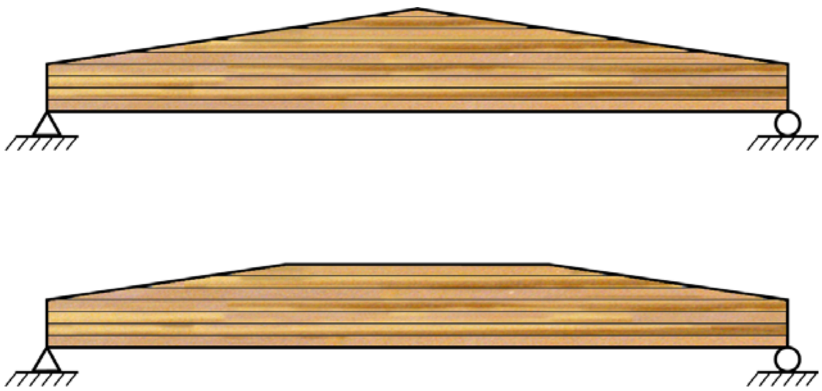

DESIGN AND PERPENDICULAR TO GRAIN TENSILE STRESS IN DOUBLE-TAPERED GLULAM BEAMS

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Abstract

Design and stress analysis of regular double-tapered beams and double-tapered beams with a flattened apex region is considered, focusing on perpendicular to grain tensile stress in apex region. Double-tapered beams with flattened apex region are analyzed since the geometry is such that the perpendicular to grain tensile stress may be reduced compared to a regular double-tapered beam of corresponding geometry.

The Eurocode 5 design criteria for regular double-tapered beams regarding bending stress, shear stress and perpendicular to grain tensile stress are reviewed. Design with respect to perpendicular to grain tensile stress is based on an approximate solution for maximum stress and Weibull type considerations for influence of size of stressed volume and heterogeneity in stress distribution on the strength.

The maximum stress, the size of the stressed volume and heterogeneity in stress distribution are analyzed by linear elastic finite element analysis for regular double-tapered beams and for double-tapered beams with a flattened apex region. For both geometry types, loading by a uniformly distributed load and point loads in the two quarter points of the beam is considered.

Based on the FE-analysis, proposals for design criterion with respect to perpendicular to grain tensile stress are also presented. The proposals consist of modifications of the Eurocode 5 design approach for regular double-tapered beams.

Keywords: glulam, tapered, perpendicular, stress, strength, design, Weibull

Sammanfattning

Dimensionering och spänningsanalys av sadelbalkar behandlas, med tyngdpunkt på tvärdragspänning i nockpartiet. Dimensioneringsvillkor enligt Eurocode 5 och till viss del även deras bakgrund presenteras och ett exempel av dimensionerande last med hänsyn till böjning, skjuvning samt tvärdrag presenteras. Exemplet visar att skjuvning har avgörande betydelse för dimensionering av sadelbalkar enligt Eurocode 5, vilket beror på den nyligen introducerade reduktionen av skjuvbärförmåga med hänsyn till befintliga sprickor. Bortses från denna reduktion är tvärdragspänning i nockpartiet ofta dimensionerande för sadelbalkar, med undantag för sadelbalkar med liten lutning då istället böjning kan vara dimensionerande. Dimensionering med hänsyn till tvärdragspänning är baserat på ett approximativt uttryck för maximal spänning i nockpartiet samt på inflytande av storleken på tvärdragsbelastad volym och heterogenitet i spänningsfördelning på bärförmågan.

Linjärelastisk spänningsanalys av konventionalla sadelbalkar och sadelbalkar med tillplattad hjässa presenteras. Sadelbalkar med tillplattad hjässa analyseras eftersom geometrin är sådan att tvärdragspänningen kan minskas jämfört med spänningen i motsvarande sadelbalk av konventionellt utförande. Resultat av spänningsanalysen presenteras i form av storlek på maximal spänning, storlek på tvärdragsbelastad volym samt heterogenitet i spänningsfördelningen. För balkgeometrier med tillplattad hjässa erhölls betydligt lägre värden på maximal tvärdragspänning jämfört med motsvarande sadelbalk av konventionell geometri. Vidare erhölls betydande skillnader i maximal tvärdragspänning och även betydande skillnader i storlek på tvärdragsbelastad volym mellan olika lastfall, vilka i denna analys var jämnt utbredd last och belastning av två punktlaster i balkens fjärdedelspunkter.

Enligt Eurocode 5 får dimenionerande tvärdragspänning reduceras för lastfall med jämnt utbredd tryckande last som verkar i balkens överkant. Motsvarande reduktion får dock inte göras av storleken på den tvärdragsbelastade volymen, som inverkar på dimensionerande hållfasthet. Den i Eurocode 5 antagna storleken av tvärdragsbelastad volym är dock mindre, i vissa fall betydligt mindre, än belastad volym enligt presenterad spänningsanalys för alla betraktade geometrier och lastfall. Trots att storleken på tvärdragsbelastad volym underskattas erhålls relativt god överensstämmelse mellan Eurocode 5 och presenterad spänningsanalys för det kombinerade inflytandet av belastad volym och heterogenitet i spänningsfördelning. Detta beror på att även heterogeniteten i spänningsfördelningen underskattas enligt Eurocode 5.

Två förslag på dimensioneringsvillkor med hänsyn till tvärdragspänningar presenteras. Dessa förslag är båda baserade på nuvarande dimensioneringsvillkor för konventionella sadelbalkar i Eurocode 5 med modifikationer av de approximativa uttrycken för maximal spänning, storlek på tvärdragsbelastad volym och inflytande av heterogenitet i spänningsfördelning. Föreslagna modifieringar är endast baserade på den presenterade spänningsanalysen och är således inte anpassade till resultat av experimentella tester på något vis.

Contents

1 Introduction

One of the most basic and most used structural elements is a simply supported beam with constant cross section carrying transverse load in bending, with shear- and normal stress distributions easily found from conventional engineering beam theory. The beam geometry can however be modified by shifting material volume from low to high stress regions to increase the efficiency of material use and maybe equally important to achieve a geometry with other desired qualities. One example is a double-tapered beam, where the beam geometry better resembles the bending stress distribution for many load cases and the geometry further gives a natural roof inclination. Modifying the beam geometry in this way does however also alter the internal shear- and bending stress distributions and furthermore introduces perpendicular to grain (hereafter abbreviated *perp-to-grain*) tensile stress in the apex region.

This report concerns design and stress analysis of double-tapered beams, focusing on finite element stress analysis of the perp-to-grain tensile stress in the apex region. Design according to Eurocode 5 and Limträhandbok regarding bending, shear and and perp-to-grain tensile stress are reviewed and the approximate solutions, on which design regarding bending and perp-to-grain tensile stress are based, are also briefly reviewed. Design with respect to perp-to-grain tensile stress is further based on Weibull type considerations of the influence of size of stressed volume and heterogeneity in stress distributions on the strength.

Two geometrical modifications according to Figure 1 of a regular double-tapered beam, aimed at reducing the perp-to-grain tensile stress, were considered. Introducing a vertical slit at the beam apex was in a preliminary analysis found to increase both stress magnitude and size of stressed volume and this approach was hence abandoned. In the following, only regular double-tapered beams and double-tapered beams with a ”flattened” apex region are considered. Stress magnitude, size of the stressed volume and heterogeneity in stress distribution are analyzed by linear elastic finite element analysis and results are compared to corresponding values according to the Eurocode 5 design criterion. Proposals for design criterion, based on modifications of the Eurocode 5 design criterion, are also presented.

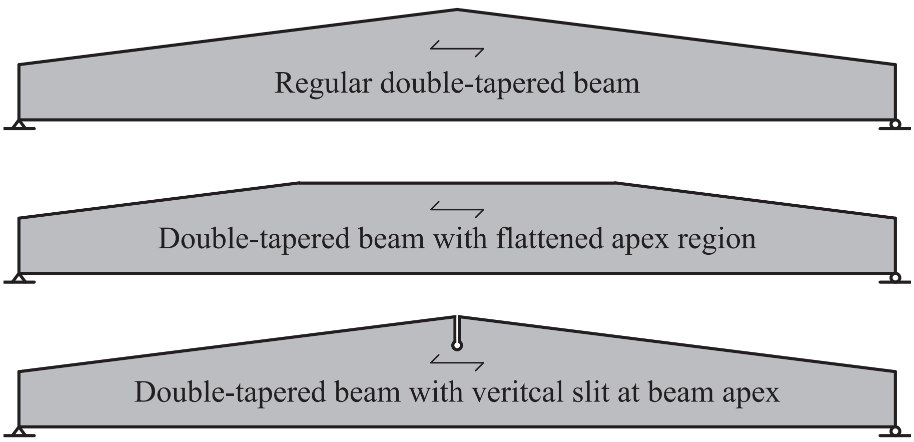

Figure 1: *Geometrical modifications aimed at reducing perp-to-grain tensile stress*

2 Approximate stress solutions

An approximate analytical solution of the bending stress distribution for a tapered beam is presented in [6]. The radial stress σ_r at a curved cross section which is neither close to support nor close to the apex, is approximated by the solution for the wedge OAB indicated in Figure 2. The solution is based on a linear elastic and orthotropic material behavior with one of the principle axis parallel to fiber direction. The extreme values of the stress σ_r, occurring at the tapered top edge and the straight bottom edge, were found to be

$$\sigma_r = \frac{6M}{bh^2} F_\alpha \qquad \text{where} \quad F_\alpha \approx \begin{cases} 1 - 3.7\tan^2\alpha & \text{for tapered edge} \\ 1 + 3.7\tan^2\alpha & \text{for straight edge} \end{cases} \tag{1}$$

and where M, b and h are bending moment, beam width and beam height respectively. The stresses at the tapered edge corresponding to material directions are then

$$\sigma_0 = \sigma_r \cos^2\alpha \tag{2}$$

$$\sigma_{90} = \sigma_r \sin^2\alpha \tag{3}$$

$$\tau = \sigma_r \sin\alpha\cos\alpha \tag{4}$$

The solution is slightly influenced by the ratio between stiffness parameters. The expressions for F_α in Equation (1) are based on stiffness parameters $E_x/E_y = 10$ and $E_x/G_{xy} - 2\nu_{xy} = 17$ with x-direction coinciding with grain direction. A slightly different expression for the tapered edge $F_\alpha = 1 - 4.4\tan^2\alpha$ is also found in literature, for example in [5] and [9].

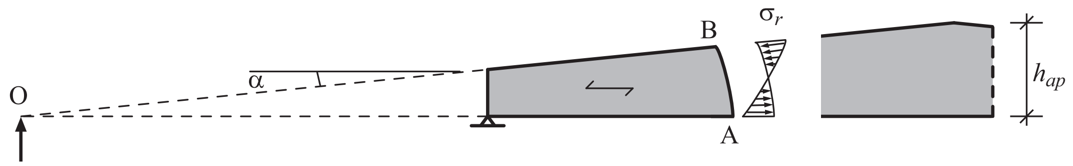

Figure 2: *Wedge considered for approximate solution of bending stress*

The perp-to-grain tensile stress in curved and tapered beams is analyzed in [4]. The maximum stress is, for several geometries, analyzed numerically by using a so called point matching technique. Based on plane stress analysis with a linear elastic and orthotropic material model, the maximum perp-to-grain tensile stress was for pure bending in apex region of a double-tapered beam found to be approximated by

$$\sigma_{90} \approx 0.2\tan\alpha \frac{6M_{ap}}{bh_{ap}^2} \tag{5}$$

where M_{ap} is the bending moment at apex and h_{ap} is the beam apex height. The results are also here slightly influenced by the elastic stiffness parameters. The approximation in Equation (5) is based on stiffness parameters $E_x/E_y = 18$ and $E_x/G_{xy} - 2\nu_{xy} = 11$ with x-direction coinciding with grain direction.

3 Weibull theory

The perp-to-grain tensile strength is for wood strongly dependent on the size of the stressed volume, which can be and often is explained by Weibull theory [10]. The basic assumptions in Weibull theory is that the weakest point in a stressed volume is decisive for the strength and that all material points are equal with respect to statistical probability distribution of strength. The global strength is hence size-dependent since the strength is limited by the weakest point and the larger the volume the more likely it is that severe defects are present. In addition to the pure volume influence, the stress distribution also effect strength. Based on the Weibull two-parameter model, a relationship between the strength f valid for a volume Ω with a heterogenous stress distribution $\sigma(x, y, z)$ and the reference strength f_{ref} valid for a homogeneous stress distribution in a reference volume Ω_{ref} is found to be

$$f = f_{ref}\left(\frac{\Omega}{\Omega_{ref}}\right)^{-1/m}\left(\frac{1}{\Omega}\int_{\Omega}\left(\frac{\sigma(x,y,x)}{\sigma_{max}}\right)^{m}\,d\Omega\right)^{-1/m} \tag{6}$$

where σ_{max} is the maximum of $\sigma(x, y, z)$ and m is the Weibull shape parameter related to the scatter in material strength, see for example [2]. The two last terms represent the influence of size of stressed volume and heterogeneity in stress distribution respectively. The relationship is in timber engineering design codes commonly expressed as

$$f = f_{ref}k_{vol}k_{dis} \qquad \text{where} \quad \begin{cases} k_{vol} &= \left(\frac{\Omega}{\Omega_{ref}}\right)^{-1/m} \\ k_{dis} &= \left(\frac{1}{\Omega}\int_{\Omega}\left(\frac{\sigma(x,y,x)}{\sigma_{max}}\right)^{m}\,d\Omega\right)^{-1/m} \end{cases} \tag{7}$$

where Ω_{ref} and m are given constant values related to the reference strength f_{ref}. The volume Ω and hence also k_{vol} is commonly given as a function of geometry while k_{dis} may be assigned a constant value or may be expressed in terms of geometry and/or load parameters for the specific application.

4 Review of code design criterion

Design criteria for regular double-tapered beams according to Eurocode 5 [7] and Limträhandbok [1] are reviewed in the following sections. A symmetric double-tapered beam of width b and with geometry according to Figure 3 is considered. The relevant equations regarding design with respect to bending stress, shear stress and tension stress perp-to-grain are reviewed. Compression at supports, combined effect of bending and axial force, lateral stability and other design issues are disregarded here.

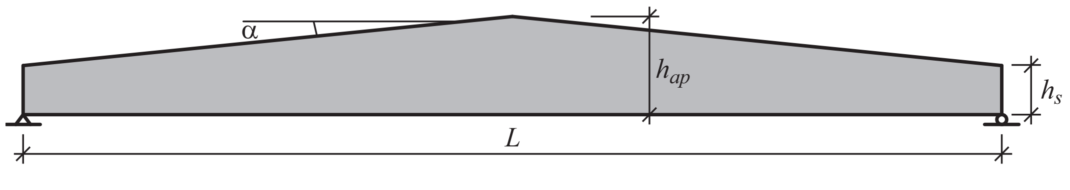

Figure 3: *Symmetric double-tapered beam*

4.1 Eurocode 5

The bending stress at the tapered edge $\sigma_{m\alpha}$ should fulfill

$$\sigma_{m\alpha} = \frac{6M}{bh^2} \leq k_{m\alpha} f_m \tag{8}$$

where f_m is the design bending strength and where for tensile stress at tapered edge

$$k_{m\alpha} = \frac{1}{\sqrt{1 + \left(\frac{f_m}{0.75 f_v} \tan\alpha\right)^2 + \left(\frac{f_m}{f_{t90}} \tan^2\alpha\right)^2}} \tag{9}$$

and where for compressive stress at the tapered edge

$$k_{m\alpha} = \frac{1}{\sqrt{1 + \left(\frac{f_m}{1.5 f_v} \tan\alpha\right)^2 + \left(\frac{f_m}{f_{c90}} \tan^2\alpha\right)^2}} \tag{10}$$

The bending stress at the apex σ_m should fulfill

$$\sigma_m = (1 + 1.4\tan\alpha + 5.4\tan^2\alpha)\frac{6M_{ap}}{bh_{ap}^2} \leq f_m \tag{11}$$

where M_{ap} is the bending moment at the apex.

The shear stress τ should fulfill

$$\tau = \frac{3V}{2b_{ef}h} \leq f_v \tag{12}$$

where f_v is the design shear strength and the effective beam width $b_{ef} = bk_{cr}$ where k_{cr} is a nationally determined parameter (NDP) related to an assumed strength reduction due to presence of cracks. The Swedish choice found in Appendix NA [7] is $k_{cr} = 0.67$. The shear force V may be determined by ignoring loads acting on the upper part of the beam, from the support and within a distance equal to the beam height at the support.

The perp-to-grain tensile stress σ_{t90} at the apex region should fulfill

$$\sigma_{t90} = 0.2\tan\alpha\frac{6M_{ap}}{bh_{ap}^2} \leq k_{dis}k_{vol}f_{t90} \tag{13}$$

$$k_{dis} = 1.4$$

$$k_{vol} = (V_0/V)^{0.2} \qquad \text{where } V = bh_{ap}^2$$

where f_{t90} is the design perp-to-grain tensile strength and $V_0 = 0.01\ \mathrm{m}^3$. As a nationally determined parameter (NDP), the design value of the perp-to-grain tensile stress σ_{t90} may alternatively be determined according to

$$\sigma_{t90} = 0.2\tan\alpha\frac{6M_{ap}}{bh_{ap}^2} - 0.6\frac{p}{b} \tag{14}$$

when there is a uniformly distributed (pressure) load p acting on top of the apex area. The Swedish choice found in Appendix NA [7] is that Equation (14) may be used.

4.2 Limträhandbok

The design criteria in Limträhandbok are in general consistent with the criteria in Eurocode 5, but there are however some exceptions:

- The first difference lies in the criterion for bending, which in Limträhandbok is expressed as

$$\sigma_{m\alpha} = k_{\sigma\alpha}\frac{6M}{bh^2} \leq k_{f\alpha}f_m \tag{15}$$

  where $k_{\sigma\alpha}$ accounts for the discrepancy in direction (at tapered edge) and in magnitude (at tapered edge and straight edge) from the linear bending stress distribution found in a beam of constant cross section

$$k_{\sigma\alpha} = \begin{cases} 1 - 4\tan^2\alpha & \text{for tapered edge} \\ 1 + 4\tan^2\alpha & \text{for straight edge} \end{cases} \tag{16}$$

  and where $k_{f\alpha}$ accounts for the difference in strength due to the stress component at an angle to grain according to

$$k_{f\alpha} = \begin{cases} \dfrac{1}{\frac{f_m}{f_{c90}}\sin^2\alpha + \cos^2\alpha} & \text{for compression at tapered edge} \\ \dfrac{1}{\frac{f_m}{f_{t90}}\sin^2\alpha + \cos^2\alpha} & \text{for tension at tapered edge} \end{cases} \tag{17}$$

- The second difference relates the design with respect to shear stress. In Limträhandbok, there is no reduction of shear strength with respect to presence of crack as stated in Eurocode 5. Design with respect to shear stress in Limträhandbok is equal to Equation (12) with $k_{cr} = 1.0$ and hence $b_{ef} = b$.

- The third difference relates to design with respect to perp-to-grain tensile stress, where in Limträhandbok nothing is stated relating to reduction of the magnitude in stress due to uniformly distributed load acting on top of the beam. The design perp-to-grain tensile stress is in Limträhandbok equal to Equation (13) although with 0.1 instead of 0.2, which however is believed to be a misprint.

4.3 Code design example

Strength design of double-tapered beams according to Eurocode 5 and Limträhandbok is in general limited due to either bending stress, shear stress or perp-to-grain tensile stress depending on beam geometry, load configuration and values of material strengths. An illustration of the design value q_d according to Eurocode 5 for a symmetric double-tapered beam exposed to a uniformly distributed load q is presented in Figure 4.

The introduction of the reduction factor $k_{cr} = 0.67$ in Eurocode 5, related to an assumed shear strength reduction due to presence of crack, has a major influence on the overall design of double-tapered beams. For the geometry and strength properties considered in Figure 4, the design load q_d is limited by shear stress for $h_{ap} > 1.37$ m ($\alpha > 1.9^o$) with the nationally determined parameter $k_{cr} = 0.67$.

Disregarding design with respect to shear stress, design is limited due to either bending stress or perp-to-grain tensile stress at apex region. For small values of beam apex height h_{ap} (small inclination α), the strength is then limit by bending stress at tapered edge. Perp-to-grain tensile stress at apex region is decisive for $h_{ap} > 1.65$ m ($\alpha > 3.7^o$) if the reduction due to uniformly distributed load on top of the beam is not accounted for. The design load q_d furthermore decreases with increasing beam apex height h_{ap}.

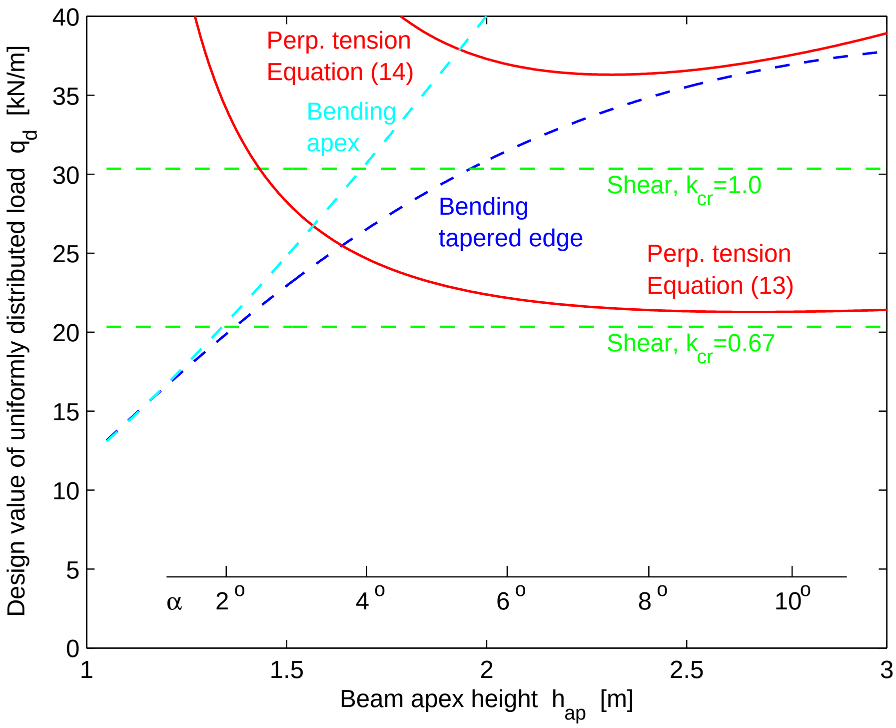

Figure 4: *Design value of uniformly distributed load q_d according to Eurocode 5 vs. beam apex height h_{ap} for $L = 20$ m, $b = 0.2$ m, $h_s = 1.0$ m with design strengths based on characteristic strengths for GL28h in [8]; $f_{mk} = 28$, $f_{c90k} = 3.0$, $f_{vk} = 3.2$, $f_{t90k} = 0.45$ MPa modified according to $k_{mod} = 0.8$ and $\gamma_m = 1.25$.*

5 Linear elastic finite element stress analysis

Two-dimensional plane stress finite element analysis was performed in order to investigate magnitude and distribution of the perp-to-grain tensile stress. An orthotropic and linear elastic material model was used with stiffness parameters $E_x = 12600$ MPa, $E_y = 420$ MPa, $G_{xy} = 780$ MPa and $\nu_{xy} = 0.35$ with x-direction coinciding with grain direction.

The analysis comprises beam geometries according to Figure 5; regular double-tapered beams and double-tapered beams with a flattened apex region. The beam length $L = 20$ m, beam width $b = 0.2$ m and beam height at supports $h_s = 1.0$ m were consistently used whereas the beam apex heights $h_{ap} = 1.25,\ 1.50,\ 1.75,\ 2.00,\ 2.25,\ 2.50$ and 2.75 m were considered. For beams with flattened apex region, the length of the flattened part was consistently $a = 2h_{ap}$ where h_{ap} refers to the theoretical beam apex height.

Two different load configurations were analyzed for both types of geometry, namely uniformly distributed load q or two point loads P acting at the quarter points of the beam length. Results of stress magnitude are based on applied loads $q = 20$ kN/m or $P = 200$ kN, which for both cases results in a bending moment at mid span of $M_{ap} = 1000$ kNm.

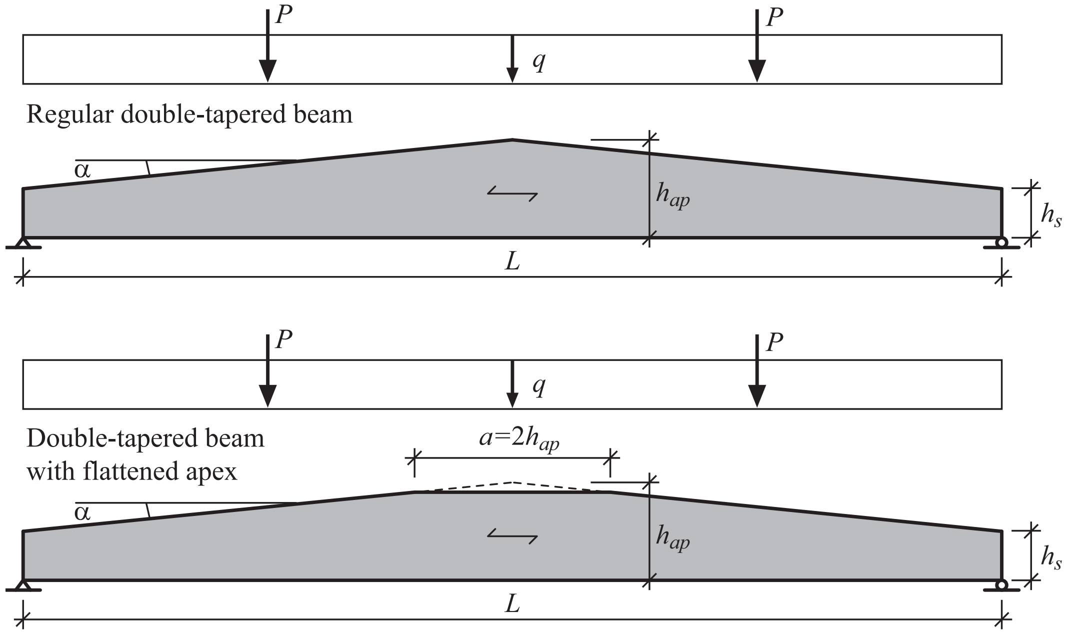

Figure 5: *Beam geometries and load configurations*

For convenient presentation of results, the following notation is adopted:

FE-q	FE analysis, uniformly distributed load q, regular double tapered beam
FE-q-a	FE analysis, uniformly distributed load q, flattened double tapered beam
FE-PP	FE analysis, two point loads P, regular double tapered beam
FE-PP-a	FE analysis, two point loads P, flattened double tapered beam
EC5	Eurocode 5, according to Equation (13)
EC5-q	Eurocode 5, according to Equation (14)

The results of the FE-analysis are presented in Figures 6-10. In Figures 7-10, only the perp-to-grain tensile stress within a beam volume of length $0.2L = 4$ m on both sides of the beam mid point is considered. This volume includes all perp-to-grain tensile stress in apex vicinity and excludes all perp-to grain stress near supports. The results relating to FE-q-a with beam apex heights $h_{ap} = 1.25$ m and 1.5 m seem to be unreliable. Both maximum stress and size of the stressed volume are very small and seem to be mesh density sensitive. These results are represented by dashed lines.

Typical examples of the perp-to-grain tensile stress are presented in Figure 6 for the four different combinations of beam geometry type and load configuration. The influence of load configuration, uniformly distributed load q or point loads P, on both stress magnitude and on size of stressed volume is obvious. Although applied loads correspond to equal bending moment at apex, uniformly distributed load results in significantly lower values of both maximum stress and size of the stressed volume. For both load configurations, the maximum stress is significantly reduced for the beams with a flattened apex region compared to the regular double-tapered geometry. The size of the stressed volume seems however to be fairly equal for the different beam geometry types, considering the two load configurations separately.

The magnitude of the maximum perp-to-grain tensile stress vs. beam apex height is presented in Figure 7. The value of the maximum perp-to-grain tensile stress for FE-PP is 6-18% greater than the stress according to Eurocode 5 stated in Equation (13). For uniformly distributed load (FE-q), the maximum perp-to-grain tensile stress is approximately 20% lower than the design stress according to Eurocode 5 stated in Equation (14). The maximum stress is significantly lower for the beams with flattened apex region (FE-PP-a and FE-q-a) compared to regular double-tapered beams with corresponding load configuration and beam apex height (FE-PP and FE-q).

Stressed volume factor k_{vol} vs. beam apex height is presented in Figure 8. For the FE-analysis, k_{vol} is determined numerically based on Equation (7). The size of the stressed volume is in the FE-analysis in general found to be greater (hence lower value of k_{vol}) than what is assumed in Eurocode 5. The size of the stressed volume is further greater for load configuration PP (FE-PP and FE-PP-a) compared to load configuration q (FE-q and FE-q-a).

Stress distribution factor k_{dis} vs. beam apex height is presented in Figure 9. For the FE-analysis, k_{dis} is determined numerically based on Equation (7). The FE-analysis suggests more heterogeneous distributions (hence greater value of k_{dis}) than assumed in Eurocode 5. The FE-analysis further indicates a slight decrease in stress distribution heterogeneity with increasing beam apex height.

The design bending moment at beam apex M_{apd} vs. beam apex height is presented in Figure 10, based on characteristic perp-to-grain tensile strength $f_{t90k} = 0.45$ MPa modified according to $k_{mod} = 0.8$ and $\gamma_m = 1.25$ giving a design strength of 0.288 MPa. The design bending moments related to the FE-analysis are determined from the previously presented equations according to Weibull theory. Although the approximations for the separate components σ_{t90}, k_{vol} and k_{dis} in Eurocode 5 differ compared to the FE-analysis for load configuration PP, the combined influence results in fairly equal design bending moments. For uniformly distributed load q, design according to Eurocode 5 is conservative compared to results of the FE-analysis. The design bending moment for FE-q-a, not shown in Figure 10, is considerably greater than FE-q, FE-PP and FE-PP-a.

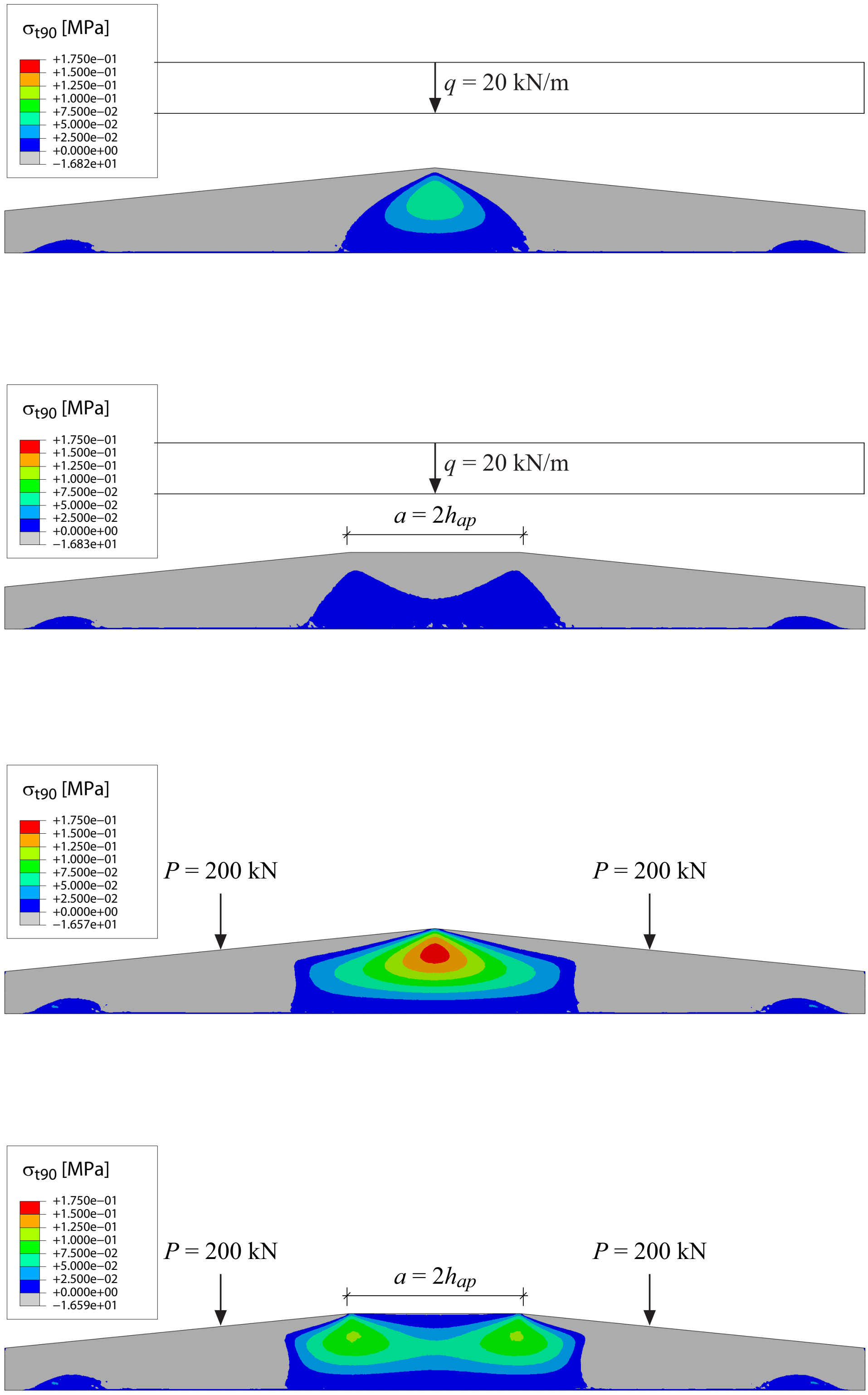

Figure 6: *Perp-to-grain tensile stress σ_{t90} at external loads corresponding to bending moment $M_{ap} = 1000$ kNm for $L = 20$ m, $b = 0.2$ m, $h_s = 1.0$ m and $h_{ap} = 2.0$ m*

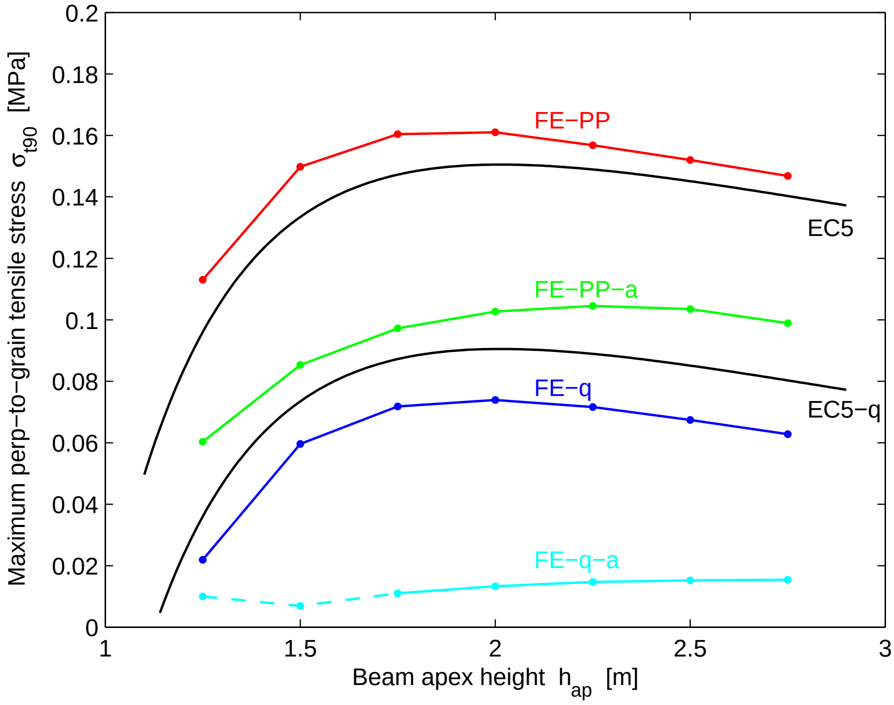

Figure 7: *Maximum perp-to-grain tensile stress vs. beam apex height h_{ap} according to Eurocode 5 and FE-analysis for applied loads corresponding to $M_{ap} = 1000$ kNm*

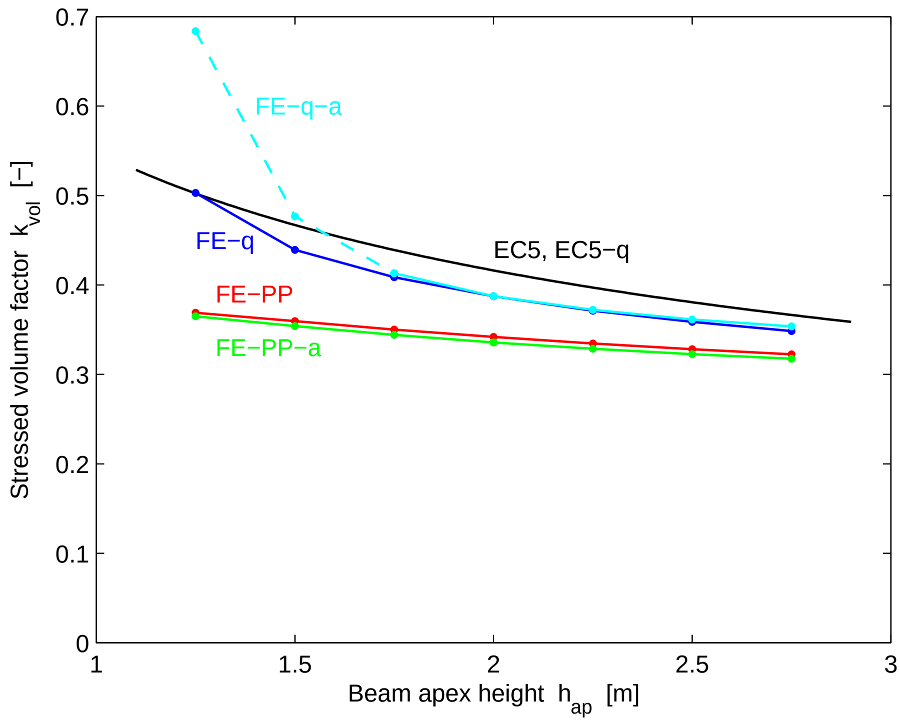

Figure 8: *Stressed volume factor k_{vol} vs. beam apex height h_{ap} according to Eurocode 5 and FE-analysis*

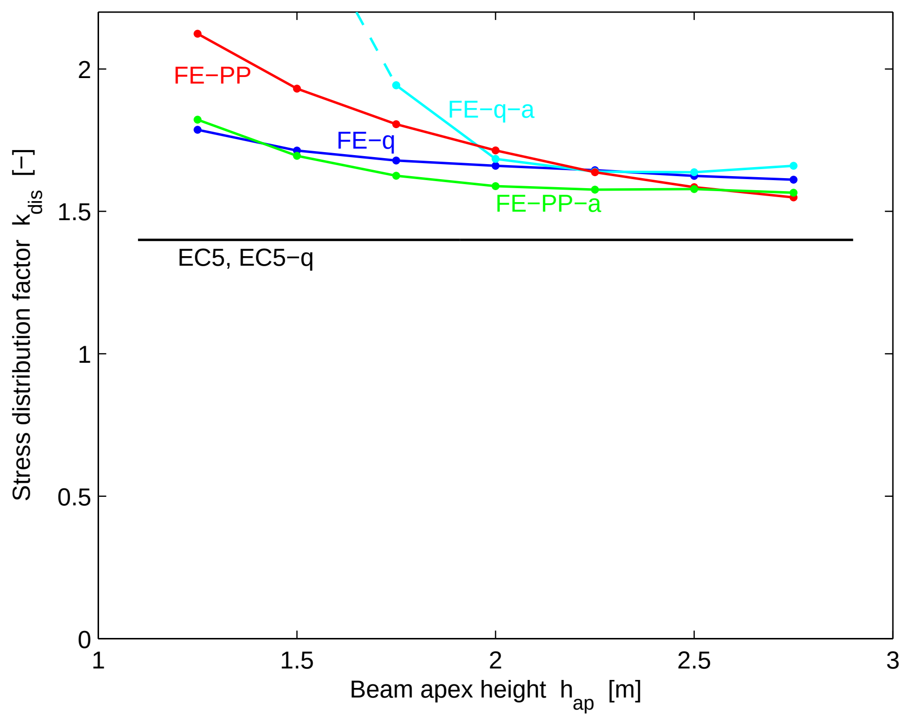

Figure 9: *Stress distribution factor k_{dis} vs. beam apex height h_{ap} according to Eurocode 5 and FE-analysis*

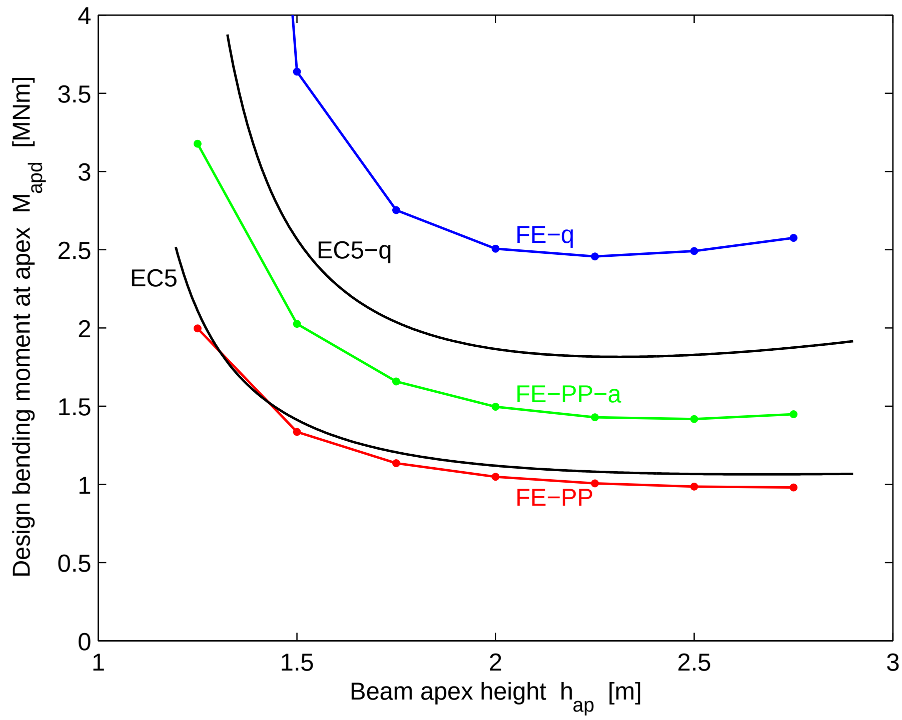

Figure 10: *Design bending moment at apex M_{apd} with respect to perp-to-grain tensile stress vs. beam apex height h_{ap} according to Eurocode 5 and FE-analysis*

6 Proposals for code design criterion

It has in the previous section been shown that the perp-to-grain tensile stress can be reduced significantly by flattening the apex region. In practical design it is however not only of importance to reduce the stress but also to accurately and conveniently predict the magnitude of stress and strength. Two proposals for design criterion are presented, both based on modifications of the Eurocode 5 approach. The introduced modifications are in no way adjusted to obtain correlation with test results but instead solely proposed based on the present FE-analysis evaluated by the Weibull two-parameter model.

Design proposal 1 includes only two minor modifications of the Eurocode 5 design equation for regular double-tapered beams aimed at making it valid for beams with a flattened apex region.

Design proposal 2 includes more modifications of the design criterion aimed at finding an overall accurate agreement between design equations and results of FE-analysis for both regular double tapered beams and beams with a flattened apex. The modifications relate to approximations of the maximum stress, the size of the stressed volume and also the level of heterogeneity in stress distribution.

6.1 Design proposal 1

Design proposal 1 is based on the assumption that the Eurocode 5 design criterion is overall accurate for strength prediction of regular double-tapered beams although the separate approximations for the maximum stress, the size of the stressed volume and the level of heterogeneity in stress distribution may be somewhat inaccurate. Striving at simplicity and consistency with the Eurocode 5 design criterion, only two minor modifications are introduced aimed at representing the overall strength well also for beams with a flattened apex region:

- For approximation of maximum stress, the angle α is by suggestion from [3] replaced by $\alpha/2$ since the change in slope for a beam with flattened apex is half that for a regular double-tapered beam
- The size of the stressed volume $V = bh_{ap}^2$ is replaced by $V = 3.0bh_{ap}^2$

Design proposal 1 for double-tapered beams with a flattened apex region of length $a = 2h_{ap}$ is hence

$$\begin{aligned} \sigma_{t90} &= 0.2\tan(\alpha/2)\,\frac{6M}{bh^2} \leq k_{dis}k_{vol}f_{t90} \\ k_{dis} &= 1.4 \\ k_{vol} &= (V_0/V)^{0.2} \qquad \text{where } V = 3.0bh_{ap}^2 \end{aligned} \tag{18}$$

where M and h refer to the bending moment and the beam height where the beam turns from tapered to straight. As in Eurocode 5, the stress may be reduced when there is a uniformly distributed (pressure) load acting on top of the beam. The maximum perp-to-grain tensile stress σ_{t90}, the stressed volume factor k_{vol}, stress distribution factor k_{dis} and design bending moment M_d with respect to perp-to-grain tensile stress is presented in Figures 11-14 for FE-analyis, Eurocode 5 and design proposal 1 (DP1).

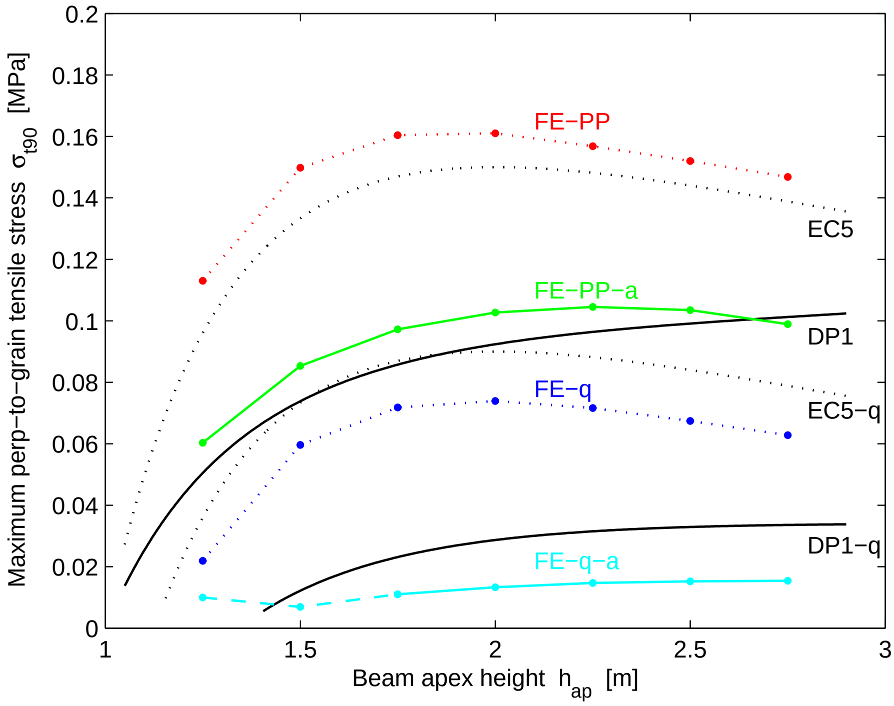

Figure 11: *Maximum perp-to-grain tensile stress σ_{t90} vs. h_{ap} according to FE-analysis, Eurocode 5 and design proposal 1 for applied loads corresponding to $M = 1000$ kNm*

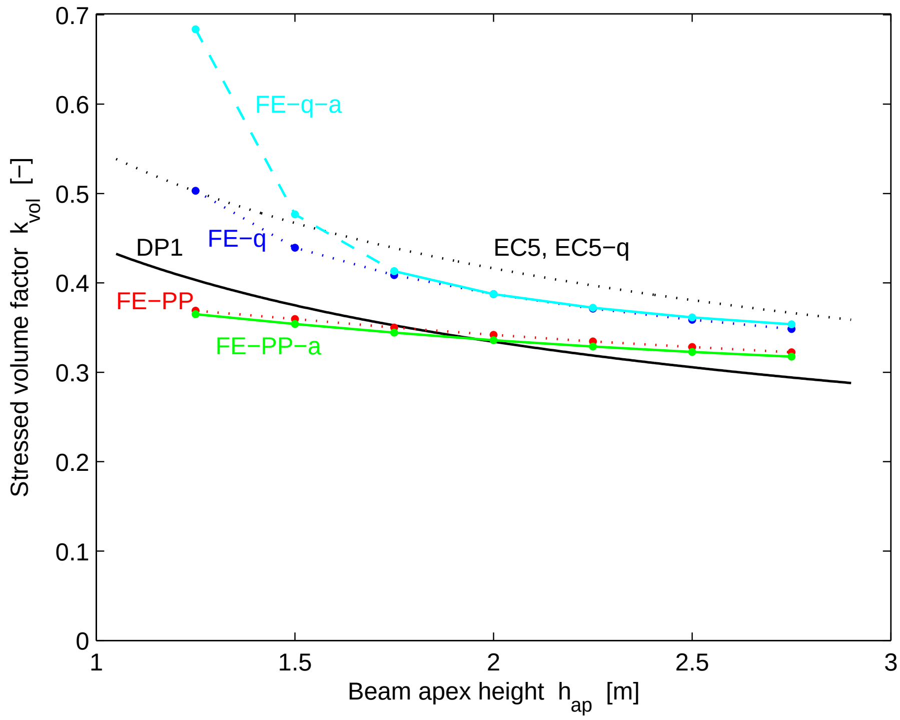

Figure 12: *Stressed volume factor k_{vol} vs. beam apex height h_{ap} according to FE-analysis, Eurocode 5 and design proposal 1*

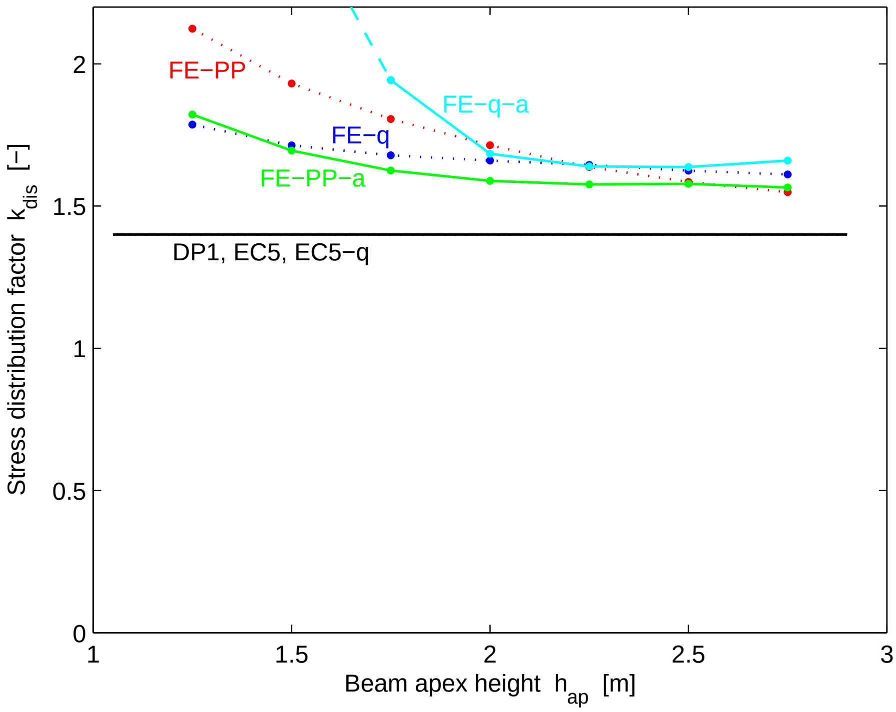

Figure 13: *Stress distribution factor k_{dis} vs. beam apex height h_{ap} according to FE-analysis, Eurocode 5 and design proposal 1*

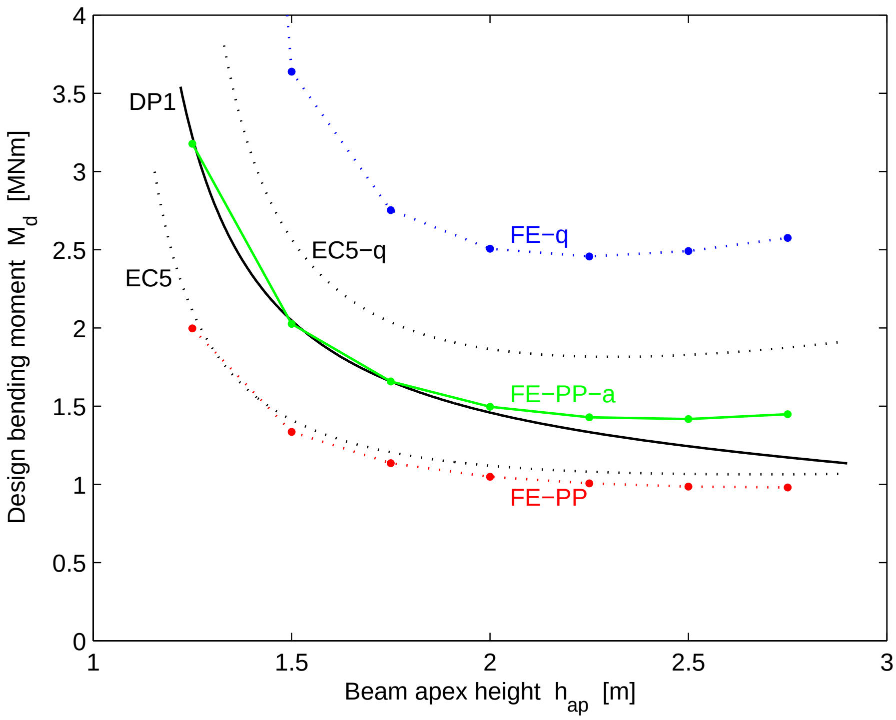

Figure 14: *Design bending moment M_d with respect to perp-to-grain tensile stress vs. beam apex height h_{ap} according to FE-analysis, Eurocode 5 and design proposal 1*

6.2 Design proposal 2

Design proposal 2 is a unified design criterion for both regular double-tapered beams and beams with flattened apex region based on the assumption that Weibull theory is applicable and that the FE-analysis is representative for various beam geometries and load configurations. The proposed design criterion is based on the Eurocode 5 approach with modifications including more accurate approximations of the maximum stress, the size of the stressed volume and the level of heterogeneity in stress distribution:

- As in design proposal 1, the angle α is replaced by $\alpha/2$ for beams with flattened apex region as suggested by [3]
- The constant 0.20 is replaced by 0.22 for better correlation of stress magnitude
- The factor $k_{dis} = 1.4$ is replaced by $k_{dis} = 1.6$ to better represent the heterogeneity in stress distribution
- The factor k_{vol} is modified by replacing the approximation for size of the stressed volume $V = bh_{ap}^2$ by $V = 0.3Lbh + 0.25Lb(h_{ap} - h_s)$
- For beams with a uniformly distributed (pressure) load acting on top of the beam, a greater reduction of the maximum stress than stated in Eurocode 5 is allowed and in addition a reduced size of the stressed volume $V = 0.25Lb(h_{ap} - h_s)$ may also be assumed

Design proposal 2 for regular double-tapered beams and double-tapered beams with a flattened apex region of length $a = 2h_{ap}$ is hence

$$\begin{aligned}
\sigma_{t90} &= 0.22 \tan \alpha^* \frac{6M}{bh^2} \le k_{dis} k_{vol} f_{t90} \qquad (19)\\
& k_{dis} = 1.6 \\
& k_{vol} = (V_0/V)^{0.2} \qquad \text{where } V = 0.3Lbh + 0.25Lb(h_{ap} - h_s) \\
& \alpha^* = \begin{cases} \alpha & \text{for regular double-tapered beams} \\ \alpha/2 & \text{for double-tapered beams with flattened apex} \end{cases}
\end{aligned}$$

where M and h refer to the bending moment and beam height at apex respectively for a regular double-tapered beam and bending moment and beam height where the beam turns from tapered to straight for a double-tapered beam with flattened apex region. Beam heights h_{ap} and h_s refer to the (theoretical) apex height and height at support respectively. The design value of the perp-to-grain tensile stress σ_{t90} and the size of the stressed volume V may alternatively be determined according to

$$\begin{aligned}
\sigma_{t90} &= 0.22 \tan \alpha^* \frac{6M_{ap}}{bh_{ap}^2} - 0.9\frac{p}{b} \qquad (20)\\
V &= 0.25Lb(h_{ap} - h_s)
\end{aligned}$$

when there is a uniformly distributed (pressure) load p acting on top of the beam.

The maximum perp-to-grain tensile stress σ_{t90}, the stressed volume factor k_{vol}, stress distribution factor k_{dis} and design bending moment M_d with respect to perp-to-grain tensile stress is presented in Figures 15-18 for FE-analysis and design proposal 2 (DP2).

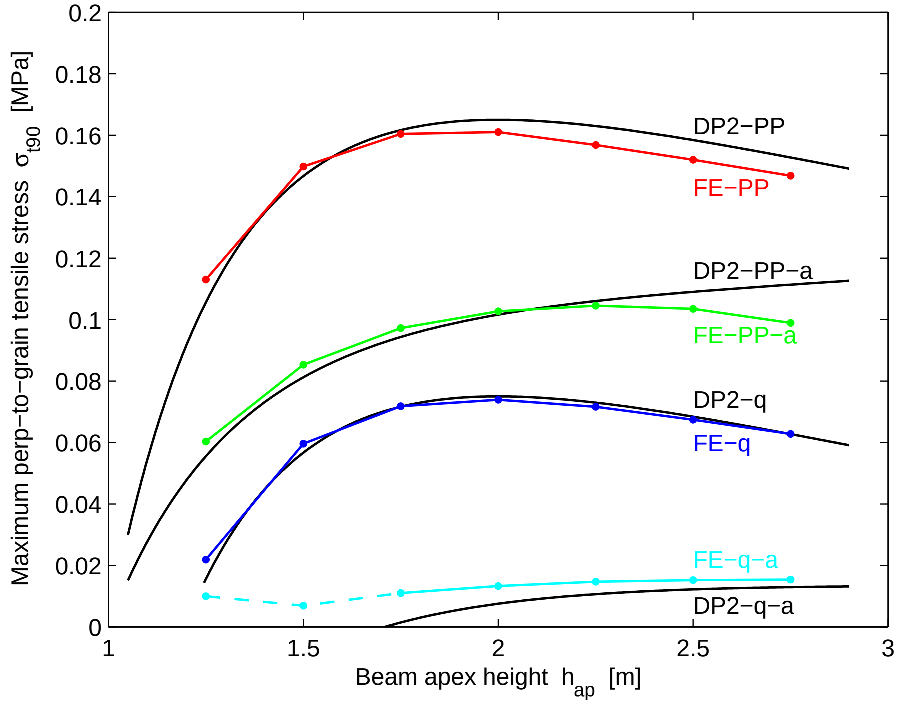

Figure 15: *Maximum perp-to-grain tensile stress vs. beam apex height h_{ap} according to FE-analysis and design proposal 2 for applied loads corresponding to $M = 1000$ kNm*

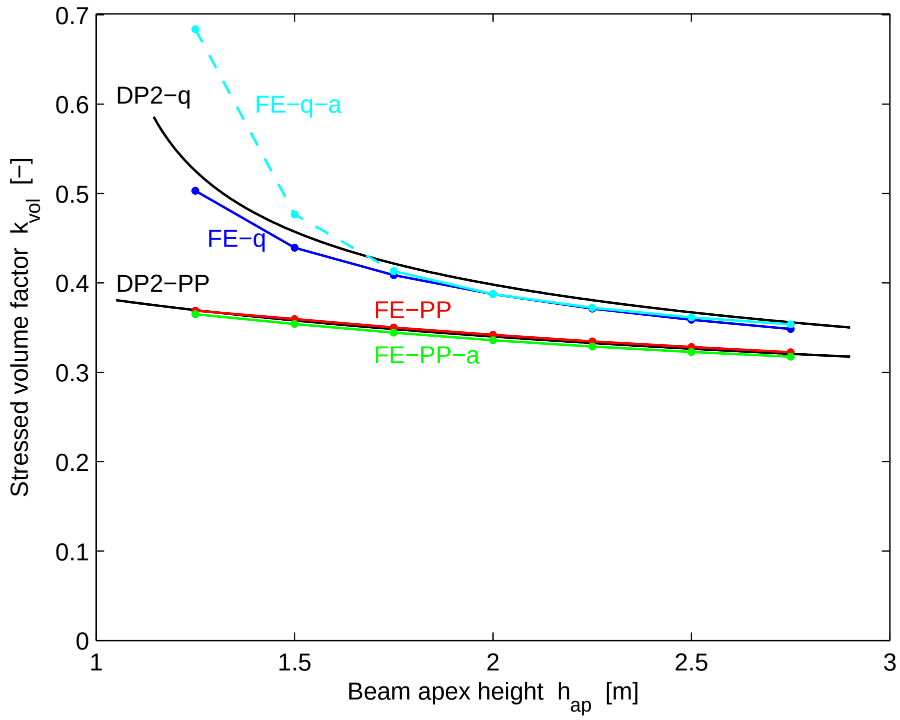

Figure 16: *Stressed volume factor k_{vol} vs. beam apex height h_{ap} according to FE-analysis and design proposal 2*

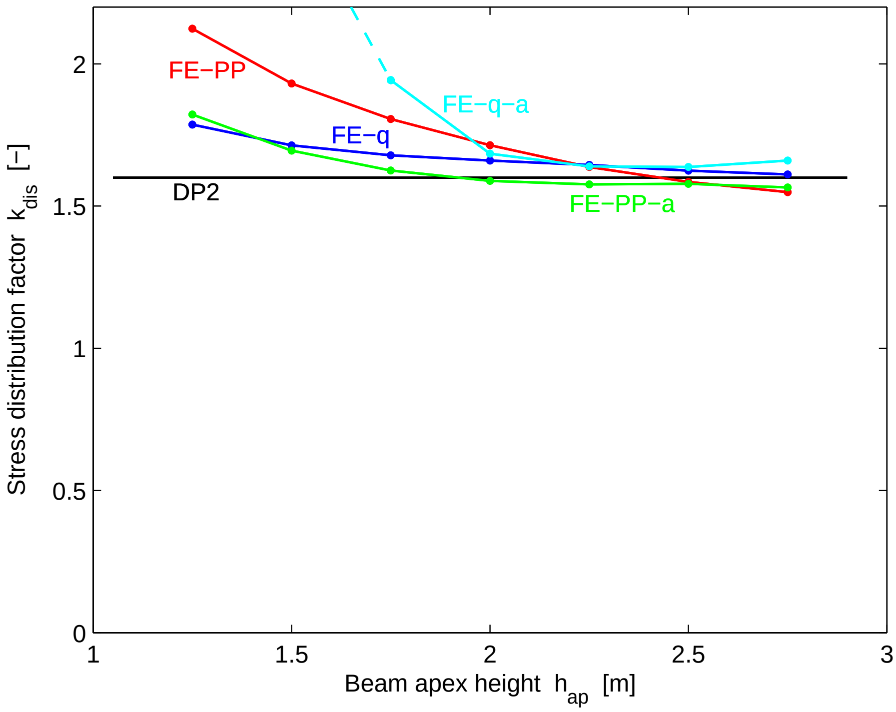

Figure 17: *Stress distribution factor k_{dis} vs. beam apex height h_{ap} according to FE-analysis and design proposal 2*

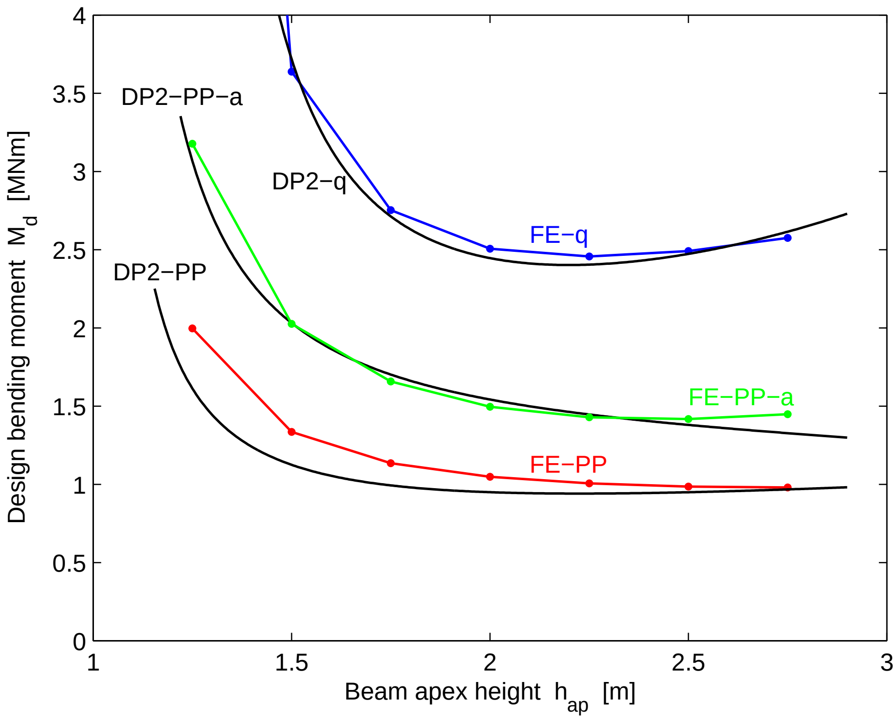

Figure 18: *Design bending moment M_d with respect to perp-to-grain tensile stress vs. beam apex height h_{ap} according to FE-analysis and design proposal 2*

7 Concluding remarks

A brief review of design criteria and finite element stress analysis, focusing on the perp-to-grain tensile stress, of double-tapered glulam beams are presented. The review of the Eurocode 5 design criteria reveals that the introduction of a reduced effective beam width (due to presence of cracks) for design with respect to shear stress has a major influence on the overall design strength of double-tapered beams. Disregarding design with respect to shear stress, the review further shows that perp-to-grain tensile stress at apex region may be decisive for design for double-tapered beams except for beams with small inclination. Design with respect to perp-to-grain tensile stress is based on an approximate solution for the magnitude of stress and Weibull type considerations relating to influence of size of stressed volume and level of heterogeneity in stress distribution on the strength.

Linear elastic finite element stress analysis of regular double tapered beams and double-tapered beams with a flattened apex region are presented. The FE-analysis concerns magnitude of stress, size of stressed volume and level of heterogeneity in stress distribution. For beam geometries with a flattened apex region, the maximum perp-to-grain tensile stress was found to be reduced significantly compared to the stress in corresponding regular double-tapered beams. The FE-analysis further revealed a considerable difference in not only the maximum stress but also in the size of the stressed volume between different load configuration, here uniformly distributed load and two point loads acting at the beam length quarter points.

In Eurocode 5, a reduction of design stress is allowed for load configurations with uniformly distributed load but the difference in size of the stressed volume, which affect the design strength, is however not accounted for. The size of the stressed volume found from FE-analysis is further greater than assumed in Eurocode 5 for all considered geometries. Although underestimating the size of the stressed volume, the overall design strength with respect to perp-to-grain tensile stress based on Eurocode 5 seems to correspond rather well with FE-analysis and Weibull theory. This is due to underestimation of the level of heterogeneity in stress distribution in Eurocode 5.

Two proposals for design criterion with respect to perp-to-grain tensile stress are presented. They are both based on the Eurocode 5 approach for regular double-tapered beams and consists of modifications of the approximate expression for determining the maximum stress, size of stressed volume and level of heterogeneity in stress distribution. All modifications are introduced solely based on the results of the FE-analysis and are in not way adjusted to experimental test results.

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