 5.5. It consists of a cylinder and a piston with orifice head. The cylinder is filled with silicone oil. The piston transmits energy entering the system to the fluid in the damper, causing it to move within the damper. The movement of the fluid within the damper absorbs the kinetic energy

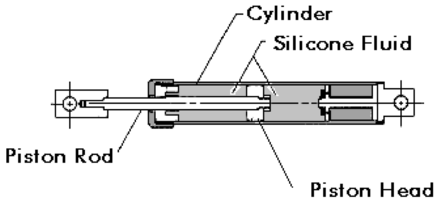

Figure 5.5: Fluid viscous damper [6]

by converting it into heat. This means that the bridge deck protected by dampers will undergo considerably less horizontal movement during applied dynamic loading [6].

Horizontal damping of the London Millennium Bridge is provided primarily by viscous dampers. After the opening of the bridge, see Section 1.2, 37 fluid viscous dampers were installed on the London Millennium Bridge mostly to suppress excessive vibrations in the lateral direction, see Fig. 5.2. As a result, the damping ratio increased from 0,5% to 20% and near-resonant accelerations were reduced by about 40 times [38].

Most of the viscous dampers are situated on top of the transverse arms every 16 meters, beneath the deck, see Fig. 5.2. Each end of the viscous damper is connected to the apex of a steel V bracing. The apex of the bracing is supported on roller bearings that provide vertical support but allow sliding in all directions. The other ends of the bracing are fixed to the neighbouring transverse arms. In this way the horizontal modal movement over 16 meters is mobilised at each damper [15].

5.2 Improved design procedures

The most commonly used design guidelines for footbridge vibration serviceability can be divided into two categories. One requires that natural frequencies which are in range coinciding with frequencies typical for human-induced dynamic loading are to be avoided. The other requires a calculation of the actual dynamic response of the bridge and checking if it is within the acceptable limits for the bridge users [38].

Resonance can be avoided by tuning the structure out of the frequency range typical for human-induced dynamic loading. The procedure for this was introduced in Section 5.1.1. There it was also noted that it is difficult to significally increase the stiffness without affecting the aesthetics of the bridge. Another drawbacks of the frequency tuning approach are the difficulties to accurately calculate the natural frequencies of a structure during the design stage [25].

It should also be noted here, that several perfectly serviceable footbridges have natural frequencies coinciding with frequencies typical for human-induced dynamic loading [25].

Therefore it is concluded, that rather than avoiding certain natural modes and frequencies, the dynamic response of the bridge deck caused by dynamic pedestrian loading, should be calculated and compared with design criteria [30].

Three things need to be done in order to calculate the dynamic response of a footbridge and check if it is within the acceptable limits for the bridge users. First, the dynamic properties of the footbridge must be known. This has been dealt with in Chapter 4. Secondly, the acceptable level of vibrations must be defined. In Chapter 3 vibration criteria for footbridges were taken from four widely used standards. Last but not least, a simple and accurate model of the dynamic pedestrian forces is required.

All pedestrian load models presented in codes of practice (Chapter 3) are based on the assumption that forces induced by pedestrians are perfectly periodic and that there is a linear relationship between the number of pedestrian traversing the bridge and the intensity of the dynamic pedestrian force.

The calculations performed in Section 4.2.5 indicate that load models based on periodic pedestrian actions are incapable of predicting the phenomenon of pedestrian-induced vibrations. Further, the assumption of a linear relationship between number of pedestrians and intensity of the dynamic pedestrian force is not in accordance with tests performed on the London Millennium Bridge. It is therefore concluded that modelling of dynamic forces induced by groups and crowds of pedestrians need to be better defined.

As discussed in the previous section, excessive pedestrian-induced vibrations and pedestrian synchronisation can be avoided by increasing the overall damping of the structure. But how much damping is needed.

In a paper published in 2002, McRobie et al. [19] suggest that a structures sensitivity to pedestrian-induced vibrations can be measured by a non-dimensional Scruton number, analogous to the Scruton number of wind engineering. In wind engineering the Scruton number is used to study the vortex-induced oscillation amplitudes.

McRobie et al. further define the pedestrian Scruton number as

$$S_p = k_1 k_2 m \tag{5.2}$$

where $k_1 = \zeta/0,005$ is the ratio of the fraction of critical damping to that of a typical mode in a lightly damped bridge structure, $k_2 = 0,6/n$ takes account of crowds of density n greater than 0,6 pers/m^2 and m is the mass per unit deck area [19]. Because a lack of data, McRobie et al. could not propose a limit for the pedestrian Scruton number.

In 2003, Newland [22] defined the pedestrian Scruton number as

$$S_p = 2\zeta M/m \tag{5.3}$$

where ζ is the modal damping ratio, M is the modal mass of the bridge and m is the modal mass of the pedestrians. Newland further concluded that for vibration serviceability the following requirement should be fulfilled

$$S_p > \alpha\beta \tag{5.4}$$

where $\alpha = 2/3$ is the ratio of movement of a person's centre-of-mass to movement of the bridge deck and $\beta = 0,4$ is the level of synchronisation of pedestrians.

For example, the pedestrian Scruton number for the second horizontal natural frequency of the London Millennium Bridge is

$$S_p = 2 \cdot 0,006 \cdot 160.848/32.400 = 0,060 < \alpha\beta = 0,267 \tag{5.5}$$

The Scruton number is to low and if the requirement is to be fulfilled the modal damping ratio need to be

$$S_p = 2 \cdot \zeta \cdot 160.848/32.400 > 0,267 \Rightarrow \zeta > 0,027 \tag{5.6}$$

As previously mentioned (see Section 2.3.2), Dallard et al. [15] proposed a primary design check requirement that the damping should exceed

$$\zeta > \frac{1}{L}\int_0^L \phi^2 \, \frac{nk}{4\pi f M} \, dx \tag{5.7}$$

For example, the required modal damping ratio for the second horizontal natural frequency of the Millennium Bridge is calculated as

$$\zeta > \frac{nk}{8\pi f M} = \frac{576 \cdot 300}{8\pi \cdot 0,971 Hz \cdot 160.848} = 0,044 \tag{5.8}$$

It is concluded that Eq. 5.7 can be used as a primary design requirement. However, more data from existing lively footbridges is needed to confirm this requirement.

Chapter 6

Conclusions

6.1 Summary

The main focus of this thesis was on the vertical and horizontal forces that pedestrians impart to a footbridge and how these loads can be modelled to be used in the dynamic design of footbridges. The work was divided into four subtasks:

- First, a literature study of dynamic loads induced by pedestrians was performed.
- Next, design criteria and load models proposed in international, European, British and Swedish standards were introduced and a comparison was made.
- Dynamic analysis of the London Millennium Bridge was performed as well as a parameter study of parameters such as pedestrian synchronisation, bridge mass and structural damping.
- Finally, available solutions to vibration problems and improvements of design procedures were studied.

The next section lists the conclusions for each subtask.

6.2 Conclusions

Pedestrians induce both vertical and horizontal dynamic loads on the structure they traverse. The frequency range of the vertical force is 1,4 - 2,4 Hz and 0,7 - 1,2 Hz for the horizontal force.

Existing vibration limits presented in standards are most probably sufficient to prevent vertical synchronisation between structure and pedestrians. However, observations indicate that horizontal synchronisation can start when the amplitude of the footbridge vibration is only a few millimeters.

Pedestrian induced vibrations are a subject of serviceability. It was therefore assumed, that structures respond linearly to applied pedestrian loads and that dynamic response can be found by solving the equation of motion.

The British standard BS 5400 requires a check of vibration serviceability in both vertical and horizontal directions. However, it only proposes a load model and a design criteria for vertical vibrations. The load modelling and the evaluation of a design criteria for horizontal vibrations are left to the designer.

The standard ISO 10137 proposes load models for calculation of vertical and horizontal vibrations due to one pedestrian. It also proposes design criteria for vertical and horizontal vibrations. It does not, however, take into account the phenomenon of pedestrian synchronisation.

Eurocode proposes load models for both vertical and horizontal loads only for simplified structures. For more complex structures, the modelling of pedestrian loads are left to the designer. Eurocode proposes frequency independent maximum acceleration limits both for vertical and horizontal vibrations.

The Swedish standard Bro 2004 proposes neither a load model nor a design criteria for horizontal vibrations. However, it requires a check for vertical vibrations and it proposes a load model as well as a design criteria for the calculation of vertical vibration serviceability.

The load models proposed by the above mentioned standards are all based on the assumptions that pedestrian loads can be approximated as periodic loads. They also seem to be incapable of predicting a structures sensitivity to excessive horizontal vibrations due to a crowd of pedestrians.

Dynamic analysis of the London Millennium Bridge according to BS 5400 showed good serviceability in the vertical direction. An attempt to model the horizontal load imposed by a totally synchronised group resulted in accelerations that exceeded the criteria proposed in Eurocode (as no criteria for horizontal vibrations are presented in BS 5400).

Dynamic analysis of the London Millennium Bridge according to ISO 10137 showed good serviceability in the vertical direction. An attempt to model the horizontal load imposed by an unsynchronised crowd resulted in accelerations that exceeded the criteria proposed by the standard.

Dynamic analysis according to Bro 2004 indicated that the London Millennium Bridge could be sensitive to large vertical vibrations. However, it was unable to detect the structures vulnerability to horizontal vibrations.

A dynamic analysis using an SDOF modelling of the London Millennium Bridge showed similar results as the analysis performed with an MDOF model. It is therefore concluded that a simple SDOF model can be adequate to perform a dynamic analysis of a structure. An accurate MDOF model is however needed in order to predict the natural frequencies and their corresponding vibration modes.

Lighter bridge structures and lower damping ratios result in higher responses to dynamic actions.

Natural frequencies which are in range coinciding with frequencies typical for human-induced dynamic loading can be avoided by increasing structural stiffness. Increasing stiffness can be an expensive measurement and will almost always have negative effects on the aesthetics of the structure.

A more effective way to solve vibration problems is to increase damping by installing a damping system. Several damping systems can be used to increase overall damping. The most commonly used are tuned mass dampers (TMD), tuned liquid dampers (TLD) and viscoelastic dampers.

Several formulas applicable in footbridge design have been set forth in order to calculate the amount of damping required to solve vibration problems. Some are based on a non-dimensional pedestrian Scruton number, analogous to the Scruton number of wind engineering. The pedestrian Scruton number is a measurement of the ratio between the mass of the bridge and the mass of the pedestrians relative to the damping ratio. However, more data from existing lively footbridges is needed to find what values of the Scruton number are sufficient for acceptable serviceability.

The trend in footbridge design over the last years, which was described in the very beginning of this thesis, has led to several cases of excessive vibrations of footbridges due to pedestrian-induced loading. It is the hope of the author of this thesis, that rather than reversing this trend, these problems will lead to improved design of footbridges in the future.

6.3 Future work

Based upon the work described in this thesis, some suggestions for future work within this field are noted.

- Investigate the level of pedestrian synchronisation as a function of the amplitude and frequency of bridge motion
- Quantify the horizontal load factor of pedestrian load as a function of the amplitude and frequency of bridge motion
- Develop a relatively simple and accurate mathematical model for horizontal pedestrian loads, which can be used in dynamic design of footbridges.

List of Tables

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Bibliography

[1] ABAQUS Version 6.5 Documentation. *ABAQUS Analysis User's Manual*. ABAQUS Inc, 2004.

[2] Bachmann, H. Lively Footbridges a Real Challenge. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 18-30.

[3] Breukelman, B. *Damping Systems*. RWDI Technotes, issue no. 10. www.rwdi.com.

[4] *Bro 2004*. Vägverkets allmänna tekniska beskrivning för nybyggande och förbättring av broar. Svensk Byggtjänst, Stockholm, Sverige.

[5] Chopra, A.K. (2001). *Dynamics of Structures, Theory and Applications to Earthquake Engineering*. Prentice Hall. Second Edition. New Jersey, USA.

[6] Constantinou, M. Application of Fluid Viscous Dampers to Earthquake Resistant Design. *Research Accomplishments, 1986-1994*. Buffalo: National Center for Earthquake Engineering Research, September 1994, pages 73-80.

[7] Dallard, P., Fitzpatrick, A.J., Flint, A., Le Bourva, S., Low, A., Ridsdill Smith, R.M. and Willford, M. The London Millennium Bridge. *The Structural Engineer* Volume 79/No 22, November 2001, pages 17-33.

[8] Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Ridsdill Smith, R., Willford, M. and Roche, M. London Millennium Bridge: Pedestrian-Induced Lateral Vibration. *ASCE Journal of Bridge Engineering*, November-December 2001, pages 412-417.

[9] Design Manual for Road and Bridges: Design Criteria for Footbridges: BD 29/04, Highway Agency, London, February, 2004.

[10] Design Manual for Road and Bridges: Loads for Highway Bridges: BD 37/01, Highway Agency, London, February, 2002.

[11] Eurocode, Basis of Structural Design - prAnnex A2. EN 1990: 2002. European Committee for Standardization, Brussels, Belgium 2002.

[12] Eurocode 1, General Actions Traffic loads on bridges. EN 1991-2: 2003. European Committee for Standardization, Brussels, Belgium 2003.

[13] Eurocode 5, Design of Timber Structures Part 2: Bridges, EN 1995-2: 2004, European Committee for Standardization, Brussels, Belgium 2004.

[14] Eyre, J. Aesthetics of footbridge design. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 96-103.

[15] Fitzpatrick, T., Dallard, P., Le Bourva, S., Low, A., Ridsdill Smith, R. and Willford, M. *Linking London: The Millennium Bridge*. The Royal Academy of Engineering. June 2001.

[16] ISO, Bases for design of structures Serviceability of buildings and pedestrian walkways against vibration, ISO/CD 10137, International Stadardization Organization, Geneva, Switzerland, 2005.

[17] Kreuzinger, H. Dynamic design strategies for pedestrian and wind actions. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 129-141.

[18] Maguire, J.R. & Wyatt, T.A. (2002). *Dynamics. An introduction for civil and structural engineers.* Thomas Telford. Second Edition. London, UK.

[19] McRobie, A., Morgenthal, G., Lasenby, J. and Ringer, M. Section model tests on human-structure lock-in. *Proceedings of the Institution of Civil Engineers, Bridge Engineering*. 2003.

[20] Nakamura, S-I. Lateral vibration on a pedestrian cable-stayed bridge. *IABSE Journal of Structural Engineering International*, volume 12, no. 4, 2002, pages 295-300.

[21] Nakamura, S-I. Model for Lateral Excitation of Footbridges by Synchronous Walking. *ASCE Journal of Structural Engineering*, January 2004, pages 32-37.

[22] Newland, D.E. Pedestrian Excitation of Bridges Recent Results. *Proceedings of the tenth International Congress on Sound and Vibration*, Stockholm 2003.

[23] Pavic, A. and Reynolds, P. *Modal testing of a 34m catenary footbridge*. Vibration Engineering Section, Department of Civil and Structural Engineering, The University of Sheffield, UK. 2001.

[24] Pimentel, R., Fernandes, H. A Simplified Formulation for the Vibration Serviceability of Footbridges. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 142-143.

[25] Pimentel, R.L., Pavic, A., Waldron, P. Evaluation of design requirements for footbridges excited by vertical forces from walking. *Canadian Journal of Civil Engineering* Volume 28, 2001, pages 769-777.

[26] Roberts, T.M. Probabilistic pedestrian lateral excitation of bridges. *Proceedings of the Institution of Civil Engineers.* Bridge Engineering 158, June 2005, pages 53-61.

[27] Roberts, T.M. Synchronised pedestrian excitation of footbridges. *Proceedings of the Institution of Civil Engineers.* Bridge Engineering 156, December 2003, pages 155-160.

[28] Steel, Concrete and Composite Bridges Part 2: Specification for Loads; Appendix C: Vibration Serviceability Requirements for Foot and Cycle Track Bridges, BS 5400. UK: British Standards Association, London, 1978.

[29] Stoyanoff, S. & Hunter, M. *Footbridges: Pedestrian induced vibrations.* RWDI Technotes, issue no. 15. www.rwdi.com.

[30] Strasky, Jiri (2005). *Stress ribbon and cable-supported pedestrian bridges.* Thomas Telford. London, UK.

[31] *The design of steel footbridges.* Corus Construction Centre. www.corusgroup.com.

[32] Weber, B. Damping of Vibrating Footbridges. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 196-207.

[33] Willford, M. Dynamic actions and reactions of pedestrians. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 66-74.

[34] Wilson, Ed. *Dynamic analysis by numerical integration.* Computers & Structures. www.csiberkeley.com.

[35] www.arup.com/milleniumbridge. The Millennium Bridge. 2005-03-18.

[36] en.structurae.de/index.cfm. Structurae: International Database and Gallery of Structures. 2005-06-20.

[37] www.vv.se. Vägverkets hemsida. 2005-09-19.

[38] Zivanovic, S., Pavic, A., and Reynolds, P. Vibration serviceability of footbridges under human-induced excitation: a literature review. *Journal og Sound and Vibration* 279 (2005).

Appendix A

Matlab files

An example of a Matlab file used for solving the equation of motion for a SDOF-model with the central difference method. In this case, load is applied using Nakamura's load model. The horizontal load factor is $k_1 = 0,04$, the pedestrian synchronisation is $k_2 = 0,33$, the pedestrian mass is $Q = 75$ kg and the density of pedestrians is 1,5 pers/m^2.

```
%------------------------------------------------------------
% MB_cdm.m
% Solve a single-degree-of-freedom dynamic model.
% Load according to Nakamura.
% The load is a function of velocity.
% Central difference method used to solve equation of motion.
%
% Fjalar Hauksson,
% 2005-09-16
%------------------------------------------------------------


% 1. Clear variables

clear all
close all
clc


% 2. Properties of SDOF-model

m=160848;        % modal mass [kg]
k=5981639;       % modal stiffness [kg/s2]
c=11771;         % equivalent damping [kg/s]

dt=0.01;         % time increment [s]
T=200;           % total time of analysis [s]
n=T/dt;          % number of time increments

k1=0.04;         % ratio of force to pedestrian weight
k2=0.33;         % percentage of pedestrians who synchronize
```

```
k3=0.01;          % saturation coefficient
dp=1.5;           % density of pedestrians [pers/m2]
Q=75;             % pedestrian mass [kg/pers]
B=4;              % deck width [m]
L=144;            % length of bridge [m]
Mp=dp*B*Q*L/2;    % modal mass of pedestrians [kg]
g=10;             % acceleration due to gravity [m/s2]


% 3. Initial values

u0=0;                                  % initial displacement
udot0=0.001;                           % initial velocity
p0=k1*k2*udot0/(k3+abs(udot0))*Mp*g;   % initial load
u2dot0=(p0-c*udot0-k*u0)/m;            % initial acceleration
u_1=u0-dt*udot0+dt^2*u2dot0/2;         % displacement at time i=-1

u=zeros(n+2,1);                        % displacement vector
u(1,1)=u_1;
u(2,1)=u0;

udot=zeros(n+2,1);                     % velocity vector
udot(1,1)=udot0;

p=zeros(n+2,1);                        % load vector
p(1,1)=p0;

kstrik=m/(dt^2)+c/2/dt;                % integration constants
a=m/(dt^2)-c/2/dt;
b=k-2*m/(dt^2);

t=zeros(n+2,1);                        % time vector
t(1,1)=-dt;
for i=2:n+2
    t(i,1)=t(i-1,1)+dt;
end


% 4. Calculations for each time step i

for i=2:n+1
    pstrik=p(i,1)-a*u(i-1,1)-b*u(i,1);
    u(i+1,1)=pstrik/kstrik;
    udot(i,1)=(u(i+1,1)-u(i-1,1))/(2*dt);
    p(i+1,1)=k1*k2*udot(i,1)/(k3+abs(udot(i,1)))*Mp*g;
    u2dot(i,1)=(u(i+1,1)-2*u(i,1)+u(i-1,1))/(dt^2);
end

u2dot(n+2,1)=0;


% 5. Plot displacement, velocity and acceleration
```

```
figure(1)
plot(t,u)
axis([0 max(t) min(u)+min(u)/10 max(u)+max(u)/10]);
title('displacement')
xlabel('time [s]')
ylabel('displacement [m]')

figure(2)
plot(t,udot)
axis([0 max(t) min(udot)+min(udot)/10 max(udot)+max(udot)/10]);
title('velocity')
xlabel('time [s]')
ylabel('velocity [m/s]')

figure(3)
plot(t,u2dot)
axis([0 max(t) min(u2dot)+min(u2dot)/10 max(u2dot)+max(u2dot)/10]);
title('acceleration')
xlabel('time [s]')
ylabel('acceleration [m/s2]')

figure(4)
plot(t,p)
axis([0 max(t) min(p)+min(p)/10 max(p)+max(p)/10]);
title('exciting force')
xlabel('time [s]')
ylabel('force [N]')

figure(5)
plot(udot,p)
title('velocity vs. force')
xlabel('velocity [m/s]')
ylabel('force [N]')

% end
```

Appendix B

ABAQUS files

An example of an ABAQUS input file used for dynamic analysis of the London Millennium Bridge. In this case, vertical dynamic load is applied according to the British standard BS 5400. The load is assumed to be represented by a pulsating load $F_p(t) = 180 \sin(2\pi\ f_n\ t)$ [N], moving across the main span of the bridge at a constant speed $v(t) = 0,9\ f_n$ [m/s].

```
**----------------------------------------------------
**----------------------------------------------------
*Heading
** Job name: BS5400_050826e Model name: Preload
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
**----------------------------------------------------
**
** PARTS
**
**----------------------------------------------------
**
*Part, name=Cable
*End Part
**
**----------------------------------------------------
**
** ASSEMBLY
**
**----------------------------------------------------
**
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**
*Instance, name=Cable-1, part=Cable
*Node
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      2,   213.623871,          -2.,   12.8005829
      3,   217.223465,          -2.,   12.6656313
         .
         .
         .
   1107,    89.477272,           2.,   14.0905113
   1108,   90.1081009,           2.,   14.1019726
   1109,     90.66008,           2.,   14.1118441
**
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2, 153,   2
3,   2, 154
```

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   .
   .
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1158, 1108, 1109
1159, 1109,  152
**
*Element, type=B33H
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39, 183, 184
40, 184, 185
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   .
   .
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1077, 1042,  137
1152, 151, 143
**
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653, 693, 694
654, 694,  83
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  352,  353,  354,  355,  373,  374,  375,  376,  377,  378,  392,  393,  394,  395,  396,  397
  398,  399,  400,  401,  402,  403,  404,  405,  406,  407,  408,  409,  410,  411,  421,  422
  423,  424,  425,  426,  435,  436,  437,  438,  439,  440,  441,  442,  443,  444,  445,  473
  474,  475,  476,  477,  478,  479,  480,  481,  482,  483,  484,  485,  486,  504,  505,  506
  507,  508,  509,  510,  511,  512,  513,  514,  515,  516,  517,  518,  519,  520,  521,  522
  523,  524,  525,  526,  527,  528,  529,  550,  551,  552,  553,  554,  555,  556,  574,  575
  576,  577,  578,  579,  580,  581,  582,  583,  584,  585,  586,  587,  588,  589,  590,  591
  612,  613,  614,  615,  616,  617,  618,  619,  620,  621,  622,  623,  624,  625,  626,  627
  643,  644,  645,  646,  647,  648,  649,  650,  651,  652,  653,  654,  684,  685,  686,  687
  688,  689,  713,  714,  715,  716,  717,  718,  719,  720,  721,  722,  723,  724,  725,  726
  727,  748,  749,  750,  751,  752,  753,  754,  755,  756,  757,  758,  783,  784,  785,  786
  787,  788,  789,  790,  791,  792,  793,  794,  795,  796,  797,  798,  806,  807,  808,  809
  810,  832,  833,  844,  853,  854,  855,  856,  857,  858,  859,  860,  861,  862,  863,  864
  865,  866,  867,  868,  869,  877,  878,  879,  880,  881,  882,  896,  897,  898,  899,  900
  901,  902,  903,  904,  905,  906,  907,  908,  909,  910,  911,  912,  913,  914,  915,  916
  917,  918,  934,  935,  936,  937,  938,  939,  940,  941,  942,  943,  944,  945,  946,  947
  948,  949,  950,  951,  952,  953,  954,  955,  956,  957,  958,  959,  960,  961,  993,  994
  995, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1048, 1049, 1050, 1051, 1066, 1067, 1068, 1069
 1070, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081, 1082, 1083, 1092, 1093, 1094, 1095
 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1129, 1130
 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146
 1147, 1148, 1149, 1150, 1151, 1153, 1154, 1155, 1156, 1157, 1158, 1159
** Region: (DeckSection:Deck)
** Section: DeckSection  Profile: DeckProfile
*Beam General Section, elset=Deck, poisson = 0.3, density=25098., section=PIPE
0.1615, 0.016
0.,0.,-1.
 2.1e+11, 8.1e+10
** Region: (TransArms:TransArms)
** Section: TransArms  Profile: TransProfile2
*Beam General Section, elset=TransArms, poisson = 0.33, density=7176., section=GENERAL
0.032756, 1., 0.001, 1., 0.005237
0.,0.,-1.
 7.2e+10, 2.6e+10
** Region: (CableSection:Picked), (Beam Orientation:Picked)
*Elset, elset=_PickedSet21, internal
   38,   39,   40,   41,   42,   43,   44,   45,   46,   47,   48,   49,   50,   74,   75,   76
   77,   78,   79,   80,   81,   82,   83,   84,   85,   86,  108,  109,  110,  111,  112,  113
  114,  115,  116,  117,  118,  119,  120,  121,  122,  123,  124,  125,  126,  198,  199,  200
  201,  202,  203,  204,  205,  206,  207,  208,  209,  210,  221,  222,  237,  238,  239,  240
  241,  242,  243,  244,  245,  246,  247,  248,  249,  250,  251,  260,  261,  270,  302,  303
  305,  306,  307,  308,  309,  310,  311,  312,  313,  314,  315,  316,  317,  327,  328,  363
  364,  379,  380,  381,  382,  383,  384,  385,  386,  387,  388,  389,  390,  391,  412,  413
  460,  461,  462,  463,  464,  465,  466,  467,  468,  469,  470,  471,  472,  495,  496,  537
  538,  539,  540,  541,  542,  543,  544,  545,  546,  547,  548,  549,  565,  566,  599,  600
  601,  602,  603,  604,  605,  606,  607,  608,  609,  610,  611,  628,  629,  630,  631,  632
  633,  634,  635,  636,  637,  638,  639,  640,  641,  642,  698,  699,  700,  701,  702,  703
  704,  705,  706,  707,  708,  709,  710,  711,  712,  735,  736,  737,  738,  739,  740,  741
  742,  743,  744,  745,  746,  747,  767,  768,  811,  812,  813,  814,  815,  816,  817,  818
```

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  819,  820,  821,  822,  823,  842,  843,  883,  884,  885,  886,  887,  888,  889,  890,  891
  892,  893,  894,  895,  919,  920,  921,  922,  923,  924,  925,  926,  927,  928,  929,  930
  931,  932,  933,  962,  963,  972,  973,  974,  975,  976,  977,  978,  979,  980,  981,  982
  983,  984, 1046, 1047, 1052, 1053, 1054, 1055, 1056, 1057, 1076, 1077, 1152
** Section: CableSection  Profile: CableProfile
*Beam General Section, elset=_PickedSet21, poisson = 0.3, density=7967., section=GENERAL
0.045239, 1e-08, 1e-09, 1e-08, 1e-08
0.,0.,-1.
 2.1e+11, 8.1e+10, 1.2e-05
*End Instance
**
*Nset, nset=_PickedSet40, internal, instance=Cable-1
 16, 35
*Nset, nset=_PickedSet75, internal, instance=Cable-1
 139, 151
*Nset, nset=_PickedSet76, internal, instance=Cable-1
 16, 35
*Nset, nset=_PickedSet86, internal, instance=Cable-1
    6,    9,   12,   16,   17,   26,   31,   41,   51,   62,   73,   80,   86,   93,   98,  107
  117,  124,  131,  139,  183,  184,  185,  186,  187,  188,  189,  190,  191,  192,  193,  194
  215,  216,  217,  218,  219,  220,  221,  222,  223,  224,  225,  226,  244,  245,  246,  247
  248,  249,  250,  251,  252,  253,  254,  255,  256,  257,  258,  259,  260,  321,  322,  323
  324,  325,  326,  327,  328,  329,  330,  331,  332,  355,  356,  357,  358,  359,  360,  361
  362,  363,  364,  365,  366,  408,  409,  410,  411,  412,  413,  414,  415,  416,  417,  418
  419,  469,  470,  471,  472,  473,  474,  475,  476,  477,  478,  479,  480,  535,  536,  537
  538,  539,  540,  541,  542,  543,  544,  545,  546,  597,  598,  599,  600,  601,  602,  603
  604,  605,  606,  607,  608,  649,  650,  651,  652,  653,  654,  655,  656,  657,  658,  659
  660,  674,  675,  676,  677,  678,  679,  680,  681,  682,  683,  684,  685,  732,  733,  734
  735,  736,  737,  738,  739,  740,  741,  742,  743,  761,  762,  763,  764,  765,  766,  767
  768,  769,  770,  771,  772,  824,  825,  826,  827,  828,  829,  830,  831,  832,  833,  834
  835,  883,  884,  885,  886,  887,  888,  889,  890,  891,  892,  893,  894,  913,  914,  915
  916,  917,  918,  919,  920,  921,  922,  923,  924,  955,  956,  957,  958,  959,  960,  961
  962,  963,  964,  965,  966, 1023, 1024, 1025, 1026, 1027
*Elset, elset=_PickedSet86, internal, instance=Cable-1
   38,   39,   40,   41,   42,   43,   44,   45,   46,   47,   48,   49,   50,   74,   75,   76
   77,   78,   79,   80,   81,   82,   83,   84,   85,   86,  108,  109,  110,  111,  112,  113
  114,  115,  116,  117,  118,  119,  120,  121,  122,  123,  124,  125,  126,  198,  199,  200
  201,  202,  203,  204,  205,  206,  207,  208,  209,  210,  239,  240,  241,  242,  243,  244
  245,  246,  247,  248,  249,  250,  251,  305,  306,  307,  308,  309,  310,  311,  312,  313
  314,  315,  316,  317,  379,  380,  381,  382,  383,  384,  385,  386,  387,  388,  389,  390
  391,  460,  461,  462,  463,  464,  465,  466,  467,  468,  469,  470,  471,  472,  537,  538
  539,  540,  541,  542,  543,  544,  545,  546,  547,  548,  549,  599,  600,  601,  602,  603
  604,  605,  606,  607,  608,  609,  610,  611,  630,  631,  632,  633,  634,  635,  636,  637
  638,  639,  640,  641,  642,  700,  701,  702,  703,  704,  705,  706,  707,  708,  709,  710
  711,  712,  735,  736,  737,  738,  739,  740,  741,  742,  743,  744,  745,  746,  747,  811
  812,  813,  814,  815,  816,  817,  818,  819,  820,  821,  822,  823,  883,  884,  885,  886
  887,  888,  889,  890,  891,  892,  893,  894,  895,  921,  922,  923,  924,  925,  926,  927
  928,  929,  930,  931,  932,  933,  972,  973,  974,  975,  976,  977,  978,  979,  980,  981
  982,  983,  984, 1052, 1053, 1054, 1055, 1056, 1057
*Nset, nset=_PickedSet87, internal, instance=Cable-1
   24,   25,   28,   34,   35,   40,   44,   50,   57,   67,   75,   85,   92,  101,  110,  123
  129,  137,  143,  151,  341,  354,  373,  407,  427,  456,  496,  563,  621,  673,  731,  788
  851,  912,  947, 1019, 1042
*Elset, elset=_PickedSet87, internal, instance=Cable-1
  221,  222,  237,  238,  260,  261,  270,  302,  303,  327,  328,  363,  364,  412,  413,  495
  496,  565,  566,  628,  629,  698,  699,  767,  768,  842,  843,  919,  920,  962,  963, 1046
 1047, 1076, 1077, 1152
*Nset, nset=_PickedSet90, internal, instance=Cable-1
  16,  35, 139, 151
*Nset, nset=_PickedSet103, internal, instance=Cable-1
 108,
*Nset, nset=_PickedSet129, internal, instance=Cable-1
 83,
*Nset, nset=_PickedSet138, internal, instance=Cable-1
 134, 140
*Nset, nset=_PickedSet213, internal, instance=Cable-1
 145, 150
```

```
*Nset, nset=_PickedSet214, internal, instance=Cable-1
 148, 152
*Nset, nset=_PickedSet215, internal, instance=Cable-1
 138, 147
*Nset, nset=_PickedSet216, internal, instance=Cable-1
 141, 142
*Nset, nset=_PickedSet217, internal, instance=Cable-1
 126, 136
*Nset, nset=_PickedSet218, internal, instance=Cable-1
 118, 135
*Nset, nset=_PickedSet219, internal, instance=Cable-1
 120, 125
*Nset, nset=_PickedSet220, internal, instance=Cable-1
 114, 119
*Nset, nset=_PickedSet221, internal, instance=Cable-1
 113, 122
*Nset, nset=_PickedSet222, internal, instance=Cable-1
 116, 128
*Nset, nset=_PickedSet223, internal, instance=Cable-1
 111, 121
*Nset, nset=_PickedSet224, internal, instance=Cable-1
 104, 115
*Nset, nset=_PickedSet225, internal, instance=Cable-1
  99, 109
*Nset, nset=_PickedSet226, internal, instance=Cable-1
  94, 105
*Nset, nset=_PickedSet227, internal, instance=Cable-1
 96,100
*Nset, nset=_PickedSet228, internal, instance=Cable-1
 87, 95
*Nset, nset=_PickedSet229, internal, instance=Cable-1
 81, 91
*Nset, nset=_PickedSet230, internal, instance=Cable-1
 82, 84
*Nset, nset=_PickedSet231, internal, instance=Cable-1
 78, 83
*Nset, nset=_PickedSet232, internal, instance=Cable-1
 68, 72
*Nset, nset=_PickedSet233, internal, instance=Cable-1
 69, 77
*Nset, nset=_PickedSet234, internal, instance=Cable-1
 63, 74
*Nset, nset=_PickedSet235, internal, instance=Cable-1
 70, 71
*Nset, nset=_PickedSet236, internal, instance=Cable-1
 64, 65
*Nset, nset=_PickedSet237, internal, instance=Cable-1
 52, 66
*Nset, nset=_PickedSet238, internal, instance=Cable-1
 53, 60
*Nset, nset=_PickedSet239, internal, instance=Cable-1
 47, 54
*Nset, nset=_PickedSet240, internal, instance=Cable-1
 45, 56
*Nset, nset=_PickedSet241, internal, instance=Cable-1
 38, 55
*Nset, nset=_PickedSet242, internal, instance=Cable-1
 32, 48
*Nset, nset=_PickedSet243, internal, instance=Cable-1
 27, 43
*Nset, nset=_PickedSet244, internal, instance=Cable-1
 21, 39
*Nset, nset=_PickedSet245, internal, instance=Cable-1
 14, 33
*Nset, nset=_PickedSet246, internal, instance=Cable-1
  7, 23
*Nset, nset=_PickedSet247, internal, instance=Cable-1
```

```
  4, 20
*Nset, nset=_PickedSet248, internal, instance=Cable-1
  2, 13
*Nset, nset=_PickedSet249, internal, instance=Cable-1
  3, 19
*End Assembly
**
**
**
*Amplitude, name=BS5400_1, definition=PERIODIC
1, 3.58274, 0., 0.
0., 1.
**
**----------------------------------------------------
**
** MATERIALS
**
**----------------------------------------------------
**
*Material, name=Aluminium
*Density
2750.,
*Elastic
 7.2e+10, 0.33
*Material, name=Steel
*Density
7850.,
*Elastic
 2.1e+11, 0.3
*Expansion
 1.2e-05,
**
**-----------------------------------------------------
**
** BOUNDARY CONDITIONS
**
**-----------------------------------------------------
**
** Name: BC-1 Type: Displacement/Rotation
*Boundary
_PickedSet90, 1, 1
_PickedSet90, 2, 2
_PickedSet90, 3, 3
** ----------------------------------------------------------------
**
**
**-------------------------------------------------------
**
** STEP: DeadLoad
**
**-------------------------------------------------------
*Step, name=DeadLoad, nlgeom=YES
*Static
0.1, 1., 1e-05, 1.
**
** LOADS
**
** Name: Gravity   Type: Gravity
*Dload
, GRAV, 9.81, 0., 0., -1.
** Name: PointLoads   Type: Concentrated force
*Cload
Cable-1.CableClamps, 3, -10546.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
```

```
**
** FIELD OUTPUT: F-Output-1
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: TempLoad
**
*Step, name=TempLoad, nlgeom=YES
*Static
0.1, 1., 1e-06, 1.
**
** FIELDS
**
** Name: Field-1   Type: Temperature
*Temperature
_PickedSet86, -101.9, 0., 0.
** Name: Field-2   Type: Temperature
*Temperature
_PickedSet87, -101.9, 0., 0.
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1, F-Output-2
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: Frequency
**
*Step, name=Frequency, perturbation
*Frequency, eigensolver=Lanczos, acoustic coupling=off, normalization=displacement, number interval=1, bias=1.
12, , , , ,
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-3
**
*Output, field, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad
**
*Step, name=DynaLoad, perturbation
*Modal dynamic, continue=NO
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_1   Type: Concentrated force
```

```
*Cload, amplitude=BS5400_1
_PickedSet138, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad2
**
*Step, name=DynaLoad2, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_2   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet213, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad3
**
*Step, name=DynaLoad3, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_3   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet214, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
```

```
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad4
**
*Step, name=DynaLoad4, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_4   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet215, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad5
**
*Step, name=DynaLoad5, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: DynaLoad5   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet216, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad6
**
*Step, name=DynaLoad6, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
```

```
** Name: BS5400_6   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet217, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad7
**
*Step, name=DynaLoad7, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_7   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet218, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad8
**
*Step, name=DynaLoad8, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_8   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet219, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
```

```
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad9
**
*Step, name=DynaLoad9, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_9   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet220, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad10
**
*Step, name=DynaLoad10, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_10   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet221, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad11
**
*Step, name=DynaLoad11, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
```

```
**
** Name: BS5400_11   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet222, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad12
**
*Step, name=DynaLoad12, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_12   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet223, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad13
**
*Step, name=DynaLoad13, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_13   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet224, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
```

```
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad14
**
*Step, name=DynaLoad14, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_14   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet225, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad15
**
*Step, name=DynaLoad15, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_15   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet226, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad16
**
*Step, name=DynaLoad16, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
```

```
** LOADS
**
** Name: BS5400_16   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet227, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad17
**
*Step, name=DynaLoad17, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_17   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet228, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad18
**
*Step, name=DynaLoad18, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_18   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet229, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
```

```
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad19
**
*Step, name=DynaLoad19, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_19   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet230, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad20
**
*Step, name=DynaLoad20, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_20   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet231, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad21
**
*Step, name=DynaLoad21, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
```

```
**
** LOADS
**
** Name: BS5400_21   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet232, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad22
**
*Step, name=DynaLoad22, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_22   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet233, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad23
**
*Step, name=DynaLoad23, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_23   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet234, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
```

```
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad24
**
*Step, name=DynaLoad24, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_24   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet235, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad25
**
*Step, name=DynaLoad25, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_25   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet236, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad26
**
*Step, name=DynaLoad26, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
```

```
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_26   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet237, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad27
**
*Step, name=DynaLoad27, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_27   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet238, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad28
**
*Step, name=DynaLoad28, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_28   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet239, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
```

```
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad29
**
*Step, name=DynaLoad29, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_29   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet240, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad30
**
*Step, name=DynaLoad30, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_30   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet241, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad31
**
*Step, name=DynaLoad31, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
```

```
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_31   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet242, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad32
**
*Step, name=DynaLoad32, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_32   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet243, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad33
**
*Step, name=DynaLoad33, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_33   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet244, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
```

```
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad34
**
*Step, name=DynaLoad34, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_34   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet245, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad35
**
*Step, name=DynaLoad35, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_35   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet246, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad36
**
*Step, name=DynaLoad36, perturbation
*Modal dynamic, continue=YES
```

```
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_36   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet247, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad37
**
*Step, name=DynaLoad37, perturbation
*Modal dynamic, continue=YES
0.01, 7.01
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_37   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet248, 3, -90.
**
** OUTPUT REQUESTS
**
**
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
** ----------------------------------------------------------------
**
** STEP: DynaLoad38
**
*Step, name=DynaLoad38, perturbation
*Modal dynamic, continue=YES
0.01, 7.02
*Modal Damping
1, 12, 0.005
**
** LOADS
**
** Name: BS5400_38   Type: Concentrated force
*Cload, amplitude=BS5400_1
_PickedSet249, 3, -90.
**
** OUTPUT REQUESTS
**
**
```

```
** FIELD OUTPUT: F-Output-4
**
*Output, field, variable=PRESELECT
**
** HISTORY OUTPUT: H-Output-2
**
*Output, history, variable=PRESELECT
*End Step
```