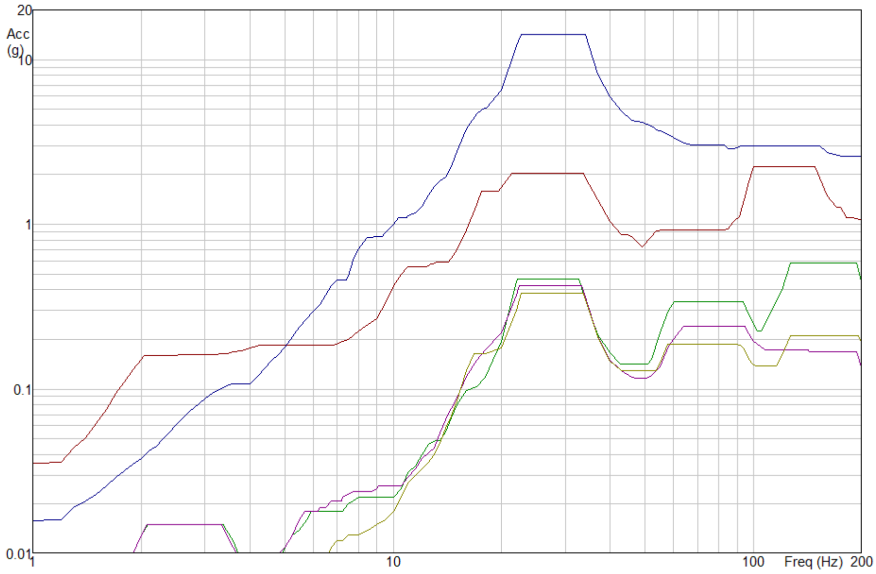

DEVELOPMENT OF METHODOLOGY FOR GENERATING RESPONSE SPECTRA FOR DECOUPLED SMALLBORE PIPING

KARL BONDESSON

Structural Mechanics

Master's Dissertation

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MASTER'S DISSERTATION

DEVELOPMENT OF METHODOLOGY FOR GENERATING RESPONSE SPECTRA FOR DECOUPLED SMALLBORE PIPING

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Preface

This master's dissertation concludes a five year civil engineering master program at Lund University. This project has sparked a further interest of mine in the field of structural analysis. I would like to thank Anders Blom at Areva NP Uddcomb in Helsingborg for his continued support and interest in this research. I would also like to thank Per-Erik Austrell and Kent Persson at the division of Structural Mechanics at Lund University.

Lund, October 2014

Abstract

In nuclear piping engineering, the response spectrum method of analysis is widely used to analyze piping systems. The matter of combining modal responses is of great interest.

Complications may occur with the response spectrum method for large systems. Examples are convergence issues and something called twin-mode effects that may overestimate the effects on the system. This master thesis investigates the possibility of separating small parts of piping systems, thus reducing the original system in size, and analyzing the separated systems independently by loading them with a modified response spectrum based around a dynamic amplification factor. The process of calculating the modified response spectrum is automated by the development of an application using Python.

The modeling of the piping systems is done in Pipestress, which is a commonly used computer software for design of pipes at nuclear power plants. Different modal combination methods and certain modeling issues are investigated and finally the stress in the original and the separated system is calculated and compared.

There is no experimental data to compare the results to, but the method of separating systems with a modified response spectrum shows some promise, even though further investigation is needed.

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1. Introduction

1.1 Background

In 2013, a master thesis work was commenced in association with Areva NP Uddcomb that should investigate different methods that could allow for the separation of piping systems with response spectrum loads. The reasons for wanting to separate small pipes from larger piping systems are numerous. For example, a large system is approximately unaffected by the removing of the small pipe if the small pipe has an area moment of inertia that is less than 1/25th than that of the larger system (Welding Research Council, 1984). Also, leaving the small pipe in the larger model can have several calculation complications; once models become too large, problems may occur with convergence. Leaving a small pipe in the model can introduce a small mass difference that can lead to something called twin-modes, which in turn lead to over-conservative responses in the analysis.

Since the larger system can be calculated separately, the question of verifying the small pipe on its own remains. When the load is applied as a response spectrum, there is no time history information in the connection point where the small pipe is connected to the large system, and a load case for the small pipe cannot be calculated in a simple, obvious manner.

The previous master thesis investigated a number of methods of approximating the load effects on the small pipe; static approximations, replacing the small pipe with a point mass, time history approximations and finally a modified response spectrum. The thesis concluded in 2014, among other things, that a modified response spectrum method could work as an approximation, but it needed further verification and deeper investigation in several areas.

1.2 Objective

The main objective is to investigate and develop calculation tools for a method to separate smaller pipes from models of larger piping systems. This thesis focuses on a method of separation based on the assumption that a small pipe will have a very small, assumed insignificant, effect on the larger systems behavior. The small pipe is simply removed from the model, and will instead require separate analysis. To analyze the small pipe, a response spectrum load is created based on the large model's response. Data needed for creating the response spectrum will be collected from results generated by the commercial FE-program PipeStress using real piping models.

To carry out the calculations and investigate the problem, an automated process for collecting the data will have to be developed as part of the thesis.

1.3 Areva NP Uddcomb

In 1970 Uddcomb AB Sweden was founded by the Swedish government, Uddeholm AB and Combustion Engineering as owners. Uddcomb AB manufactured many reactor pressure vessels to the nuclear power plants in Sweden**,** but also exported to Finland and Germany. The company went through some changes over the years, and finally in 2005 Uddcomb Engineering AB became a subsidiary of AREVA NP.

AREVA is a French multinational company specializing in nuclear and renewable energy. AREVA has around 45 000 employees as of 2013. The company is divided into 5 business areas:

- Mining
- Front End
- Reactors and Services
- Back End
- Renewable energy

AREVA NP Uddcomb has around 170 employees and the revenue for 2013 was around 540 million SEK. The company is located in Helsingborg and Karlskrona.

1.4 Limitations

The following limitations apply

- Linear elastic isotropic material properties are assumed
- Damping constants ≤ 0.05
- Cut-off frequency chosen as 100 Hz in modal analysis
- Four modal superposition methods investigated; absolute summation, square root of sum of squares, NRC 10 % grouping method, complete quadratic combination
- No coupled stiffnesses considered in modelling of anchor points
- No reference time-history calculations are executed

2. Theory of dynamics

2.1 Equation of motion

The basis for the theory of dynamics is the equation of motion. The equation of motion for a single degree of freedom (SDOF) mass-stiffness system with damping is

$$m\ddot{u} + c\dot{u} + ku(t) = p(t) \tag{2.1}$$

u, $\dot{u}$, $\ddot{u}$ is displacement, velocity and acceleration as a function of time and m, c and k is mass, damping and stiffness, and $p(t)$ is the applied force.

Dividing this equation by m gives another useful formulation of the equation of motion

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = p(t) \tag{2.2}$$

$\omega_n = \sqrt{\frac{k}{m}}$ is the natural frequency and $\zeta = \frac{c}{2m\omega_n}$ is the critical damping ratio.

2.2 Modal analysis

For a multi-degree of freedom system (MDOF) without damping, free vibration (no applied loads) is described by Eq. 2.3.

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{0} \tag{2.3}$$

$\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices and $\boldsymbol{u}$ and $\ddot{\boldsymbol{u}}$ are the displacement and relative acceleration vectors

When assuming harmonic motion, the mode shape displacement can be written as

$$u = Acos(\omega_n t)\boldsymbol{\Phi} \tag{2.4}$$

ω_n holds the natural frequencies and $\boldsymbol{\Phi}$ is the mode shape matrix.

Any deformed shape of a structure can be made up of a linear combination of these mode shapes. Eq. 2.5 can then be formulated as

$$(\boldsymbol{K} - \omega_n^2\boldsymbol{M})\boldsymbol{\Phi} = \boldsymbol{0} \tag{2.5}$$

Eq. 2.5 is an eigenvalue problem and can be solved by

$$det(\boldsymbol{K} - \omega_n^2\boldsymbol{M}) = \boldsymbol{0} \tag{2.6}$$

Several eigenfrequencies are computed from Eq. 2.6, and the corresponding modes are calculated from Eq. 2.5. The analysis of a mass-stiffness system without loading that generates, among other things, the eigenfrequencies and eigenmodes is called modal analysis.

The displacement $\boldsymbol{u}(t)$ can be described as a superposition of all modes of vibration, each corresponding to a natural frequency.

$$\boldsymbol{u}(t) = \sum q_i(t)\phi_i \tag{2.7}$$

$q_i(t)$ is a scalar time function and ϕ_i is the mode shape vector

Any motion can be represented by a linear combination of the systems modes and their time functions, as long as the system is considered linear.

The equation of motion for a MDOF system can be rewritten by using Eq. 2.7 and premultiplying the terms with ϕ_i^T. It can be shown that $\phi_j^T \boldsymbol{M} \phi_i = 0$ and $\phi_j^T \boldsymbol{K} \phi_i = 0$ when $i \neq j$ (Chopra, 1995). In other words, the modes are orthogonal. Since only the terms with $i = j$ remain, the equations become uncoupled.

$$\sum \phi_i^T \boldsymbol{M} \phi_i \ddot{q}_i(t) + \sum \phi_i^T \boldsymbol{C} \phi_i \dot{q}_i(t) + \sum \phi_i^T \boldsymbol{K} \phi_i q_i(t) = \phi_i^T \boldsymbol{p}(t) \tag{2.8}$$

Each equation can be solved on its own after a modal analysis has been performed, and only q_i and its derivatives are unknown. The equations can be solved either by a time stepping method like Newmark's method or the Central Difference Method, or using a convolution integral like Duhamel's integral. Once solved, simply transform back from q to $\boldsymbol{u}$ by using Eq. 2.7 to get the displacement.

An advantage of this formulation is the fact that a systems behavior can be described accurately without all the mode shapes. The ones with low frequencies govern the motion to such a degree that many higher frequency modes can be left out of the calculation.

For further information on these subjects please refer to (Clough & Penzien, 1993) or (Chopra, 1995).

2.3 Response spectrum

A response spectrum is a plot of the maximum response of a number of idealized single-degree-of-freedom oscillators (see Figure 2.1) with certain natural frequencies, subjected to a dynamic load, e.g. ground acceleration $\ddot{u}_g$.

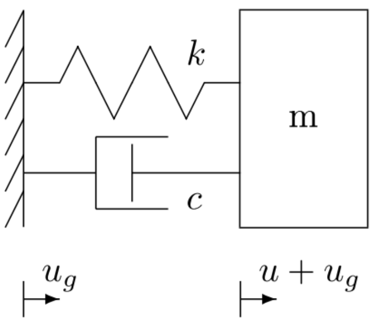


Figure 2.1. Oscillator

The SDOF system in Figure 2.1 is described by

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g \tag{2.9}$$

The equation be can be rewritten (see Eq. 2.2) as

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g \tag{2.10}$$

By solving the equation of motion for systems with different natural frequencies ω_n and damping constants ζ, and analyzing the maximum response (acceleration, velocity or displacement), a response spectrum can be created.

To create a ground motion deformation response spectrum, $S_d(\zeta, \omega)$, for a given ground motion, one follows the steps below (Chopra, 1995):

1. Numerically define the ground acceleration $\ddot{u}_g(t)$
2. Select the natural frequency, ω_n, and the damping constant, ζ, of the SDOF system
3. Use a numerical method to calculate the deformation response as a function of time
4. Register the peak value of the deformation response for the SDOF system
5. Repeat steps 2-4 for other damping constants and natural frequencies to create a range of interest
6. Plot the results

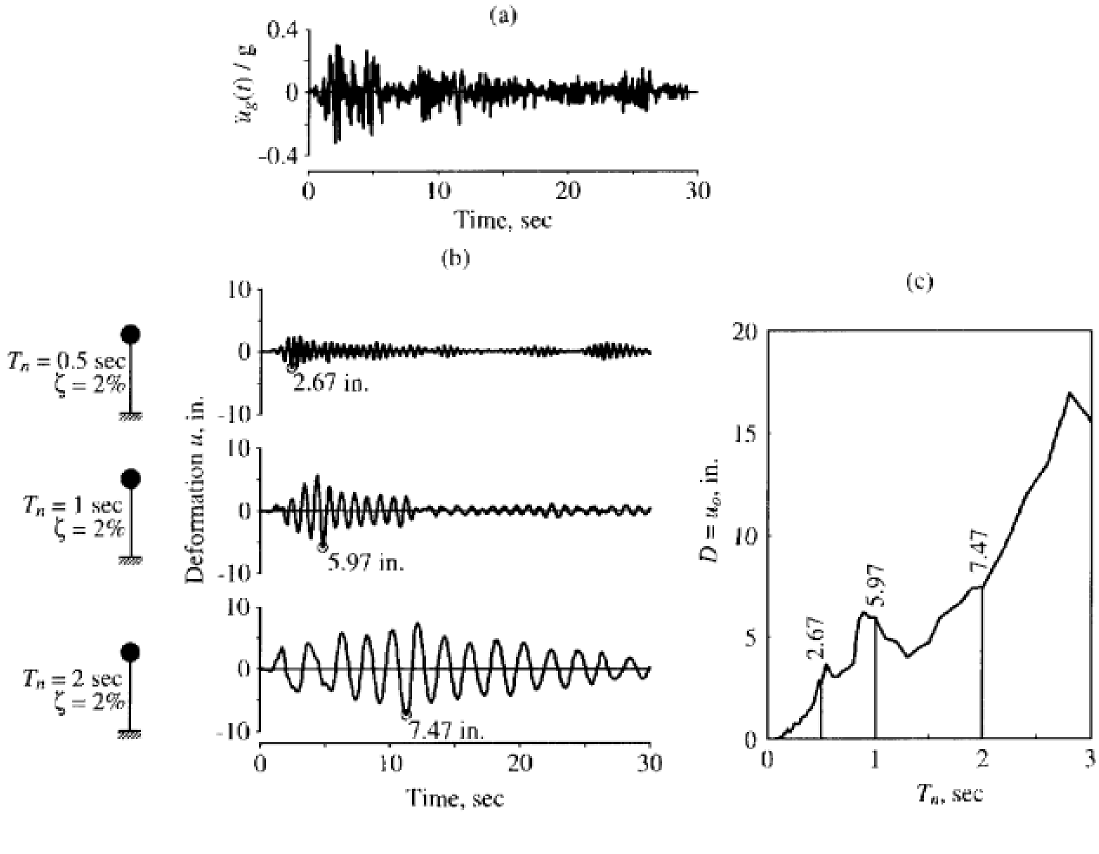


Figure 2.2. Creation of deformation response spectrum (Chopra, 1995)

Figure 2.2(b) show the numerically calculated deformation response history, u(t), for a number of SDOF oscillators with certain natural periods and the damping constant 2%, for the measured ground motion from the El Centro earthquake Figure 2.2(a). The deformation response spectrum is also presented in Figure 2.2(c).

Using Eq. 2.11 the corresponding pseudo-velocity and pseudo-acceleration response spectrum is created. Figure 2.3 show these spectra.

$$\frac{1}{\omega^2} \cdot S_{pa}(\zeta, \omega) \approx \frac{1}{\omega} \cdot S_{pv}(\zeta, \omega) \approx S_d(\zeta, \omega) \tag{2.11}$$

This not the actual velocity or acceleration, since they are based on the relative displacement between the moving ground and the moving mass of the oscillator, and they are instead called pseudo-velocity and pseudo-acceleration response spectrum respectively.

Note that while the spectra from (Chopra, 1995) are plots of response against natural period, the response can also be plotted against natural frequency using the relationship $T_n = \frac{2\pi}{\omega_n}$.

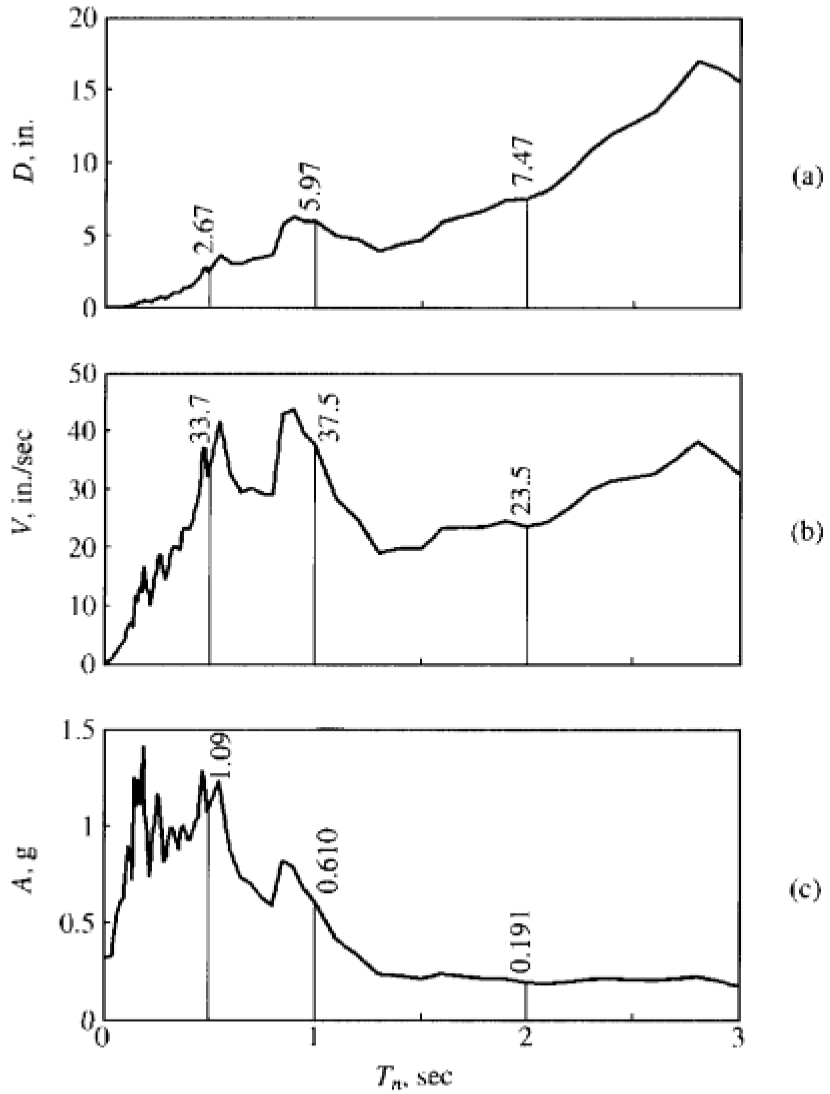


Figure 2.3. (Chopra, 1995)

There are different types of response spectra referred to in nuclear piping design. The *ground motion response spectrum* refers to the response of a building standing on the ground. The building response is simulated by idealized SDOF oscillators while the ground is subjected to dynamic loads such as earthquake loads.

The *in-building response spectrum* refers to the response in structures attached to the building. The piping system is simulated by idealized single-degree-of-freedom oscillators while the building is subjected to dynamic loads. See Figure 2.4.

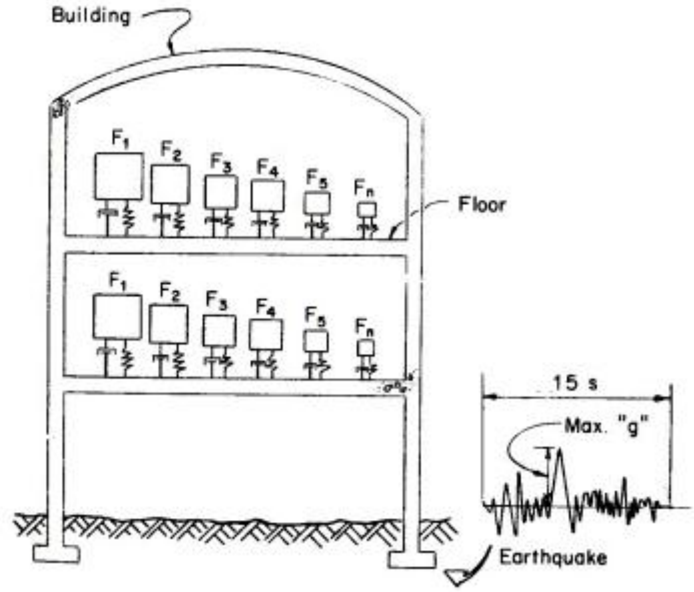


Figure 2.4. The earthquake can be represented by a ground motion response spectrum. The theoretical SDOF oscillators inside a building are used to create an in-building response spectrum. (Smith & Van Laan, 1987)

2.4 The response spectrum method of analysis

The equation of motion for an MDOF system with ground motion acceleration is

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{Ku} = -\boldsymbol{MI}\ddot{u}_g \tag{2.12}$$

$\boldsymbol{I}$ is the identity matrix.

Using the mode shapes to perform modal transformation, Eq. 2.12 can be rewritten according to (Datta, 2010) as

$$\ddot{u}_i + 2\zeta_i\omega_i\dot{u}_i + \omega_i^2 u_i = -\frac{\phi_i^T \boldsymbol{MI}}{\phi_i^T \boldsymbol{M}\phi_i}\ddot{u}_g \tag{2.13}$$

The equations can be solved by the numerical methods described before. A response spectrum can be created from the results. The maximum response for a mode with a certain natural frequency ω_i can then be given by the response spectrum. The maximum displacement for a single mode is calculated according to Eq. 2.14 (Datta, 2010).

$$\boldsymbol{u}_{i,max} = \frac{\phi_i^T \boldsymbol{MI}}{\phi_i^T \boldsymbol{M} \phi_i} \cdot S_d(\xi, \omega_i) \tag{2.14}$$

Looking at this expression, since the response spectrum gives the maximum response for the given frequency and damping, the ratio $\frac{\phi_i^T \boldsymbol{MI}}{\phi_i^T \boldsymbol{M}\phi_i}$is an expression for how much each mode participates. $L_{\boldsymbol{i}} = \phi_i^T \boldsymbol{MI}$ is the *modal participation factor*, $M_i = \phi_i^T \boldsymbol{M}\phi_i$ is the *generalized mass*.

The magnitude of the participation factors usually diminish with increasing mode number. This is the reason why all the modes do not need to be used in a response spectrum analysis. To know when enough modes are used in the analysis to represent the structure properly, a method of effective mass can be utilized. Effective mass for a mode is $\frac{L_i^2}{M_i}$. The sum of the effective masses for all modes is equal to the total mass of the structure. The sum of the included modes' effective mass should be no less than 90% of the total mass (Carr, 1994).

American Society of Mechanical Engineers only requires that all significant modes below 33 Hz are accounted for in the modal analysis for seismic loads. For other building vibration loads, the frequency content could be higher, usually up to 100 or 200 Hz. Higher frequency mode responses are then applied as a static load using a method called “left-out force”.

2.5 Summation methods

It is clear that a response spectrum load give a certain response for each mode. The total response of the system can be calculated by combining the response from each mode into one single response. When combining the response of the modes to calculate the total response, there are a number of different summation methods to consider.

ABSSUM – Absolute sum of maximum values

$$x = \sum_{i=1}^{m} |x_i|_{max} \tag{2.15}$$

This is the maximum theoretical value of the response and thus a conservative way of combining the responses. It is highly unlikely that the maximum value of each mode will occur at the same time. Therefore there are theoretical reasons to avoid this method.

SRSS – Square root of sum of squares of response

$$x = \sqrt{\sum_{i=1}^{m} x_{i,max}^2} \tag{2.16}$$

This method assumes completely uncoupled maximum responses. When the frequencies of the modes are well separated this assumption works well. However, if there are closely spaced modes it may give results that are not conservative when compared to time history calculations of a corresponding load.

NRC 10 % grouping method

$$x = \sqrt{\sum_{k=1}^{m} x_k^2 + 2\sum \left|x_i x_j\right|} \qquad i \neq j \tag{2.17}$$

This is a SRSS summation but with a contribution for closely spaced modes (U.S. Nuclear Regulatory Commission, 1976). If the natural frequencies ω_i and ω_j of modes i and j are within 10 % of each other, they are assumed to be coupled.

$$\frac{\omega_j - \omega_i}{\omega_i} \leq 0.1 \tag{2.18}$$

The absolute value of this product is used to assure conservatism.

CQC – Complete quadratic combination

$$x = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{m} \rho_{ij} x_i x_j} \tag{2.19}$$

This is a SRSS summation but with an algebraic contribution for closely spaced modes. ρ_{ij} is a coefficient between 0 and 1 that describes the effect of how closely spaced peak modal responses correlate. The correlation coefficient ρ_{ij} can be formulated in different ways. One suggestion is (Rosenblueth & Elorduy, 1969):

$$\rho_{ij} = \frac{\xi^2 \left(1 + \beta_{ij}\right)^2}{\left(1 - \beta_{ij}\right)^2 + 4\xi^2 \beta_{ij}} \tag{2.20}$$

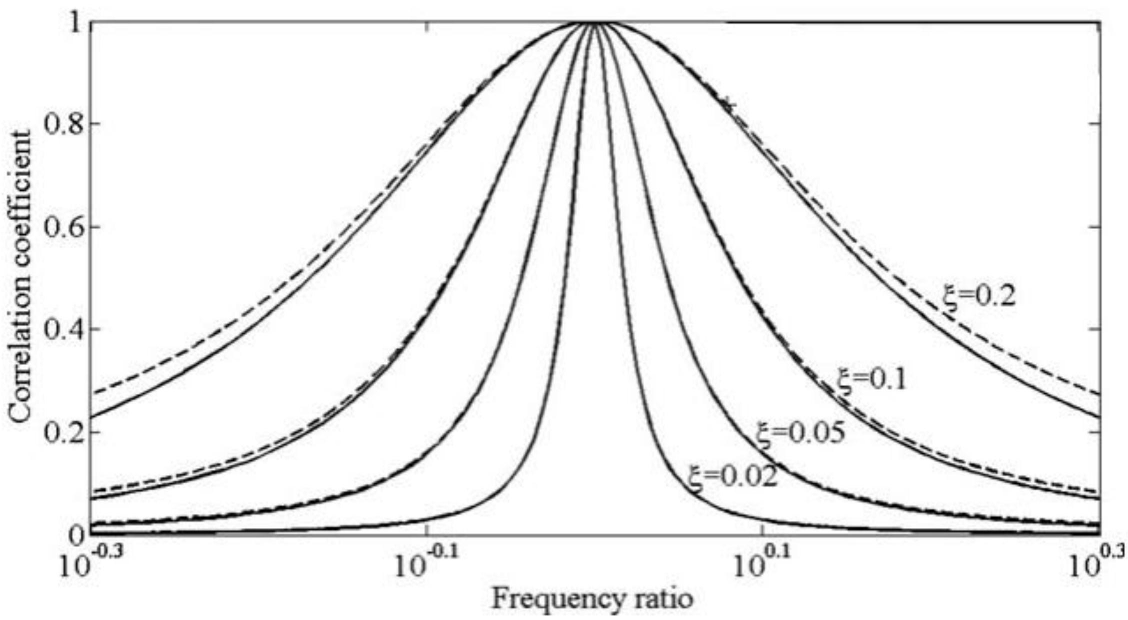
Where

$$\beta_{ij} = \frac{\omega_i}{\omega_j}$$

Another suggestion for a correlation coefficient is (Der Kiureghian, 1981):

$$\rho_{ij} = \frac{8\xi^2 \left(1 + \beta_{ij}\right) \beta_{ij}^{\frac{3}{2}}}{\left(1 - \beta_{ij}\right)^2 + 4\xi^2 \beta_{ij} \left(1 + \beta_{ij}\right)^2} \tag{2.21}$$

The value of ρ_{ij} decrease rapidly as β_{ij} moves away from 1, see Figure 2.5.



Figur 2.5. Plot of correlation coefficients (Datta, 2010). The dotted line is Eq. 2.20 and the constant line is Eq. 2.21.

2.6 Twin mode problem

Twin modes are correlated, closely spaced modes in systems with unevenly distributed mass. When an eigenfrequency of a small mass subsystem matches an eigenfrequency of a heavier system, the response may be overestimated by 100 to 1000 % and obviously become meaningless.

The effect of analyzing the two subsystems together creates an over-estimation of the response of the small mass system which occurs often in cases when algebraic summation is not possible. It is the use of sign-free modal superposition methods such as absolute summation, SRSS summation and the NRC 10 percent grouping method that causes the unrealistically high response. An explanation of the twin mode phenomenon and a way of overcoming it by rotating twin modes is given by (Houdart, Hennart, & Urbano, 1997).

The most important features of twin modes are as follows:

- Large mass ratio in system
- Modes with close frequencies
- Similar mode shapes (equal or opposite)
- Large and opposite response in small mass subsystem

2.7 Choice of superposition method

As mentioned previously, the choice of summation method is not insignificant when there are closely space frequencies. Some studies on the subject have been made; one such study, by Maison, Neuss and Kasai, reprinted in (Gupta, 1990), was on a multistory building model that was subjected to earthquake loading in two directions, and the results are presented in Table

2.1. By comparing the results of different summation methods to a theoretically correct time history calculation, some conclusions can be drawn.

		CQC	SRSS	ABSSUM
East-west response				
	Average error (%)	6	18	27
	Maximum error	17	26	67
North-south response				
	Average error	32	251	491
	Maximum error	67	350	800

Table 2.1. Error in results compared to true time-history for a multi-story building (Gupta, 1990)

In the north-south direction, closely spaced frequencies occur and the results from the SRSS and ABSSUM methods become unacceptable. This holds true for other earthquake loads (Gupta, 1990) and the conclusion is that coupled modes must be handled.

The NRC 10 % grouping method is widely used in nuclear piping design as it handles closely spaced modes, but it is a sign-free method since it takes the absolute value of the contribution (see Eq. 2.17). To limit the effects of twin mode occurrences, a form of algebraic summation is needed. The CQC method is therefore of most interest when continuing with these investigations.

3. Method

The goal is to find a method which can be used to separate small pipes from a larger piping system. According to (Welding Research Council, 1984) a small pipe can be allowed to simply be removed if the area moment of inertia ratio is 25:1. Sometimes, even lower ratios are accepted; as low as 7:1 has been accepted by the Nuclear Regulatory Commission (Antaki, G.A., 1995). Usually, in dynamic and flexibility analysis, the small pipe is removed and the large piping run is used as an anchor point for the branch line.

When a response spectrum is applied to a MDOF system, only the maximum static response can be calculated for a single point. There is no time history information, since the response spectrum does not give this information. Therefore it is not possible to calculate the response history in a point and apply this as a time history load to a removed pipe.

3.1 Modified response spectrum

The method investigated in this thesis is a way of calculating a modified response spectrum based on the information that can be obtained from a modal analysis of a MDOF system. The total response is given by a summation of each mode's response. The response spectrum contains information that eventually gives the maximum amplitude of each mode. These responses are combined in a chosen way to give a maximum value.

However, the modes are harmonic oscillations with a corresponding frequency. Consider the vibrating string in Figure 3.1; there is time history information in each mode. Knowing the amplitude and frequency of a mode, it can be considered a function of time.

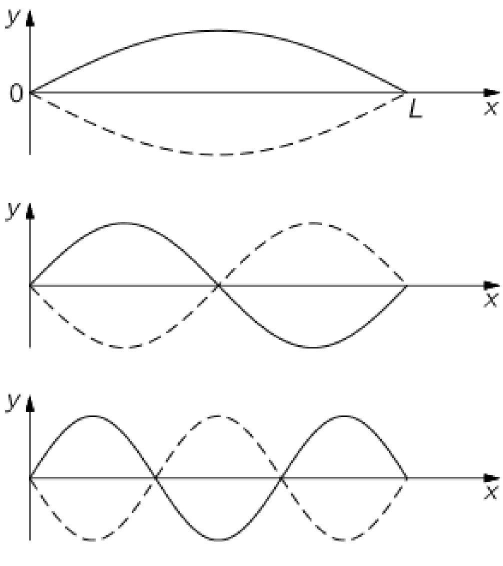


Figure 3.1. The first three modes of a vibrating string. The maximum amplitude given by the response spectrum is u_0. The modes move with a given natural frequency and u(t) for each mode can be established.

Now consider the connection point between the small pipe and the larger system in the following Figure 3.2.

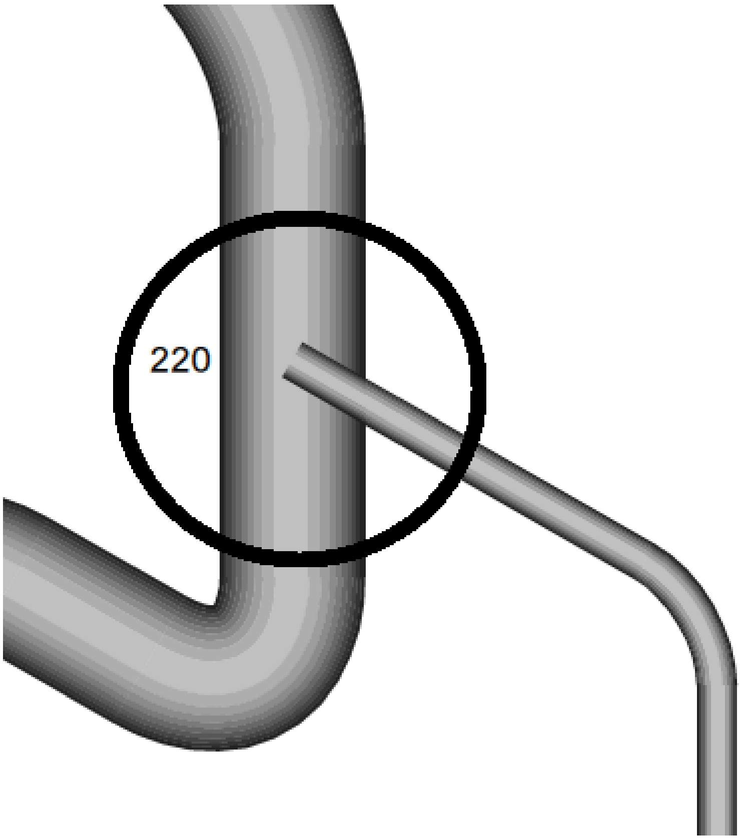


Figure 3.2. Connection point in model 1.

The connection point could be approximated as an anchor point to the small pipe that is being displaced by the movement of the larger system. This movement can be described by the response of the modes in the large system and a summation of them. This will give a bounded solution for the displacement of the anchor point without time history. Consider for each mode a dynamic displacing load, with a frequency and amplitude, acting on the small pipe. A dynamic amplification factor can be calculated for each mode's effect on the small pipe.

In Dynamics of Structures (Chopra, 1995) the following dynamic amplification factor for ground motion is derived:

$$\frac{\ddot{u}_0^t}{\ddot{u}_{g0}} = \frac{\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \tag{3.1}$$

where $\ddot{u}_0^t = \ddot{u}_g(t) + \ddot{u}(t)$.

This is the relation between the ground motion and the total motion of the mass. Of more interest is the amplification factor for relative displacement, $\ddot{u}(t)$, since only the relative displacement or deformation is of interest when considering stress.

Beginning with this equation:

$$u(t) = \frac{-m\ddot{u}_{g0}}{k} R_d \sin(\omega t - \varphi) \tag{3.2}$$

Differentiate twice

$$\ddot{u}(t) = \omega^2 \frac{-m\ddot{u}_{g0}}{k} R_d \sin(\omega t - \varphi) \tag{3.3}$$

Maximum at sin(x) = 1 and with the knowledge that $\frac{m}{k} = \frac{1}{\omega_n^2}$ a dynamic amplification factor for relative displacement is given by:

$$\frac{\ddot{u}_0}{\ddot{u}_{g0}} = \frac{\frac{\omega^2}{\omega_n^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \tag{3.4}$$

$\ddot{u}_0$ is the maximum value of $\ddot{u}(t)$ and $\ddot{u}_{g0}$ is the ground motion amplitude

A plot of this function is presented in Figure 3.3.

The dynamic amplification factor is dependent on the ratio between the frequency of loading and natural frequency of the loaded system. In this case, the frequency of loading is the natural frequencies of the modes from the large system, and the loaded system is the small pipe.

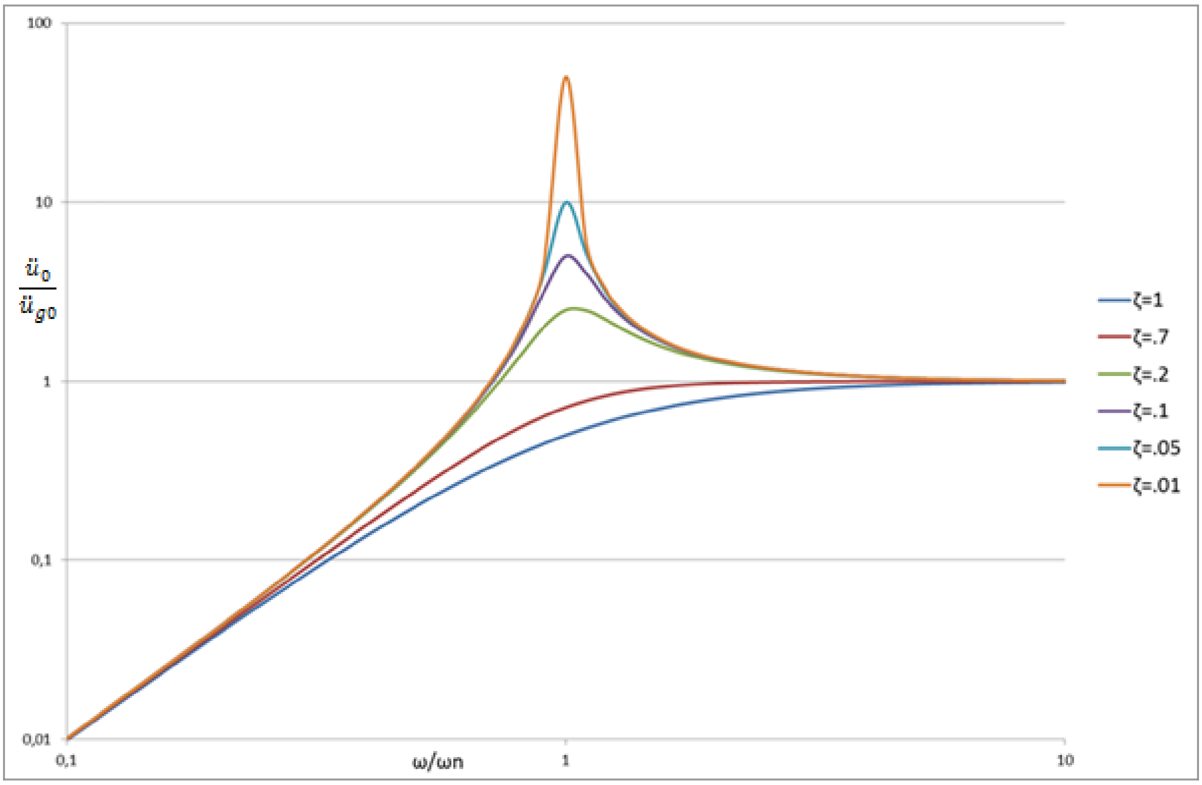


Figure 3.3. Plot of the dynamic amplification factor.

3.2 Using the dynamic amplification factor to create a spectrum

A response spectrum is created by calculating the maximum response for a number of SDOF oscillators. Consider each theoretical eigenmode for the small pipe as an oscillating system attached to the large pipe. The movement of the large pipe in the connection point can be considered as a ground acceleration $\ddot{u}_{g0}$ acting on the small pipe. This ground acceleration is dependent on the response of each mode in the large system.

For every combination of eigenmodes in the large pipe and attached oscillator there is a dynamic amplification factor that describes how large the response in the oscillator will be in relation to the response of a mode in the large pipe. Calculate the maximum response of a single oscillator by applying the dynamic amplification factor to each mode's response in the large pipe and combine these responses with a certain summation method. For an oscillator with the natural frequency ω the maximum acceleration is calculated, using the absolute summation method, with Eq. 3.5. This is the basis of the method investigated within this thesis.

$$\ddot{u}_{max,oscillator}(\omega) = \sum_{i=1}^{N} |A(\omega, \omega_i) \cdot \ddot{u}_i| \tag{3.5}$$

Here, ω_i are the natural frequencies of the large pipe, $\ddot{u}_i$ are the maximum acceleration responses for each mode in the large pipe and $A(\omega, \omega_i)$ is the dynamic amplification factor from Eq. 3.4.

To create a response spectrum for the small pipe, assume that the oscillators can have natural frequencies that range between 0 Hz and, for example, 100 Hz. Numerically define a range of frequencies, e.g. $\omega = [0,1,2,3,\ldots,100\ Hz]$, and combine the amplified response for each mode ω_n in the large pipe in a certain way; ABSSUM, SRSS, CQC or other. Presented in Table 3.1 is an outline of the method. $A(\omega, \omega_n)$ is the dynamic amplification factor, ω are the numerically defined natural frequencies of the oscillators, ω_n are the natural frequencies of the large pipe calculated from a modal analysis.

ω / ω_n	$\omega_{n=1}$	$\omega_{n=2}$		ω_n	$Combined$
ω_1	$A(\omega_{n=1}, \omega_1) \cdot \ddot{u}_1$	$A(\omega_{n=2}, \omega_1) \cdot \ddot{u}_2$	$\cdots$	$A(\omega_n, \omega_1) \cdot \ddot{u}_n$	$u_{1,tot}$
ω_2	$A(\omega_{n=1}, \omega_2) \cdot \ddot{u}_1$	$A(\omega_{n=2}, \omega_2) \cdot \ddot{u}_2$	$\cdots$	$A(\omega_n, \omega_2) \cdot \ddot{u}_n$	$u_{2,tot}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
ω_i	$A(\omega_{n=1}, \omega_i) \cdot \ddot{u}_1$	$A(\omega_{n=2}, \omega_i) \cdot \ddot{u}_2$	$\cdots$	$A(\omega_n, \omega_i) \cdot \ddot{u}_n$	$u_{i,tot}$

Table 3.1. Method of creating a response spectrum using a dynamic amplification factor.

Plot the combined responses $u_{i,tot}$ of the oscillators against their frequencies ω_i and the modified response spectrum is created.

To summarize, described above is a method of creating a response spectrum load on the small pipe without any time history information from the original response spectrum applied to the larger system. By assuming the modes in the large system are dynamic loads on the small pipe, and applying an amplification factor to the response in each mode generated by the response spectrum analysis, a load from the connection point can be created. The dynamic amplification factor is dependent on the natural frequencies of the large system and the theoretical range of frequencies of the small pipe. This way, a response spectrum of the load is created.

3.3 Developing an automated process of calculating the modified response spectrum

When a modal analysis of a piping system is done with a cut-off frequency of 100 Hz, it is not unusual that the number of modes exceed 100, or even 1000. As described previously, the maximum response for one certain frequency of the separated pipe is calculated by amplifying the response of all the modes from the modal analysis, and then combining these by some summation method. It is obvious that these calculations would be unreasonable to do by hand for each new case.

For the purposes of these calculations, as part of this thesis, an application has been developed using Python, which is a general-purpose, high-level programming language. It is also free and open source software. The application collects data from the modal analysis by PipeStress and then calculates the modified response spectrum in a chosen way using the method and theories described in the previous sections.

4. Modelling in PipeStress

4.1 Investigating the method

The goal is to investigate whether the modified response spectrum, when applied to the separated smaller piping system, will yield the same results with respect to stress, as when the smaller system is modeled onto the large piping system. The method chosen for verifying this is quite simple. First, calculate the stresses in the smaller pipe caused by a response spectrum load on the whole system. Then, remove the smaller pipe and calculate a modified response spectrum in the connection point where the smaller pipe was previously connected. Finally, apply the modified response spectrum to a model of only the smaller pipe and compare the calculated stresses in the pipe to the ones where the larger and smaller system were modeled as a single unit. See Figure 4.1.

If the small pipe has another anchor point, the movement of the large system will deform the small pipe and impose secondary stresses. These stresses are far greater than those caused by the dynamic load of the modified response spectrum. This fact is noteworthy, and it makes analyzing the modified response spectrum's validity difficult. Therefore, no anchor points exist in the small pipes used to investigate the method. That way, no other stresses than those caused by the modified response spectrum occur.

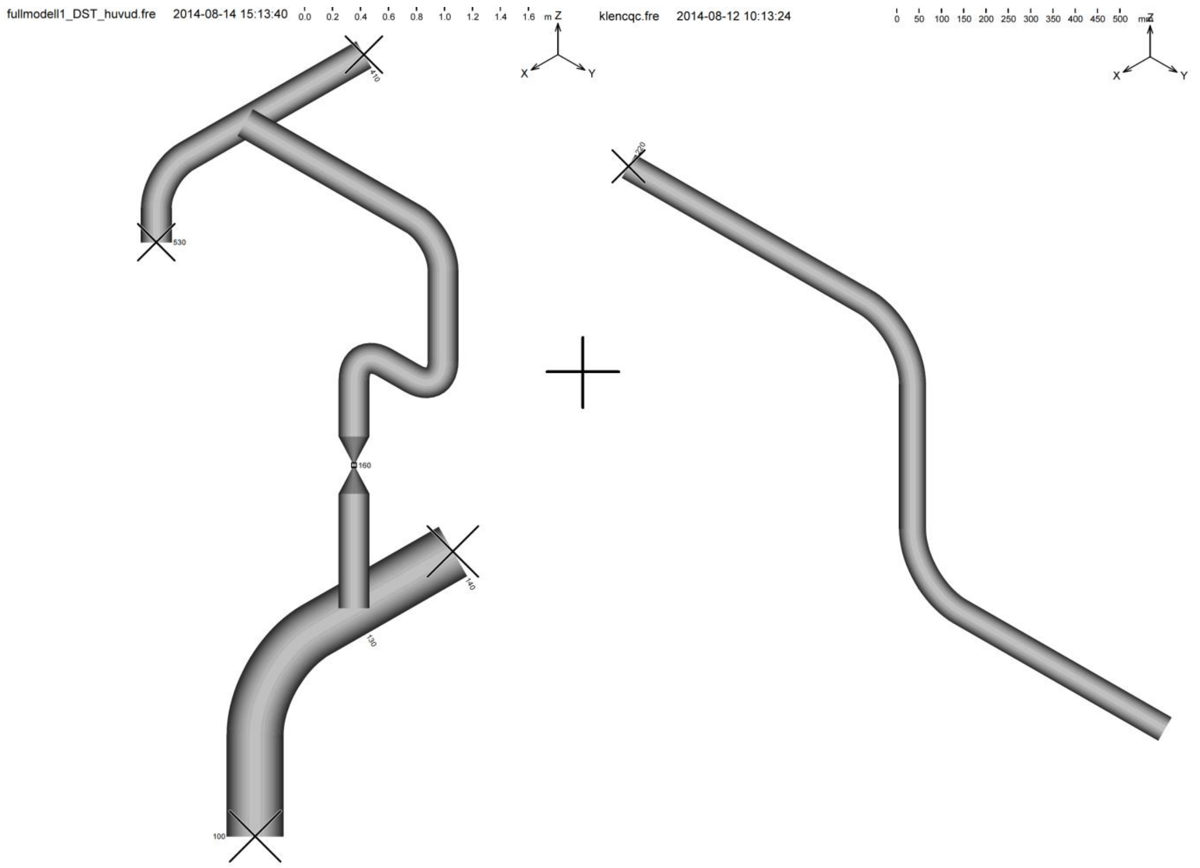


Figure 4.1. The small bore pipe separated from model 1.

4.2 Models used

The finite element program PipeStress has been used to carry out the modal analysis of the piping systems and the response to the different response spectra loads. PipeStress uses beam elements with mass, damping and stiffness to represent the pipes, and can use modal superposition to calculate the response due to response spectra loads. The use of these methods allow for fast, efficient calculations of large and complex piping systems.

For the calculations in this thesis, a fraction of critical damping based on the natural frequency for each mode has been given instead of a damping matrix.

4.2.1 Model 1

The first model is presented in Figure 4.2. It has a small pipe connected at point 220. Spectrum 1 (see Section 4.3) is applied in the two higher anchor points, and spectrum 2 in the two lower anchor points (marked by X).

The large pipe is a 8” Sch 10S pipe with an outer diameter of 219.07 mm and a wall thickness of 3.76 mm. The small pipe is a 2” Sch 10 10 S pipe with an outer diameter of 60.32 mm and 2.77 mm wall thickness.

$$I_{large} = t\pi r^3 = 3.76 \cdot \pi \cdot \left(219.07 - \frac{3.76}{2}\right)^3 = 121 \cdot 10^6\ mm^4$$

$$I_{small} = 2.77 \cdot \pi \cdot \left(60.32 - \frac{2.77}{2}\right)^3 = 2 \cdot 10^6\ mm^4$$

The ratio of area moment of inertia is approximately 60:1 which fulfills the WRC criteria of 25:1.

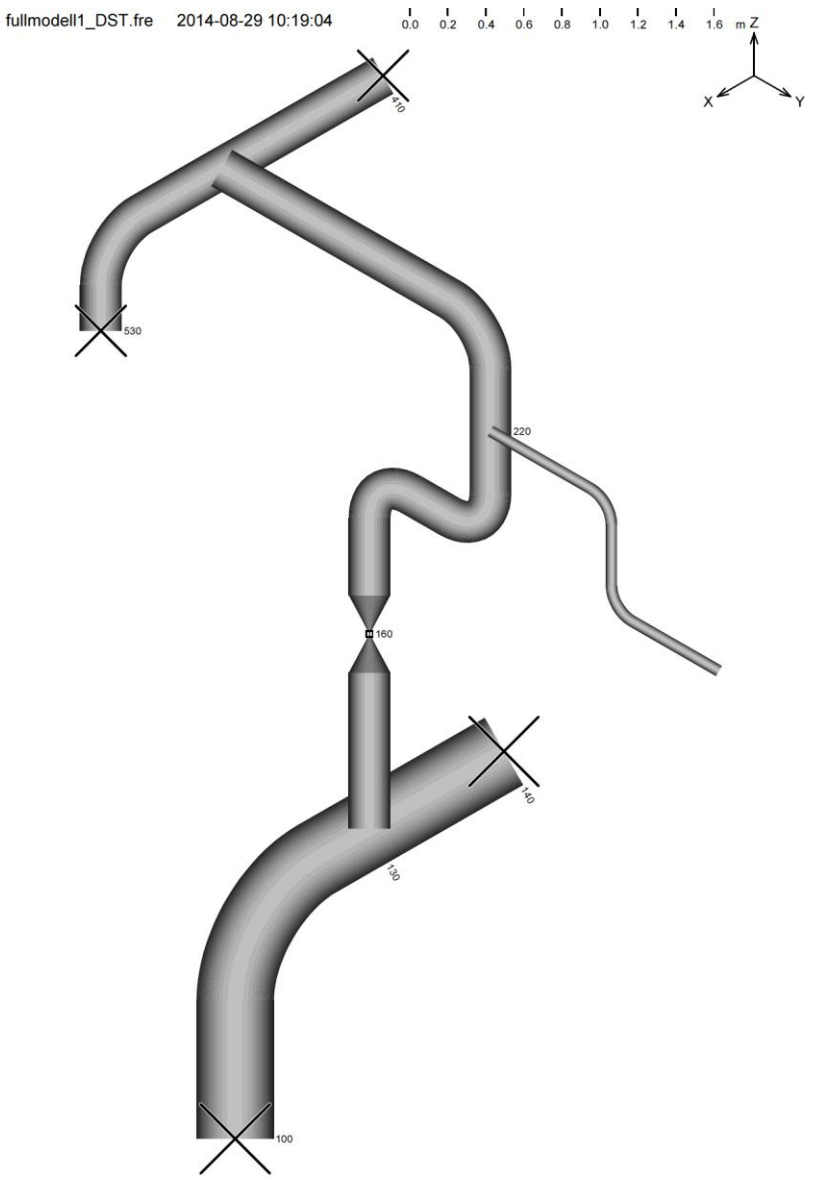


Figure 4.2. The geometry of the first of two models used to investigate the modified response spectrum.

4.2.2 Model 2

The geometry of the second model is presented in Figure 4.3. It is considerably larger and more complex than model 1. The examined small pipe is connected at point 642.

The large pipe has an outer diameter of 580.0 mm and a wall thickness of 34.0 mm. The small pipe has an outer diameter of 68 mm and 13.45 mm wall thickness.

$$I_{large} = t\pi r^3 = 34 \cdot \pi \cdot \left(580 - \frac{34}{2}\right)^3 = 19 \cdot 10^9\ mm^4$$

$$I_{small} = 13.45 \cdot \pi \cdot \left(68 - \frac{13.45}{2}\right)^3 = 10 \cdot 10^6\ mm^4$$

The ratio of area moment of inertia is approximately 2000:1.

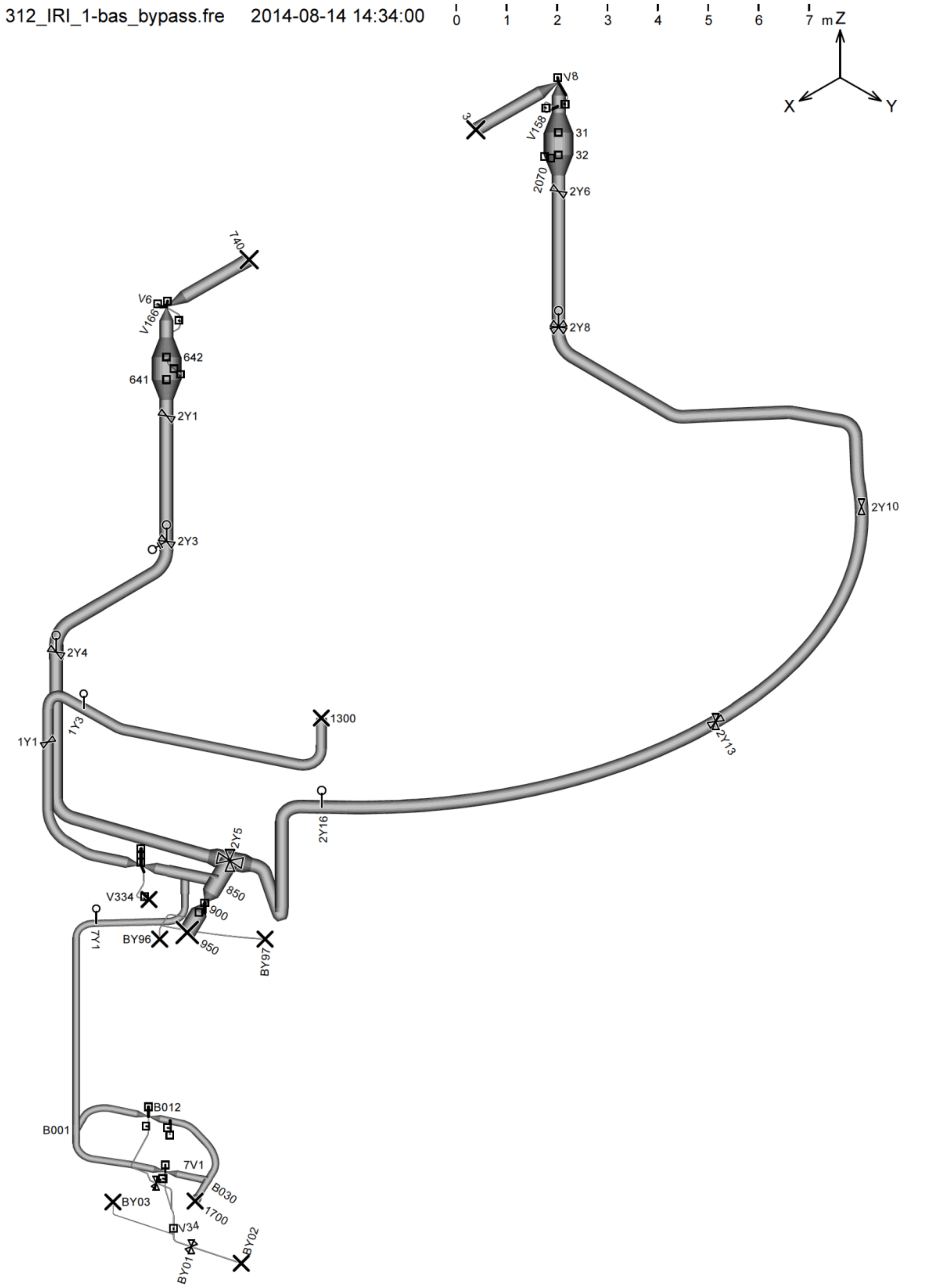


Figure 4.3. The geometry of the second model used to investigate the modified response spectrum.

4.3 Response spectra used

The response spectra used are the same in both models. They are shown in Figure 4.4 and 4.5.

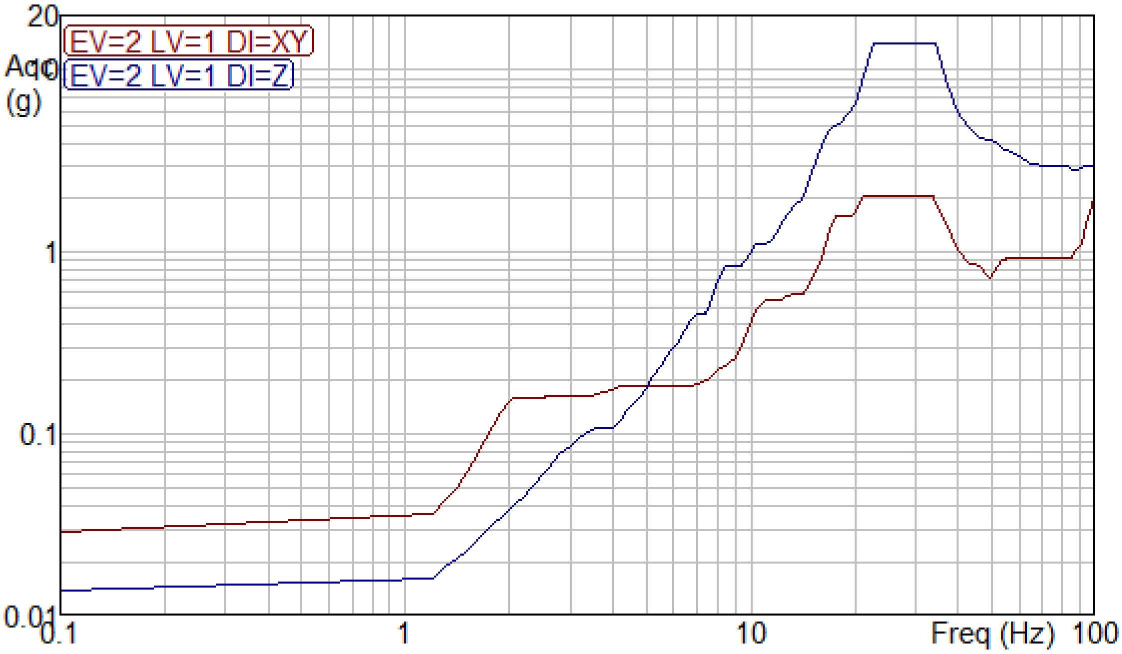


Figure 4.4. The response spectra applied on higher positioned anchor points.

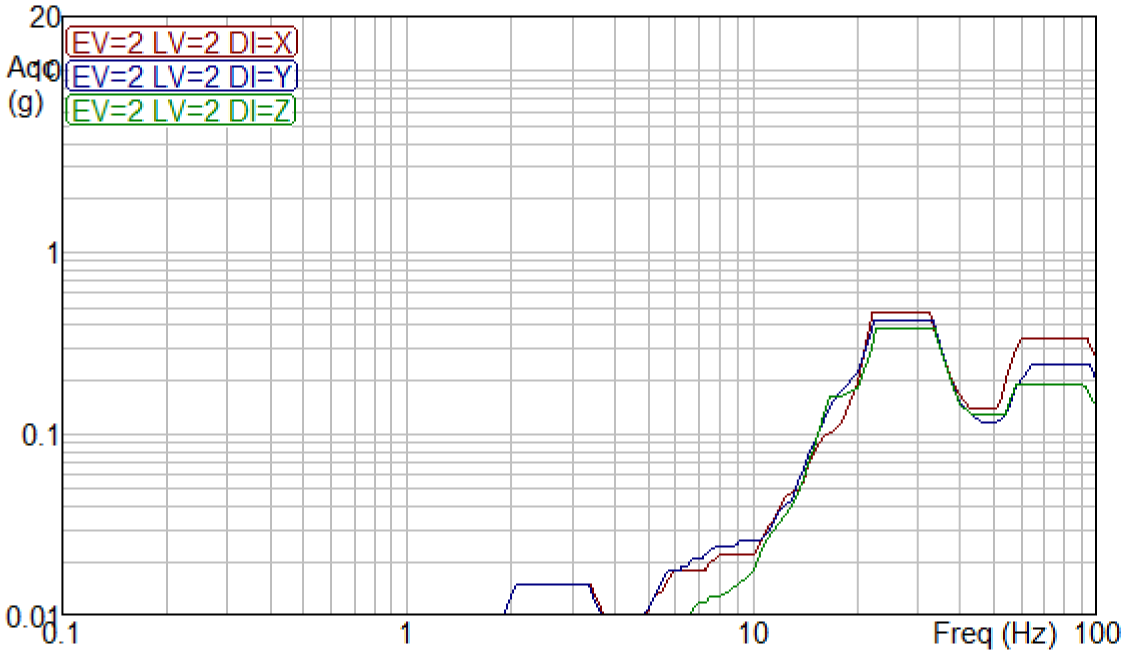


Figure 4.5. The response spectra applied on lower positioned anchor points.

4.4 Modelling of connection point

A point of interest is the modeling of the connection point. Two different kinds of stiffnesses are considered in this thesis; a rigid anchor point, and a spring support. The rigid anchor point is considered fixed in all 6 degrees of freedom. The spring support has a spring stiffness in all 6 degrees of freedom.

The stiffness in the spring support is calculated by applying a unit load in the connection point, with the small pipe removed, in all 6 degrees of freedom separately. A displacement or rotation occurs in each degree of freedom in the connection point. Take the six displacement values for each of these six unit loads and construct the matrices in the relationship

$$\boldsymbol{K}^{-1} \cdot \boldsymbol{f} = \boldsymbol{u} \tag{4.1}$$

For the unit load $f_1 = 1$ and all others set to zero the first column of the inverted stiffness matrix is calculated. Repeat for all six unit loads and the whole matrix is determined.

$$\begin{pmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ k_{12} & 0 & 0 & 0 & 0 & 0 \\ k_{13} & 0 & 0 & 0 & 0 & 0 \\ k_{14} & 0 & 0 & 0 & 0 & 0 \\ k_{15} & 0 & 0 & 0 & 0 & 0 \\ k_{16} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{pmatrix} \rightarrow k_{11} = u_1, k_{12} = u_2 \dots$$

The inverse of $\boldsymbol{K}^{-1}$ is the stiffness matrix for the spring support.

To simplify the calculation so that the inverse of a 6x6 matrix is not needed, only use the displacement or rotation in the degree of freedom that the unit load is acting in, as an approximation (Möller, 2007).

$$\begin{pmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ k_{12} & 0 & 0 & 0 & 0 & 0 \\ k_{13} & 0 & 0 & 0 & 0 & 0 \\ k_{14} & 0 & 0 & 0 & 0 & 0 \\ k_{15} & 0 & 0 & 0 & 0 & 0 \\ k_{16} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \boldsymbol{K}^{-1} = \begin{pmatrix} u_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & u_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & u_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & u_6 \end{pmatrix}$$

Hooke's law can be used to calculate the stiffness, since the equations become uncoupled.

$$k_i = \frac{f_i}{u_i} \tag{4.2}$$

This approximation is useful since PipeStress allows spring stiffnesses in the modelling of its anchor points.

4.5 Example of summation with results from PipeStress

There are three "dimensions" that have to be combined; levels, intermodal and interspatial. The order of the summation matters, especially when using CQC or other non-sign-free methods.

The response spectrum in direction X will excite all modes that have a participation factor in the X direction that is $\neq 0$. The response in these modes as a result of the response spectrum is calculated as the modal response multiplied by the mode shape. Since a mode shape can have values in all directions, a response spectrum in only one dimension can give a response in all three dimensions. The total response in direction X is the sum of the responses triggered in direction X by the spectra applied in direction X, Y and Z.

4.5.1 ABSSUM example

From model 1, presented in Section 4.2.1, an example calculation of the response in node 220 from a response spectrum load, using absolute summation on all dimensions, will be presented here. The mode shape for the structure can be found in the files generated by PipeStress. For this case, only 5 modes have been calculated. The normalized mode shapes for point 220 in direction X, Y, Z:

Freq (Hz)	x	y	z
16.214	0.87446	-0.50421	-0.21835
16.295	0.8788	0.32111	0.24782
23.015	0.07121	-0.2758	0.99945
24.466	0.48212	0.07591	-0.28825
45.060	0.15918	0.01549	0.01459

Table 4.1. Natural frequencies and mode shapes for the first five modes in model 1.

The modal response for each mode and direction is also calculated. The modal response for a mode *i* is calculated by taking the acceleration value from the response spectrum for the natural frequency of the mode, and multiplying it by the participation factor for mode X.

Modes	Modal response (g)		
Freq (Hz)	x	y	z
16.21	0.35	-0.209	-0.046
16.30	0.494	0.193	0.068
23.02	-0.117	-0.067	0.384
24.47	-0.271	0.015	-0.097
45.06	-0.258	0.007	0.044

Table 4.2. The modal response for the first five modes in model 1.

The acceleration response in direction X as a result of the spectrum in direction X is

$$x_{accX} = \sum |mode\ shape\ X\ (i) * \ \ modal\ response\ X\ (i)| =$$

$$|0.87 \cdot 0.35| + |0.88 \cdot 0.49| + |0.07 \cdot (-0.12)| + |0.48 \cdot (-0.27)| + |0.16 \cdot (-0.26)| =$$

$$0.92\ g$$

To get the total response in direction X, calculate the response in direction X as a result of each spectrum (X, Y and Z) and combine these values by absolute summation.

$$x = |x_{accX}| + |x_{accY}| + |x_{accZ}|$$

The same calculations are performed to obtain values for Y and Z. The results from the ABSSUM combination are the theoretical maximum value.

$$x = 1.47\ g, y = 0.74\ g, z = 1.00\ g$$

4.5.2 SRSS example

Using the same order of summation, but simply replacing every ABSSUM with SRSS

$$x_{accX} = \sqrt{\sum_{i=1}^{m} \left(mode\ shape\ X\ (i) \cdot\ modal\ response\ X\ (i)\right)^2}$$

the following response for node 220 is obtained.

$$x = \sqrt{x_{accX}^2 + x_{accY}^2 + x_{accZ}^2}$$

$$x = 0.61\ g, y = 0.29\ g, z = 0.44\ g$$

It is obvious already that using different summation methods result in very different responses for the same system and loading.

4.5.3 CQC example

For the given frequencies the correlation factor given by Der Kiureghian becomes

$$\rho_{ij} =$$

$$\begin{matrix} 1.0000 & 0.9937 & 0.0192 & 0.0138 & 0.0018 \\ 0.9937 & 1.0000 & 0.0197 & 0.0140 & 0.0018 \\ 0.0192 & 0.0197 & 1.0000 & 0.2995 & 0.0032 \\ 0.0138 & 0.0140 & 0.2995 & 1.0000 & 0.0040 \\ 0.0018 & 0.0018 & 0.0032 & 0.0040 & 1.0000 \end{matrix}$$

Using this in the combination of modes the acceleration response results are

$$x = 0.75, y = 0.21, z = 0.43$$

Note that compared to the SRSS method, the response in direction X is noticeably higher, and Y is lower. This serves to show the difference that the summation methods can give using just a simple model with only 5 modes.

5. Results

A summary of the results are presented in this section. The complete results are presented in Appendix A.

5.1 Model 1

The connection point for model 1 was presented in Figure 3.2. The spectra from Section 4 were applied in PipeStress.

5.1.1 Modified response spectrum

The modified response spectrum is calculated for the connection point (node 220) using the three summation methods; ABSSUM, SRSS and CQC. The three spectra are presented in Figure 5.1 through 5.3.

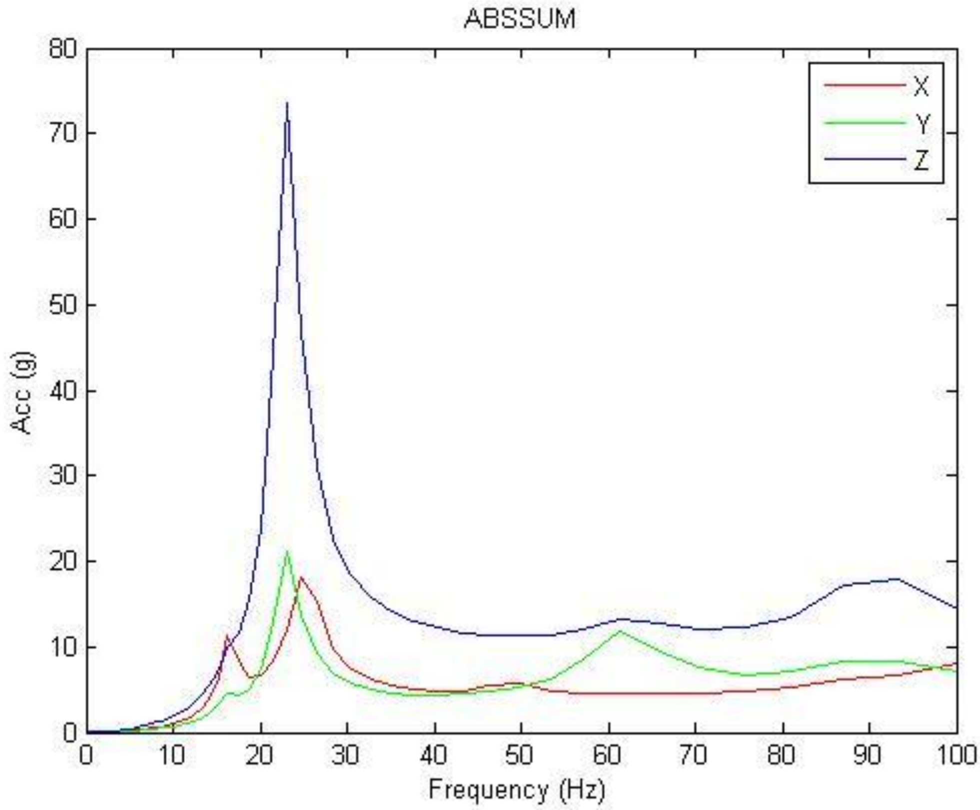


Figure 5.1. Modified response spectrum for node 220 in model 1 using absolute summation.

The response spectrum in Figure 5.1 is calculated using the ABSSUM method and has a maximum value near 20 Hz. The amplitude in the Z-direction is especially large. If a structure has natural frequencies near 20 Hz with meaningful participation factors, a large response is expected in the Z-direction.

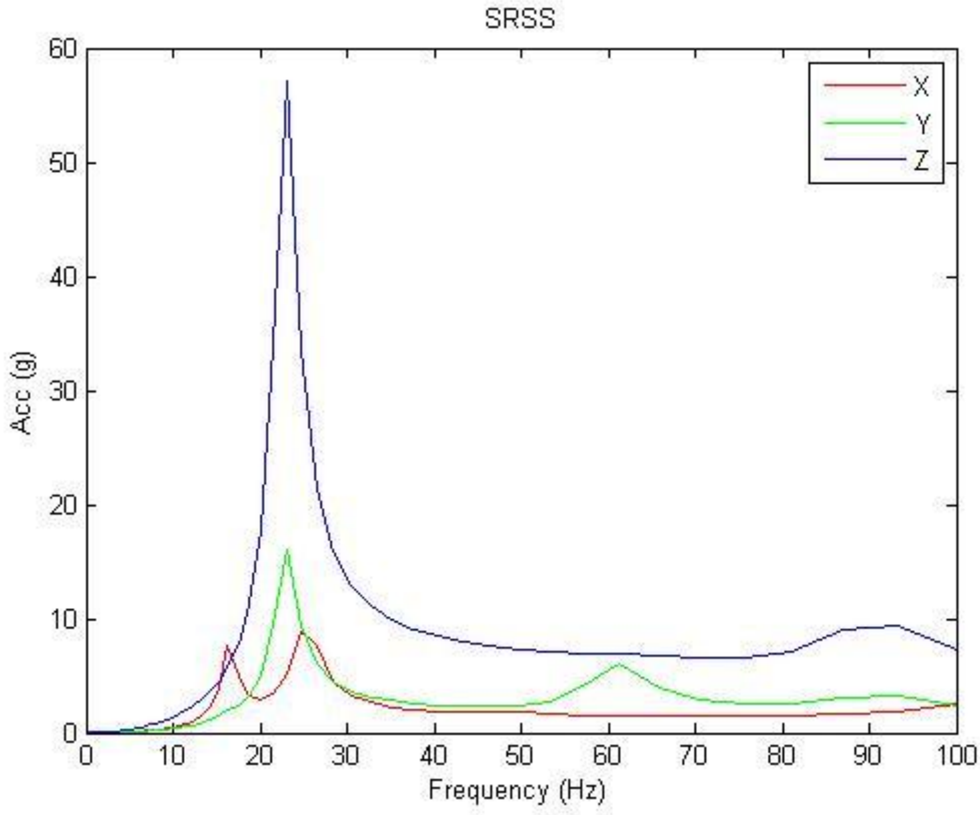


Figure 5.2. Modified response spectrum for node 220 in model 1 using SRSS summation.

The response spectrum from SRSS modal combination in Figure 5.2 also has a maximum value near 20 Hz. The amplitude in the Z-direction is 50 g, smaller than the ~70 g from the ABSSUM spectrum.

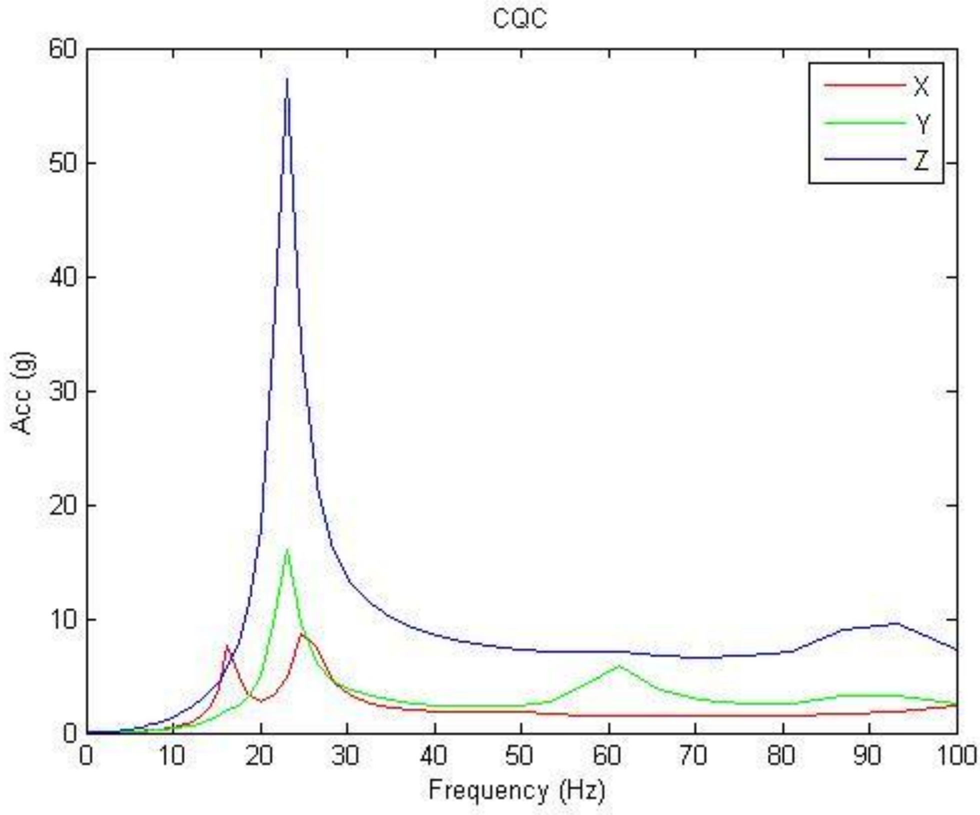


Figure 5.3. Modified response spectrum for node 220 in model 1 using complete quadratic combination (CQC).

The response spectrum from CQC summation is similar to the one calculated using SRSS summation. The effects of closely spaced modes are not great.

5.1.2 Modified response spectrum comparison

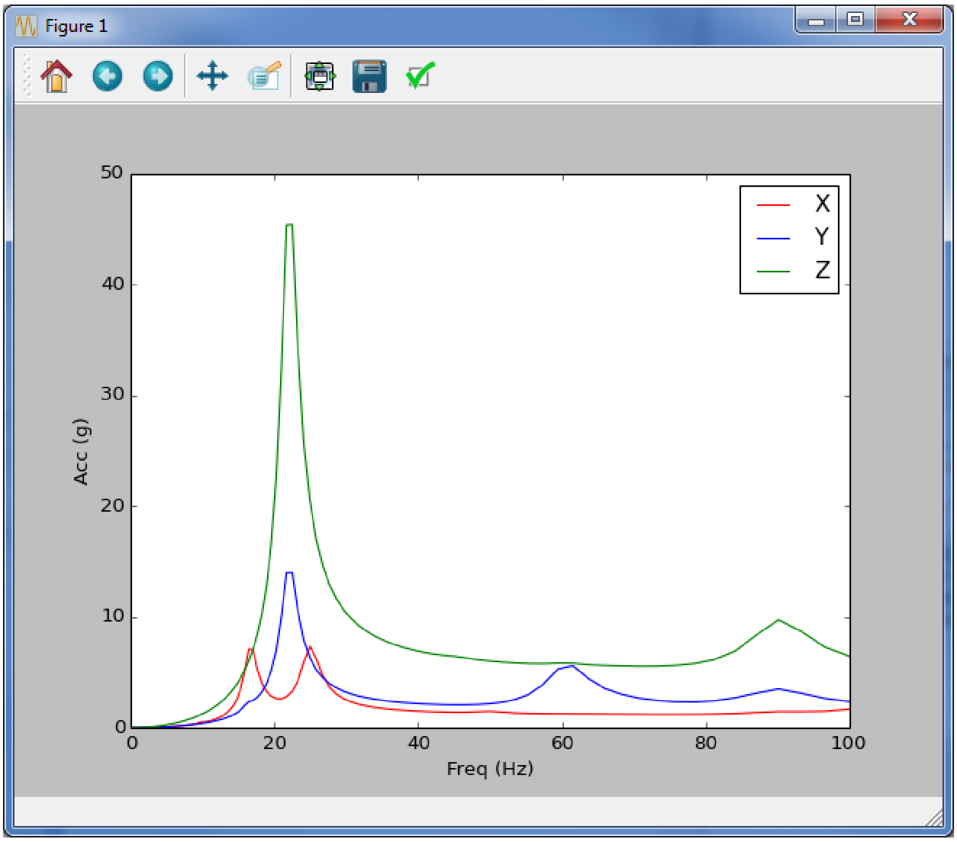


Figure 5.4. Modified response spectrum using the full model (small bore pipe included) and CQC for model 1.

To investigate the possible inertia effects of removing the small pipe from the system, the modified response spectrum was also calculated for the full model 1. The response spectrum is presented in Figure 5.4. Compared to Figure 5.3 there principal appearance is the same but there is a difference in the peak response at 20 Hz.

5.1.3 Anchor with spring stiffness

The flexibility matrix, $\boldsymbol{K}^{-1}$, was established by applying loads of 1 kN or 1 kNm in each degree of freedom in the connection point.

$$\boldsymbol{K}^{-1} = \begin{pmatrix} 0.761 & -0.013 & 0.024 & 0 & -0.025 & -0.574 \\ -0.013 & 0.188 & -0.10 & 0.226 & -0.004 & 0.007 \\ 0.024 & -0.10 & 0.399 & -0.001 & 0.015 & -0.01 \\ 0 & 0.226 & -0.001 & 0.492 & 0.004 & 0.002 \\ -0.025 & -0.004 & 0.015 & 0.004 & 0.641 & 0.003 \\ -0.574 & 0.007 & -0.01 & 0.002 & 0.003 & 1.196 \end{pmatrix}$$

The approximate method gives the stiffnesses

Stiffnesses	
Kx	1.314 kN/mm
Ky	5.319 kN/mm
Kz	2.506 kN/mm
Mx	2034 kNm/rad
My	1560 kNm/rad
Mz	836 kNm/rad

Table 5.1. Approximate stiffness in anchor point for model 1

The natural frequencies of the small bore pipe with and without the stiffnesses are presented in Table 5.2 below.

Natural frequncies (Hz)	
Fixed anchor	Spring stiffness
9.855	9.445
10.001	9.815
38.889	34.793
43.272	40.686
49.424	47.575
54.212	48.272
270.306	131.702

Table 5.2. Natural frequencies of the small bore pipe with and without spring stiffness for model 1.

The springs replace the completely rigid anchor point and reduce the natural frequencies of the small bore pipe. This changes the maximum response for each mode since the response is collected from the response spectrum for each natural frequency.

5.1.4 Summary Model 1

Presented in Table 5.3 is the ratio of calculated stress in the separated small pipe model compared to calculated stress in the full model. If the ratio is 1 the modified response spectrum caused the same stress in that node as the full model. A ratio > 1 means the modified response spectrum produced conservative results, ratio < 1 mean non-conservative. Node 220 is the connection point, 310 is the end of the pipe.

The results in Table 5.3 show that the stresses in the small pipe are quite different in the full model and the removed model. The stresses in the removed small pipe are higher in some parts and lower in others. The highest over-estimation of the stresses is 77 % (node 280, CQC with spring) too high and the lowest under-estimation is 24 % (node 300, SRSS and CQC) too low.

	Model 1 – Summary of stress ratio in separated pipe					
	ABSSUM		SRSS		CQC	
Node	Fixed anchor	Spring	Fixed anchor	Spring	Fixed anchor	Spring
220	1.329	1.416	1.292	1.433	1.309	1.442
Z001	1.159	1.116	1.224	1.184	1.224	1.184
Z001	1.159	1.116	1.224	1.184	1.224	1.184
270	1.305	1.484	1.314	1.588	1.333	1.608
270	1.307	1.487	1.330	1.612	1.340	1.621
280	1.285	1.466	1.443	1.722	1.454	1.742
280	1.287	1.467	1.468	1.745	1.468	1.766
290	0.806	0.903	0.818	0.909	0.818	0.909
290	0.801	0.911	0.822	0.889	0.844	0.889
300	0.898	1.031	0.762	0.881	0.762	0.881
300	0.889	1.032	0.762	0.857	0.800	0.900
Z002	0.842	1	0.833	0.833	1	1
Z002	0.842	1	0.833	0.833	1	1
310	1	1	1	1	1	1

Table 5.3. Ratio of stress in small bore pipe from model 1 nodes using the modified response spectrum compared to the original response spectra.

5.2 Model 2

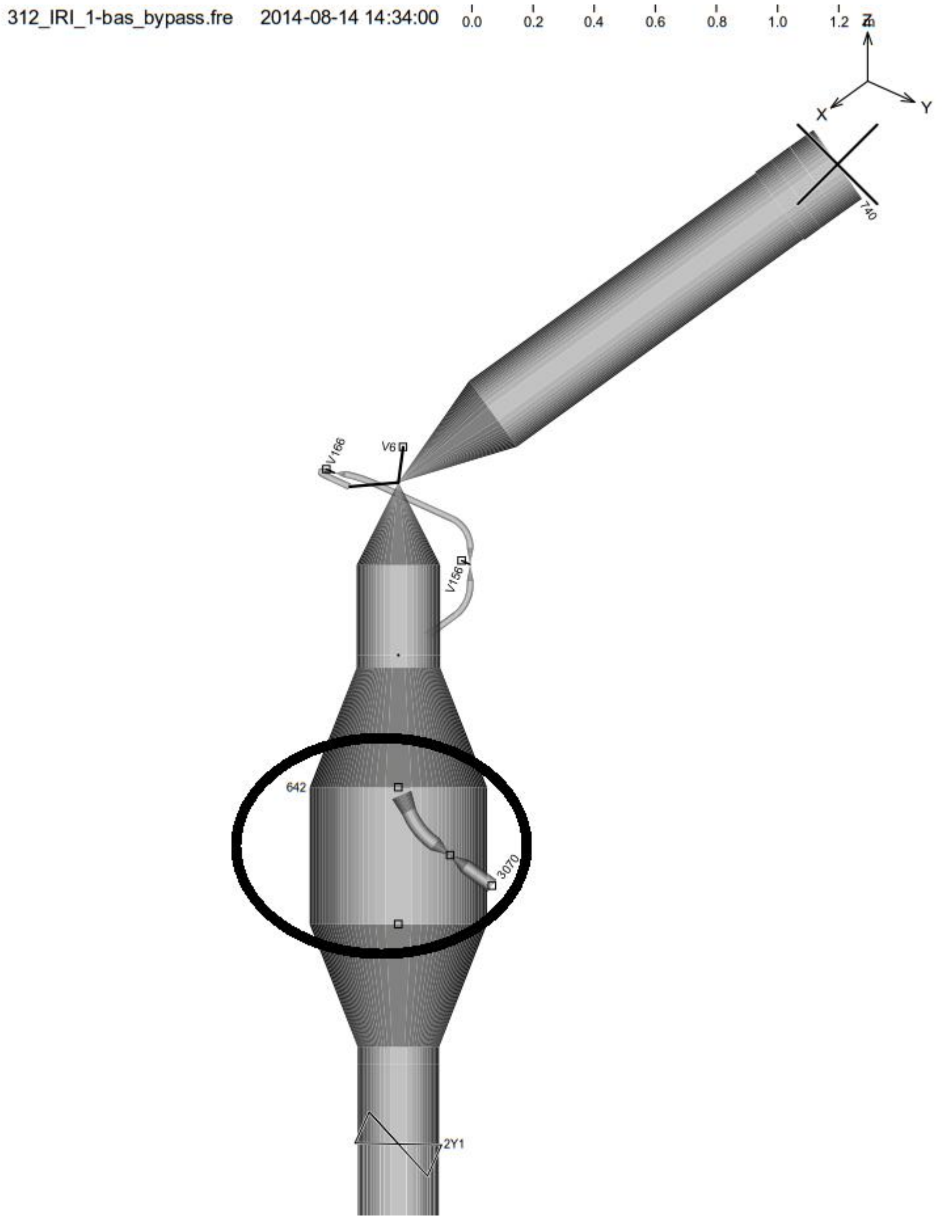


Figure 5.5. Small bore pipe and connection point (node 642)

The connection point for model 2 is shown in Figure 5.5. The small bore pipe that is studied begins in node 642 and ends in node 3070. The spectra from Section 4 were applied to the model in PipeStress.

5.2.1 Modified response spectrum

The modified response spectrum is calculated for the connection point (node 642) using the three summation methods; ABSSUM, SRSS and CQC. The three spectra are presented in Figure 5.7 through 5.9. The maximum response occurs at 20 Hz. In this case, the ABSSUM method has a local maximum response for both the X- and Y-direction between 60-70 Hz. This is less noticeable when using SRSS and almost completely removed with the CQC summation.

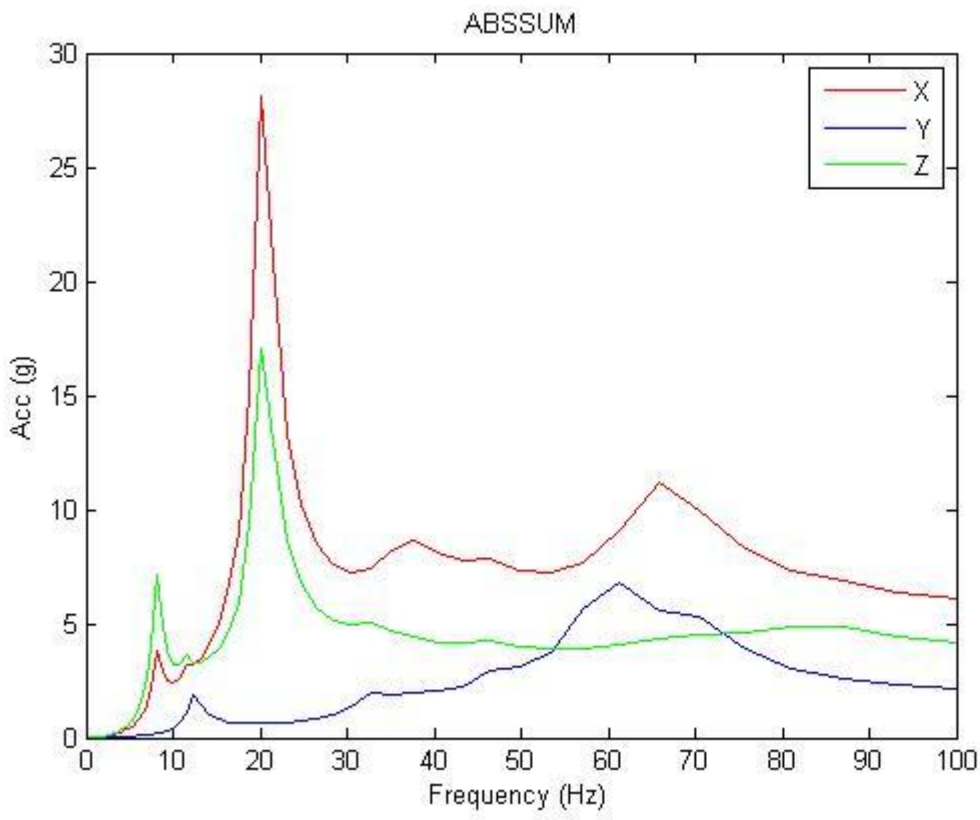


Figure 5.6. Modified response spectrum for node 642 in model 2 using absolute summation.

The response spectrum in Figure 5.6 is calculated using the ABSSUM method and has a maximum value near 20 Hz. The amplitude in the X-direction is especially large, but also in the Z-direction. There are also considerable responses at relatively high frequencies (50-80 Hz) in all three directions.

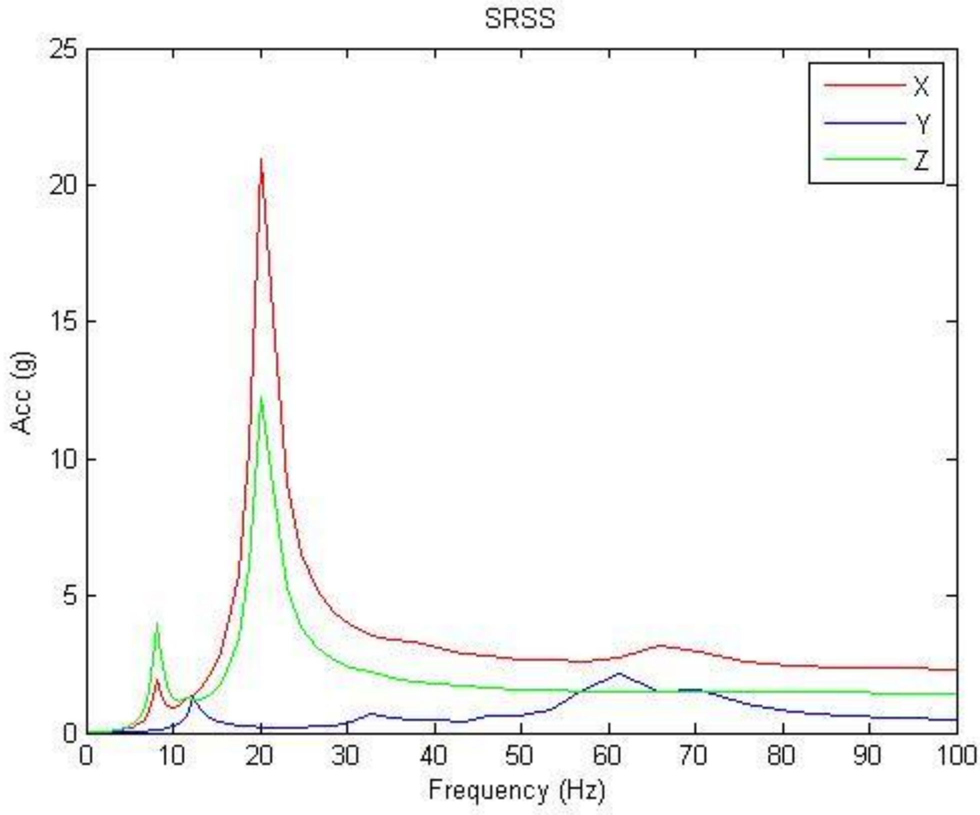


Figure 5.7. Modified response spectrum for node 642 in model 2 using SRSS summation.

The response spectrum in Figure 5.7 is calculated using the SRSS method and has a maximum value near 20 Hz. The amplitudes are reduced as expected compared to the ABSSUM response. The peak responses at the frequencies 50-80 Hz are gone.

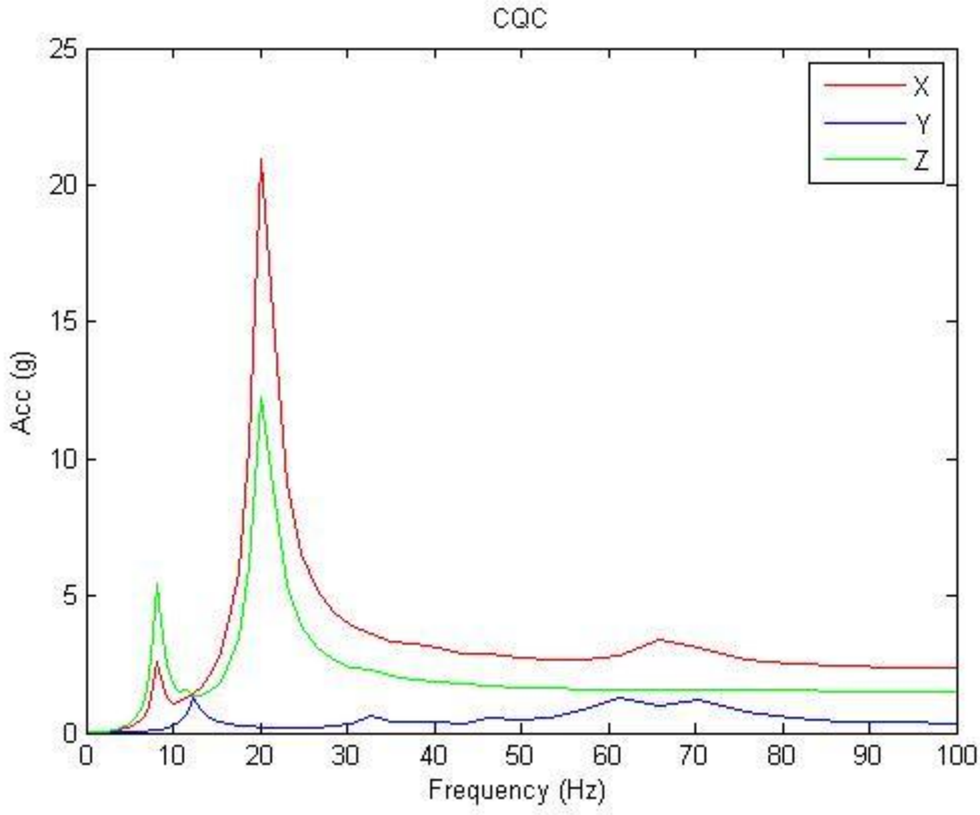


Figure 5.8. Modified response spectrum for node 642 in model 2 using CQC summation.

The response spectrum in Figure 5.8 is calculated using the CQC method and also has a maximum value near 20 Hz. The amplitudes are reduced as expected compared to the ABSSUM response. The peak responses at the frequencies 50-80 Hz are gone. The SRSS spectrum from Figure 5.7 is very similar to this spectrum.

5.2.2 Modified response spectrum comparison

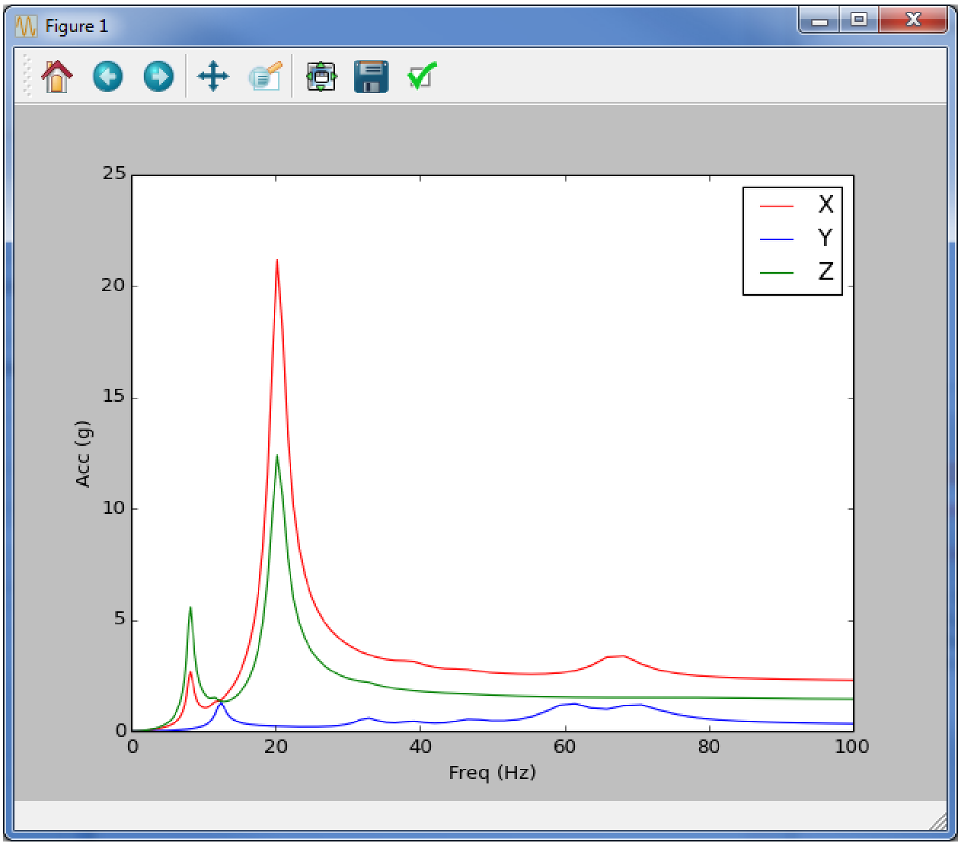


Figure 5.9. Modified response spectrum using the full modell (small bore pipe included) and CQC for model 2.

To investigate the possible inertia effects of removing the small pipe from the system, the modified response spectrum was also calculated for the full model 2. The response spectrum is presented in Figure 5.9. Compared to Figure 5.8 there is basically no difference, and removing the small pipe has no inertia effect on the large system.

5.2.3 Spring stiffness

The flexibility matrix, $\boldsymbol{K}^{-1}$, was established by applying loads of 1 kN or 1 kNm in each degree of freedom in the connection point.

$$\boldsymbol{K}^{-1} = \begin{pmatrix} 0.035 & 0 & 0.037 & 0 & -0.225 & 0.001 \\ 0 & 0.048 & 0.001 & -0.027 & 0 & 0.038 \\ 0.372 & 0.001 & 0.094 & 0 & -0.021 & 0 \\ 0 & -0.027 & 0 & 0.024 & 0 & -0.026 \\ -0.225 & 0 & -0.021 & 0 & 0.023 & 0 \\ 0.001 & 0.038 & 0 & -0.026 & 0 & 0.087 \end{pmatrix}$$

The approximate method gives the stiffnesses in Table 5.4.

Stiffnesses	
Kx	28.58 kN/mm
Ky	20.69 kN/mm
Kz	10.61 kN/mm
Mx	42150 kNm/rad
My	44210 kNm/rad
Mz	11444 kNm/rad

Table 5.4. Approximate stiffness in anchor point for model 2

The natural frequencies of the small bore pipe with and without the stiffnesses are presented in Table 5.5 below.

Natural frequncies (Hz) for model 2	
Fixed anchor	Spring stiffness
49.451	47.0066
51.074	47.5343
322.33	131.637

Table 5.5. Natural frequencies of the small bore pipe with and without spring stiffness for model 2.

The springs replace the completely rigid anchor point and reduce the natural frequencies of the small bore pipe. This changes the maximum response for each mode since the response is collected from the response spectrum for each natural frequency.

5.2.4 Summary Model 2

Presented in Table 5.6 is the ratio of calculated stress in the separated small pipe model compared to calculated stress in the full model. If the ratio is 1 the modified response spectrum caused the same stress in that node as the full model. Ratio > 1 means the modified response spectrum produced conservative results, ratio < 1 non-conservative. Node 642 is the connection point, 3070 is the last node.

The results in Table 5.6 show that the stresses in the small pipe are described well by the use of the modified response spectrum. The stresses in the removed small pipe are lower near the connection point, and near the end of the pipe the stresses are the same. The modified response spectrum with the CQC method with spring stiffness at the anchor point give stresses that are 92% - 101% of those calculated with the full model.

	Model 2 – Summary of stress ratio in separated pipe					
	ABSSUM		SRSS		CQC	
Node	Fixed anchor	Spring	Fixed anchor	Spring	Fixed anchor	Spring
642	0.76	0.83	0.89	0.89	0.89	0.92
3000	0.76	0.82	0.90	0.90	0.89	0.92
3010	0.75	0.80	0.90	0.91	0.89	0.93
3020	0.76	0.80	0.91	0.92	0.90	0.93
Z001	0.77	0.80	0.92	0.93	0.92	0.95
3030	0.79	0.82	0.95	0.95	0.94	0.97
3040	0.80	0.82	0.96	0.96	0.95	0.98
3050	0.88	0.88	1	0.99	0.99	1.01
3060	0.91	0.91	1	1	1	1
3070	1	1	1	1	1	1

Table 5.6. Ratio of stress in small bore pipe from model 1 nodes using the modified response spectrum compared to the original response spectra.

6. Conclusions

There is no experimental data to compare the results to at this time. This makes the modified response spectrum hard to validate from a practical standpoint. The thesis only examines two models using certain computer software and has a very limited scope. Still, the results from the two models can be looked at in a number of ways from a theoretical and norm-based standpoint.

It can be concluded that the method of a modified response spectrum does not provide consistently conservative results if they are compared to ASME-accepted methods such as modeling the complete system as one. The reason that conservatism is being considered is that there is a desire for a simple method that allows for the parting of small bore pipes. If the results are shown to be conservative, small pipes could be allowed to be removed and analyzed separately using the modified response spectrum. However, conservative results should perhaps not be expected. There is nothing in the approach that should lead to theoretical conservatism. There are no safety factors or other approximations, only the dynamic amplification factor which is merely the theoretical increase of response due to coinciding loading- and natural frequencies. In practical use the peaks of the modified response spectra would be broadened to achieve more conservatism.

The stresses in the separated pipe have been compared to the stresses in the same pipe in the original large system. Recall that one of the reasons for removing small bore pipes is twin modes, the effects of which are reduced when systems are downsized. With this in mind, the modified response spectrum could (perhaps even should) lead to non-conservative results when it is generated from a reduced model since the effects of twin modes are reduced. However, this should not influence the results that calculate response with the CQC method.

In the first model, the stresses are conservative in some parts and non-conservative in other parts of the small pipe. The modified response spectrum does not recreate the conditions that the small pipe is being subjected to when still connected in an adequate way. This could possibly, in part, have its origin in an approximation being made when the small pipe is removed. An assumption is made that the inertia of the small pipe is insignificant. The inertia of the small pipe is, however, affecting the system in this model, and this can be illustrated by calculating the modified response spectrum before and after the small pipe is removed. If the assumption is correct, the spectra should be identical. This is not the case as is seen in Section 5.1.2.

The second model is much larger in absolute size, but also relative to the removed pipe. The assumption that the small pipe is not affecting the system is more accurate than in the first model, and thus the spectra in Section 5.2.2 are the same. Both the model with the fixed connection point and the spring connection point give interesting results that gives reason to think that the modified response spectrum can recreate the conditions of the original system in a satisfactory way.

Using spring stiffness in the anchor point of the small pipe does seem to represent the connection point better, and in turn the results. It is reasonable to believe that a more realistic model of the anchor point would lead to even better resemblance. A full matrix formulation of the stiffness would likely yield even more similar results.

The results show that the method of a modified response spectrum can give non-conservative results. However, the “worst” non-conservative node still accounted for 88 % of the stress calculated with the small bore pipes left in the models. Even if the results are non-conservative, the comparisons made in this thesis might not tell the whole story. The response spectrum method is already making use of a lot of approximations and is accepted by ASME since it is considered conservative. Although not conservative, the results generated by the modified response spectrum could in fact be closer to the “true” response; true time-history or experimental response.

7. Future research

There are possible ways forward to theoretically validate the modified response spectrum method as conservative compared to ASME regulations.

The peaks of the modified response spectra could be broadened to include more conservatism.

A response spectrum load corresponds to several time history loads, and there are many time history loads applied on a system that gives the same response spectra reaction at the supports. ASME states that if three independent time history loads can be derived from a response spectrum and these are enveloped to ensure conservatism, it is permitted to adopt this time history load as a replacement for the original response spectrum. A system can be parted and the removed pipe is analyzed with the enveloped time history load. If the stresses found in the small pipe loaded by the time history load are found to be less than those generated by the modified response spectrum (generated from the same original response spectrum), it is reasonable to think that the modified response spectrum can also be accepted by ASME and other authorities on the matter.

If it cannot be proven conservative compared to methods currently accepted by ASME, the effect on a system's inertia by removing a pipe, and the subsequent effects on a modified response spectrum and its caused response, could be examined. Perhaps the modified response spectrum can give accepted and usable results if the inertia is not changed by a certain fraction.

The modeling of the connection point could be studied further. Using spring stiffness in all 6 degrees of freedom improved the model, and it is reasonable to think that a full stiffness matrix would improve the model even more. Also, the springs could be modeled on the connection point in the large pipe as well, to include some effects of the small pipe.

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Appendix A

Calculation results

Model 1

The first model analyzed is the one from Figure 4.1. There are two response spectra loads acting on two different levels. The modified response spectrum is calculated and can be found in Section 5. Table A.1 show the stress ratio in each node in the separated pipe.

Model 1 - ABSSUM comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
220	0.275	0.365	1.328658
Z001	0.138	0.16	1.15942
Z001	0.138	0.16	1.15942
270	0.128	0.167	1.304688
270	0.261	0.341	1.306513
280	0.249	0.32	1.285141
280	0.122	0.157	1.286885
290	0.072	0.058	0.805556
290	0.146	0.117	0.80137
300	0.128	0.115	0.898438
300	0.063	0.056	0.888889
Z002	0.019	0.016	0.842105
Z002	0.019	0.016	0.842105
310	0	0	1

Table A.1

These are the results using absolute summation on both levels and modes, in the order L/M/I.

Model 1 - SRSS comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
220	0.121	0.156	1.292471
Z001	0.049	0.06	1.22449
Z001	0.049	0.06	1.22449
270	0.051	0.067	1.313725
270	0.103	0.137	1.330097
280	0.097	0.14	1.443299
280	0.047	0.069	1.468085
290	0.022	0.018	0.818182
290	0.045	0.037	0.822222
300	0.042	0.032	0.761905
300	0.021	0.016	0.761905
Z002	0.006	0.005	0.833333
Z002	0.006	0.005	0.833333
310	0	0	1

Table A.2

Model 1 - CQC comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
220	0.121	0.158	1.309041
Z001	0.049	0.06	1.22449
Z001	0.049	0.06	1.22449
270	0.051	0.068	1.333333
270	0.103	0.138	1.339806
280	0.097	0.141	1.453608
280	0.047	0.069	1.468085
290	0.022	0.018	0.818182
290	0.045	0.038	0.844444
300	0.042	0.032	0.761905
300	0.02	0.016	0.8
Z002	0.005	0.005	1
Z002	0.005	0.005	1
310	0	0	1

Table A.3

The stress ratios in the separated pipe with a spring stiffness anchor loaded with a modified response spectrum are presented in Table A.4.

Model 1 – with spring support

ABSSUM		SRSS		CQC	
Separated pipe - Stress ratio	% of stress	Separated pipe - Stress ratio	% of stress	Separated pipe - Stress ratio	% of stress
0.389	1.416	0.173	1.433	0.174	1.442
0.154	1.116	0.058	1.184	0.058	1.184
0.154	1.116	0.058	1.184	0.058	1.184
0.19	1.484	0.081	1.588	0.082	1.608
0.388	1.487	0.166	1.612	0.167	1.621
0.365	1.466	0.167	1.722	0.169	1.742
0.179	1.467	0.082	1.745	0.083	1.766
0.065	0.903	0.02	0.909	0.020	0.909
0.133	0.911	0.04	0.889	0.040	0.889
0.132	1.031	0.037	0.881	0.037	0.881
0.065	1.032	0.018	0.857	0.018	0.900
0.019	1	0.005	0.833	0.005	1
0.019	1	0.005	0.833	0.005	1
0	1	0	1	0	1

Table A.4

Model 2

Model 2 (312) - ABSSUM comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
642	0.168	0.128	0.761905
3000	0.159	0.121	0.761006
3000	0.159	0.121	0.761006
3010	0.53	0.4	0.754717
3010	0.53	0.4	0.754717
3020	0.517	0.391	0.756286
3020	0.517	0.391	0.756286
Z001	0.435	0.333	0.765517
Z001	0.435	0.333	0.765517
3030	0.343	0.271	0.790087
3030	0.343	0.271	0.790087
3040	0.327	0.26	0.795107
3050	0.223	0.196	0.878924
3060	0.161	0.146	0.906832
3060	0.161	0.146	0.906832
3070	0.084	0.084	1

Table A.5

Model 2 (312) – SRSS comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
642	0.065	0.058	0.892308
3000	0.062	0.056	0.903226
3000	0.062	0.056	0.903226
3010	0.198	0.179	0.90404
3010	0.198	0.179	0.90404
3020	0.194	0.176	0.907216
3020	0.194	0.176	0.907216
Z001	0.168	0.155	0.922619
Z001	0.168	0.155	0.922619
3030	0.139	0.132	0.94964
3030	0.139	0.132	0.94964
3040	0.134	0.128	0.955224
3050	0.124	0.124	1
3060	0.106	0.106	1
3060	0.106	0.106	1
3070	0.084	0.084	1

Table A.6

Model 2 (312) - CQC comparison

Node	Pipe still in model -Stress ratio	Separated pipe - Stress ratio	% of stress
642	0.064	0.057	0.890625
3000	0.061	0.054	0.885246
3000	0.061	0.054	0.885246
3010	0.196	0.175	0.892857
3010	0.196	0.175	0.892857
3020	0.192	0.172	0.895833
3020	0.192	0.172	0.895833
Z001	0.166	0.152	0.915663
Z001	0.166	0.152	0.915663
3030	0.137	0.129	0.941606
3030	0.137	0.129	0.941606
3040	0.132	0.125	0.94697
3050	0.123	0.122	0.99187
3060	0.106	0.105	0.990566
3060	0.106	0.105	0.990566
3070	0.084	0.084	1

Table A.7

The stress ratios in the separated pipe with a spring stiffness anchor loaded with a modified response spectrum are presented in table A.8.

Model 2 – with spring support

ABSSUM		SRSS		CQC	
Separated pipe - Stress ratio	% of stress	Separated pipe - Stress ratio	% of stress	Separated pipe - Stress ratio	% of stress
0.139	0.83	0.058	0.89	0.059	0.92
0.130	0.82	0.056	0.90	0.056	0.92
0.130	0.82	0.056	0.90	0.056	0.92
0.425	0.80	0.181	0.91	0.183	0.93
0.425	0.80	0.181	0.91	0.183	0.93
0.414	0.80	0.178	0.92	0.179	0.93
0.414	0.80	0.178	0.92	0.179	0.93
0.348	0.80	0.156	0.93	0.157	0.95
0.348	0.80	0.156	0.93	0.157	0.95
0.281	0.82	0.132	0.95	0.133	0.97
0.281	0.82	0.132	0.95	0.133	0.97
0.269	0.82	0.128	0.96	0.129	0.98
0.197	0.88	0.123	0.99	0.124	1.01
0.146	0.91	0.106	1	0.106	1
0.146	0.91	0.106	1	0.106	1
0.084	1	0.084	1	0.084	1

Table A.8