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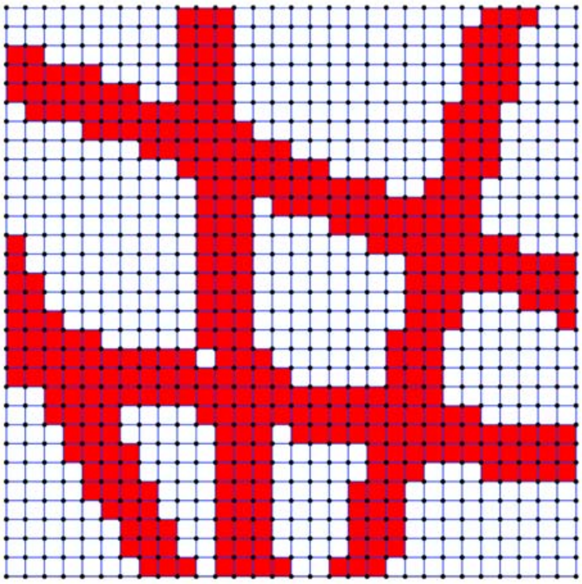

NUMERICAL INVESTIGATION OF THE HOMOGENEOUS STRAIN ASSUMPTION FOR WOOD FIBRE COMPOSITE MATERIALS

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Abstract

The stiffness and hygroexpansion properties of a wood fibre composite material was calculated by means of finite element analysis based homogenisation. The purpose was to investigate if the often used assumption of homogeneous strain is valid when analysing paper based composite materials. The representative volume element, the RVE, consists of in-plane randomly oriented orthotropic fibres, fibre crossings and an isotropic matrix material in the complementary area. The RVE was subjected to cyclic boundary conditions and plane stress. For a fibre volume fraction of 60 percent, which is typical for paper based composite products, the assumption of homogeneous strain was found to overestimate the composite stiffness by approximately 15 percent and underestimate the hygroexpansion by approximately 15 percent as compared to the results of the numerical simulations. These deviations produced by the homogeneous strain assumption can be considered to be within an acceptable range of engineering accuracy for analytical models for simplified analysis of stiffness and hygroexpansion properties of paper based composite materials.

Introduction

High pressure laminate, HPL, is a wood fibre composite composed of craft paper impregnated by phenolic or melamine resin. An important issue concerning wood composites is that of shape stability in relation to moisture content gradients and changes [1]. This entails a need for models that enable analysis and prediction of the hygroexpansion and stiffness properties of wood fibre composites such as HLP.

Various analytical models for analysis of the stiffness properties of fibre composite materials have been developed in recent decades [2]. A common approach is to model the composite as a single fibre surrounded by matrix material [3,4,5]. These models assumes there to be no interaction between the fibres. Such an assumption is, however, questionable in connection with HPL since the particle phase of this composite material is paper, which is a network of long fibres that are bonded to each other. Another approach that strongly simplifies calculations and may be relevant for HPL composites is to assume homogeneous strain. Such a simplification makes it possible to model also the complex material behaviour of the wood fibres, taking into account creep and mechano-sorptive strains [6]. It has been shown, however, that the homogeneous strain assumption will always overestimate the stiffness of a composite material [7,8]. The aim of this investigation is partly to find out if this overestimation is sufficiently small for the assumption to be acceptable in modelling. In the investigation is moreover the homogeneous strain based prediction of hygroexpansion properties studied.

Developments in the area of numerical simulations by the finite element method have made it possible to model complicated fibre composite material structures, consisting of more than one fibre [9,10,11]. Most of these studies have dealt with estimation of the stiffness properties of plane fabric composites [12,13,14,15,16]. These materials have a weaved fibre phase in which the fibres are crossed over and under each other at right angles. Since the fabric geometry is regular, homogenisation of a single fibre crossing is sufficient to provide a satisfactory estimate of the composite material stiffness. Fibre networks of a more random type have also been studied using numerical simulations [17,18]. There has been no study at a micro mechanical level, however, of the behaviour of network composite materials of irregular geometry. It seems that numerical simulations of hygroexpansion properties of composite materials with an irregular material structure are lacking.

The present investigation is performed by comparing the results from a homogenisation performed under the assumption of homogeneous strain and a numerical homogenisation of the same representative volume element (RVE), a unit cell. The unit cell is a representation of a wood fibre composite with an irregular fibre network, such that of paper. The parameters of the model are the mechanical properties of the constituents, the fibre geometry, the fibre orientation distribution and the fibre volume fraction. The configuration of the fibre network, number and orientation of the fibres, and is created by a Monte Carlo simulation process. This enables a good representation of the micro-structure of the fibre composite material.

The numerical results on stiffness and hygroexpansion are presented for a wide range of fibre volume fractions. The results are compared to those obtained but the homogeneous strain assumption and also to those obtained by the homogeneous stress assumption. To determine a sufficient size of the RVE, calculations were made for a range of ratios between fibre width and unit cell side length. To determine how fine the finite element mesh needs to be for good numerical accuracy, calculation were made for a range of ratios between size of the finite elements and size of the RVE. The numerical results versus the corresponding results of the homogeneous strain assumption made it possible to make an estimation of the validity of that later assumption when used in modelling of deformations of HPL materials as affected by stress, moisture, moisture changes and time of loading.

The Numerical Model

Figure 1 shows an example of an RVE. In this example there are 5 fibres in the RVE and the RVE is modelled by 30x30 finite elements. The RVE is a square with unit a constant thickness and the finite elements are plane stress square shaped elements of equal size and of the Melosh type with 4 nods. The fibre material is modelled as orthotropic linear elastic and the matrix material as isotropic linear elastic. Also the hygroexpansion properties of the fibres and the matrix are assumed to be orthotropic and isotropic, respectively. The geometry and location of a fibre is defined by pre-defined constant width and randomly selected orientation and location in the RVE. A finite element is assigned the properties of a fibre if more than 50 percent of its volume is within a fibre, and vice versa. In areas of fibre crossings are the finite elements assigned the average stiffness properties of the crossing fibres. Cyclic boundary conditions are employed. This means that the RVE acts as being surrounded by similar RVE:s and assumed to be a small part of an infinitely large volume of composite material. Estimations of material properties of the cellulose fibre and melamine resin are from [19] and [20] and is presented in Table 2 and Table 3, respectively.

The stiffness and hygroexpansion properties of the composite are evaluated by exposing the square unit cell to 4 different loadings: global tensile strain in the *x*-direction, global tensile strain in the *y*-direction, global shear strain in the *x*-*y* plane and increase in moisture content in the fibres and the matrix. The response of the unit cell to these loads is obtained by solving the finite element system of equations. The stiffness tensor of the unit composite cell is the calculated from the mean strains and the boundary traction vectors and the hygroexpansion parameters of the composite cell are obtained from the average strains in the *x*- and *y*-directions when exposing the cell to increased moisture.

The model was implemented in Matlab-code [21], using the free-ware finite element toolbox CALFEM [22] for the definition of the elements and the material properties and for assembling and solving of the equation system. Evaluation of the composite material properties was then made by means of Matlab.

Table 1: Model parameters

Side length of RVE (mm) = $l_x = l_y$	10 mm
Number of elements/side	20, 30, 50, 75, 100, 150
Fibre width / side length	0.02, 0.04, 0.05, 0.06, 0.08, 0.1, 0.125, 0.15, 0.2, 0.25, 0.3, 0.4
Number of fibres/RVE	Randomly from 1 to 20

Table 2: Fibre material parameters in local coordinates, *x* being the longitudinal direction of the fibre, *y* and *z* being the transverse directions, respectively:

E_x (GPa)	40	ν_{xy} (-)	0.2	G_{xy} (GPa)	4	β_x (-)	$0.01 \cdot 10^{-2}$
E_y (GPa)	5	ν_{xz} (-)	0.2	G_{xz} (GPa)	4	β_y (-)	$0.26 \cdot 10^{-2}$
E_z (GPa)	5	ν_{yz} (-)	0.3	G_{yz} (GPa)	2	β_z (-)	$0.26 \cdot 10^{-2}$

Table 3: Matrix material parameters:

E (GPa)	5.75
ν (-)	0.3
β (-)	$0.01 \cdot 10^{-2}$

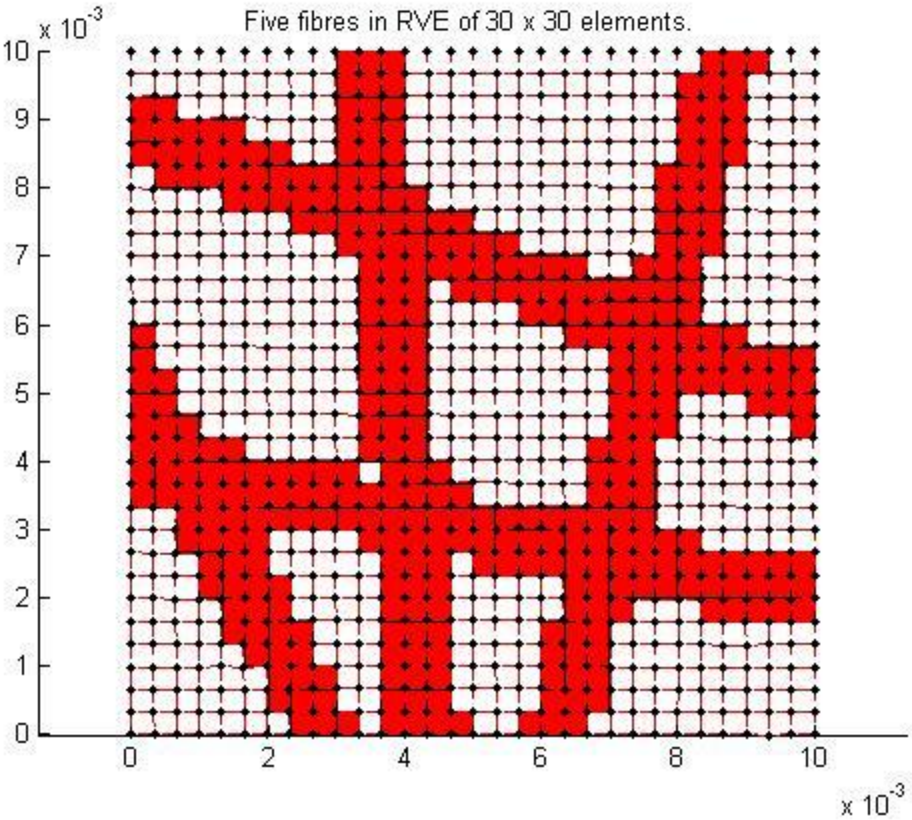

Figure 1: A square unit cell with physical dimensions 10 x 10 mm, with 30x30 elements and 5 fibres with a width relative to unit cell side length of 0.1.

Choice of modelling parameters

A sufficient size of the RVE and a sufficiently fine finite element mesh are needed for representative and accurate numerical results. These modelling parameters were therefore studied before investigating the homogeneous strain assumption.

Figure 2 shows the calculated normalized stiffnesses in the *x*- and *y*-directions as a function of the number of finite elements, n, along a side of the RVE. The number of degrees of freedom is $2n^2$ and calculations were made for $n = 20$ to $n = 150$. Simulations with three different fibre configurations showed an accuracy better than 1 percent for $n \geq 50$ for all three different fibre configurations, both in the *x*- and *y*-directions. $n = 50$, corresponding to 2500 elements and 5000 degrees of freedom was estimated to be sufficiently large and was thus used throughout in the subsequent calculations.

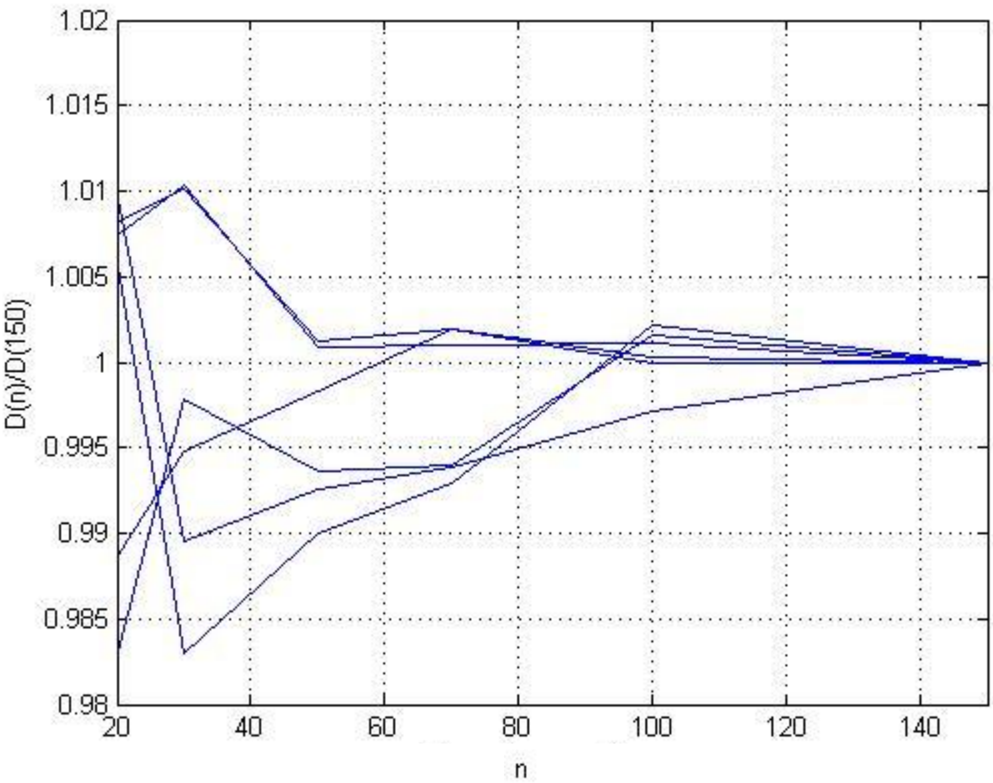

Figure 2. Convergence – the ratio between stiffness at n elements per side and stiffness at 150 elements per side.

The influence of ratio fibre width to cell side length was investigated by performing a large number of simulations for different fibre volume fractions. The simulations were divided into eight groups with a volume fraction within the following intervals: [15-25], [25-35], [35-45], [45-55], [55-65], [65-75], 7[5-85] and [85-95] percent. Different fibre volume fraction was for constant fibre width obtained by change of the number of fibres. This means that any certain interval of fibre volume fraction comprise fibre networks made up of few wide fibres as well as networks made up of many narrow fibres.

Figure 3 shows the stiffnesses found for the fibre volume fraction interval 55-65 percent. The figure also shows a linear regression for stiffness versus fibre width with an estimated inclination k = -3.08 GPa per 100 percent, that is less than the estimated inclination's standard deviation σ = 3.12 GPa per 100 percent.

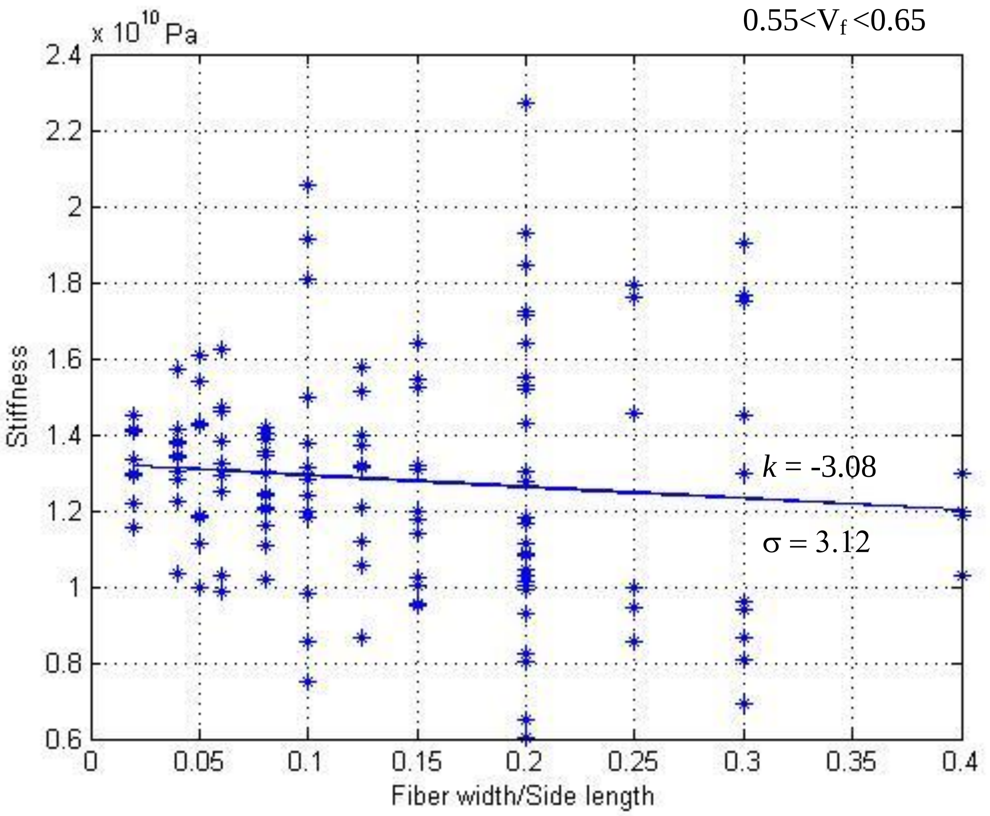

Figure 3. Unit cell stiffness as a function of fibre width relative to side length. Volume fraction of the unit cells are in the interval between 55 and 65 percent.

Figure 4 shows the regression lines for all above mentioned intervals. Higher fibre volume fraction implicates higher stiffness, which means that the regression line from Figure 3 is the fifth line from the bottom and is the line with the largest inclination. The results indicate that there is no significant influence of the ratio fibre width to unit cell length if less than about 0.3. A fibre width of 0.1 times the cell side length was used in the subsequent simulations.

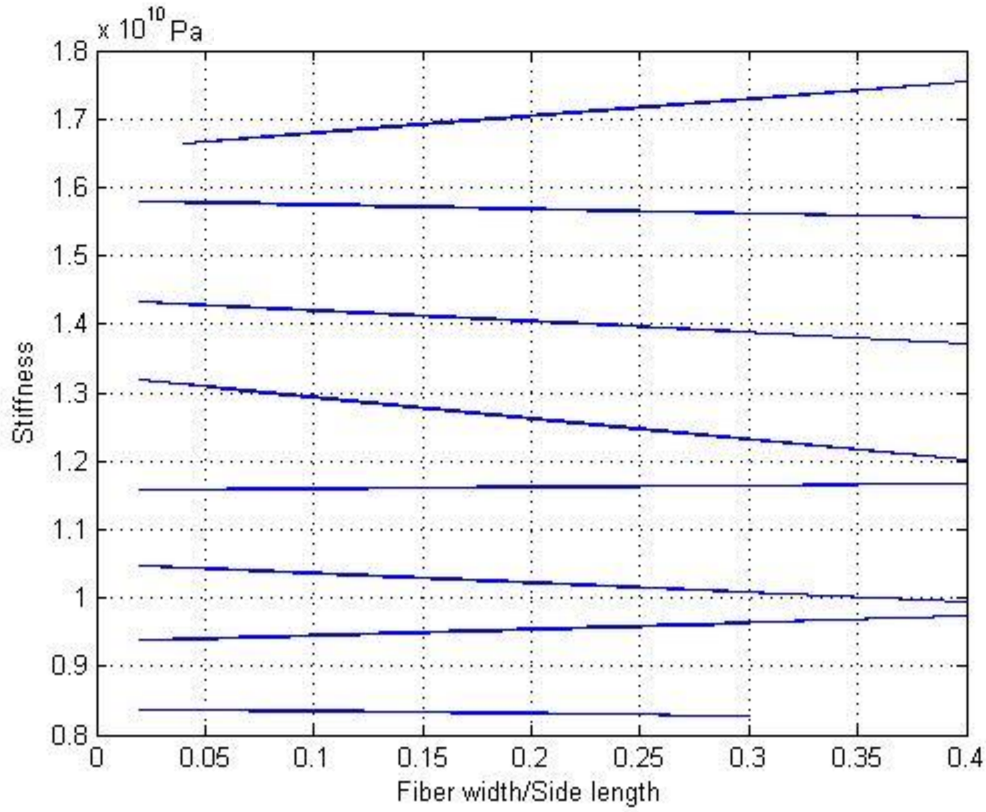

Figure 4. Unit cell stiffness as a function of fibre width relative to side length. All intervals of volume fractions, from bottom and up: 15-25 percent, 25-35 percent up to 85-95 percent.

Results

For investigation of the homogeneous strain and stress assumptions was 100 unit cells generated with a random number of fibres in the range from 1 to 20, oriented and located at random. The stiffness and moisture expansion properties of these cells were determined as described in the above and then, for each individual unit cell, compared to the corresponding results as produced by the homogeneous strain and homogeneous stress assumptions. The homogeneous strain composite material stiffness was calculated as the average material stiffness matrix of the 2500 finite elements and homogeneous stress stiffness was calculated as the inverse of the average compliance matrix for the finite elements. The homogeneous strain and stress composite material moisture expansion properties were calculated in the corresponding manner. The stiffness results are shown in Figure 5 where the 2x3x100=600 stiffnesses are plotted as a function of fibre volume fraction. The upper and lower bounds of the stiffness are for each fibre cell configuration given by homogeneous strain and stress assumptions. In figure 6 is the ratio between the homogeneous strain stiffness and the numerically simulated stiffness shown for the 2x100 stiffness values. This ratio is, as expected, for both directions and for all unit cells greater or equal to 1.0.

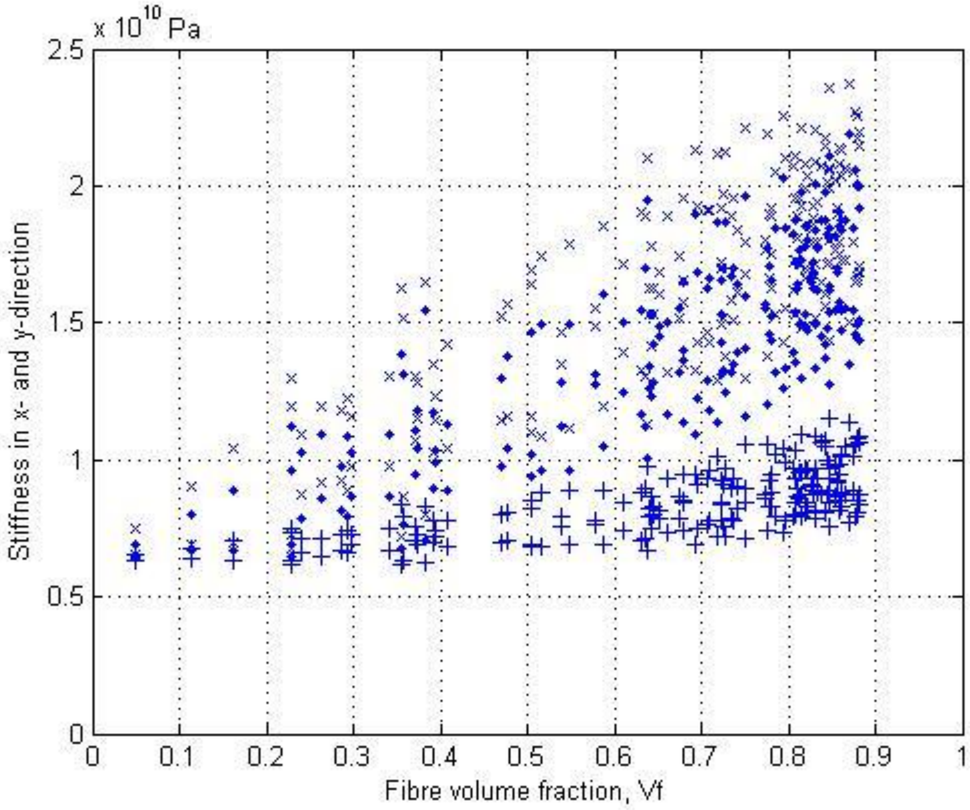

Figure 5. Homogenised stiffness in Pa in *x*- and *y*-directions according to: × – homogeneous strain, • – finite element simulation, + – homogeneous stress assumption.

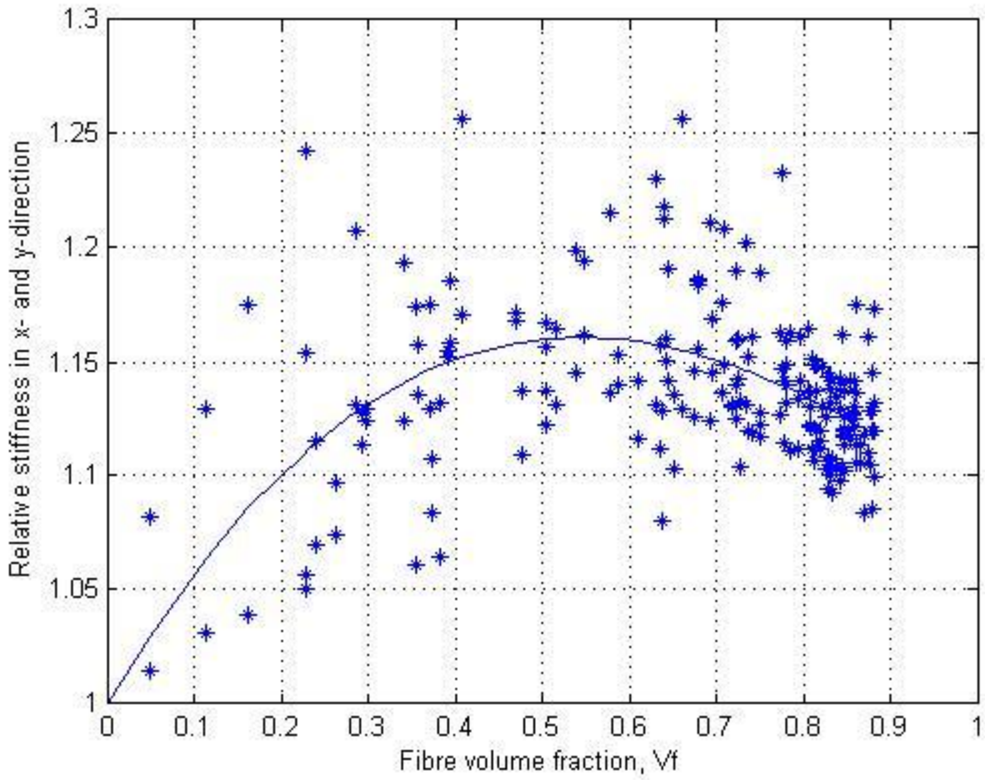

Figure 6. Relative stiffness = stiffness according to homogeneous strain divided by simulated stiffness for each simulated unit cell with a regression line according to least square method.

The free hygroexpansion results are shown i Figure 7. Here the homogeneous strain and stress assumptions are no longer giving bounds for the material properties as is the case of stiffness. This is evident in Figure 8, showing the hygroexpansion of the homogeneous strain assumption divided by the hygroexpansion of the simulations. For fibre volume fractions less than about 0.3 is the hygroexpansion ratio greater than 1, and for V_f greater than about 0.3 less than 1.

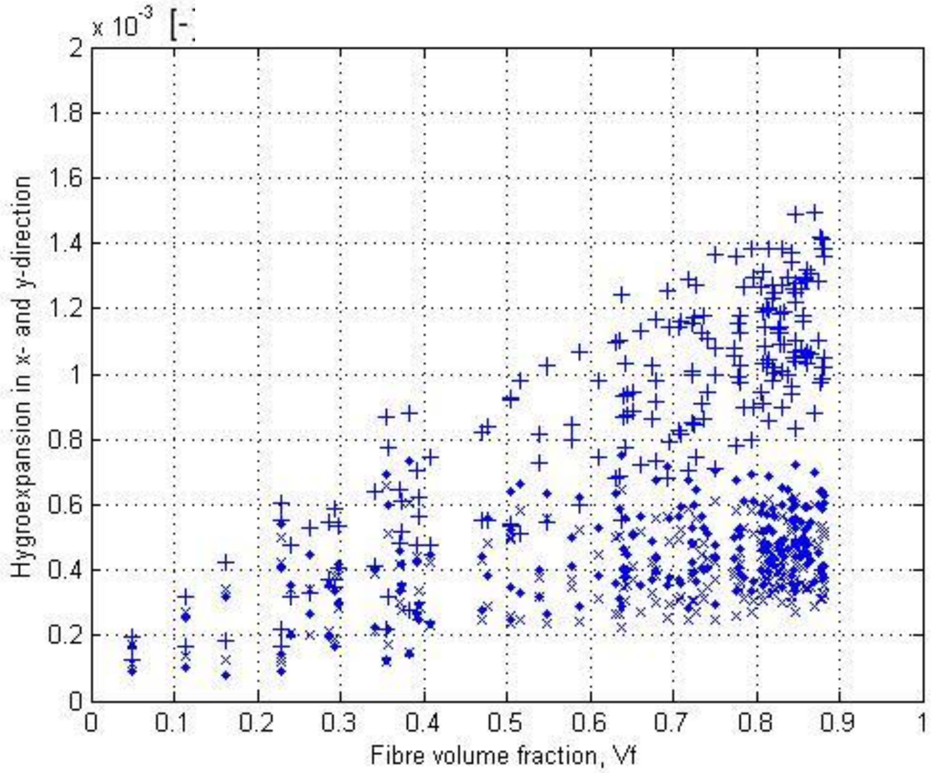

Figure 7. Homogenised hygroexpansion in *x*- and *y*-directions according to: x – homogeneous strain, · – finite element simulation, + – homogeneous stress assumption.

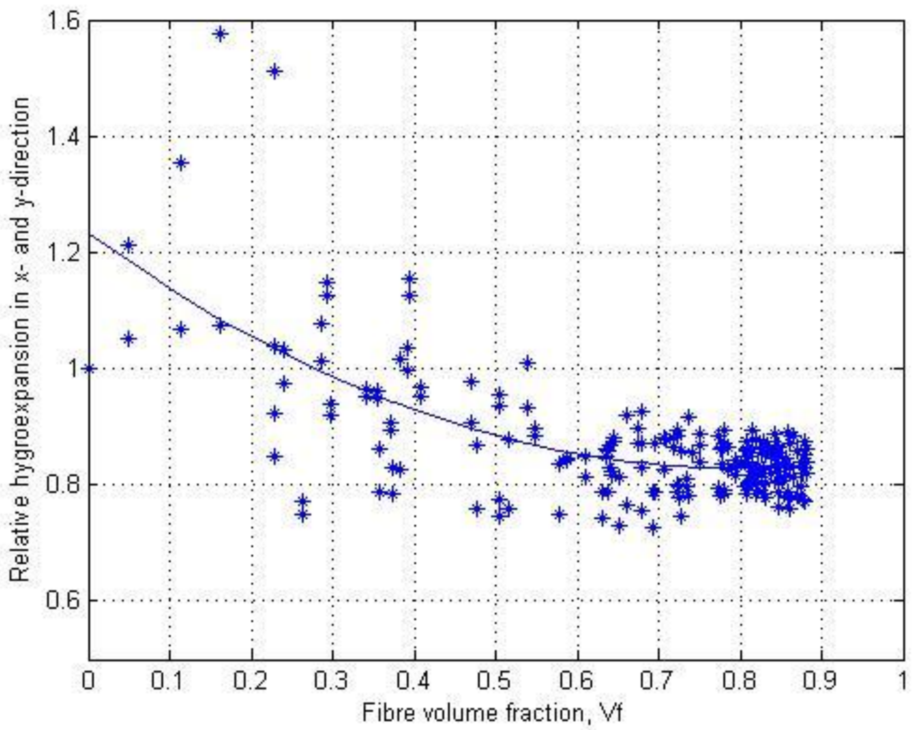

Figure 8. Relative hygroexpansion = hygroexpansion according to homogeneous strain divided by simulated hygroexpansion, with a regression line, calculated according to least square method.

Conclusion

This paper presents a homogenisation of a wood fibre network composite material. The homogenisation was performed using finite element analysis and cyclic boundary conditions and the aim of the analysis was to investigate if the assumption of homogeneous strain is valid for estimating stiffness and hygroexpansion of wood fibre composite materials with an in-plane random fibre orientation distribution.

First modelling parameters in terms required size and required finite element mesh of the RVE were determined. An RVE with a side length 10 times the fibre width and represented by a uniform finite element mesh with 50x50 elements were found to be sufficient. In the subsequent analysis were 100 RVE:s with various fibre volume fraction and with random orientation and location of the fibres analysed with respect to stiffness and hygroexpansion.

The results from the homogenisations show that the homogeneous strain assumption overestimates the simulated stiffness by approximately 15 percent and underestimates the simulated hygroexpansion by approximately 15 percent at the most interesting fibre volume fraction, which is around 60 percent. The deviation of the results of the homogeneous strain assumption from the results of the simulations is not greater than it can be justified to use the assumptions in analytical modelling of HPL materials. There is also a possibility to introduce a correction factor to an analytical model to take the deviation into account.

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